



UNDERSTAND



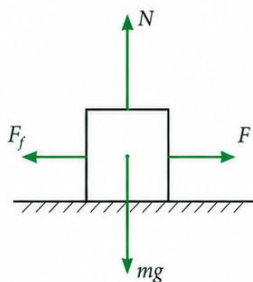
APPLY



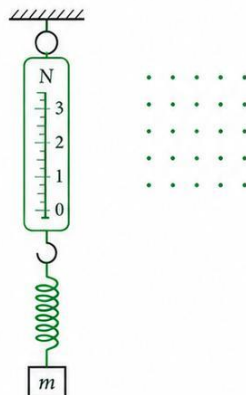
PRACTISE



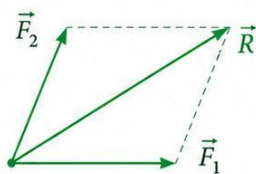
IMPROVE



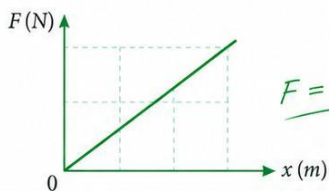
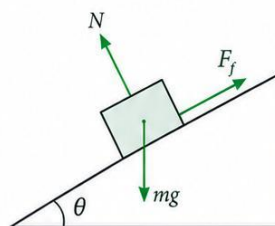
$$F = ma$$



Forces *in* Action



$$W = mg$$



$$F = kx$$



OWNER DETAILS



NAME:

GROUP:

TEACHER:





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5-A-Day

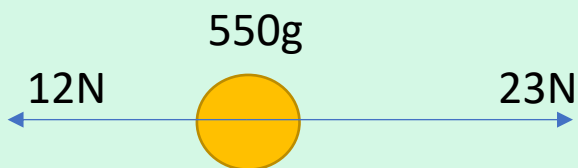
An object moves up a slope with an initial velocity of 6m/s and a deceleration of 2m/s^2 .

1. Calculate the time taken to reach the maximum distance travelled up the slope.
2. Calculate the distance travelled up the slope.
3. Sketch a labelled graph of $x-t$ showing the motion of the ball up and down the slope.
4. Sketch a labelled $v-t$ graph of the motion of the object up and down the slope.
5. What does the deceleration depend upon?

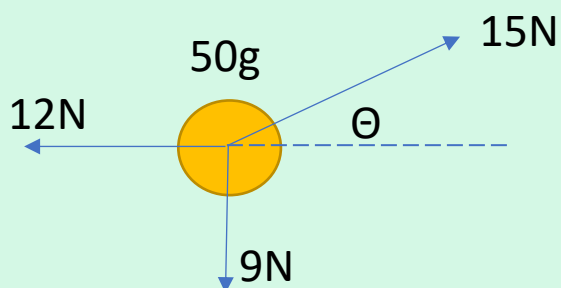
5-A-Day

1. State Newton's second law of motion.
2. A **resultant** force of 15N is experienced by a mass of 400g. Calculate the acceleration of the mass.
3. A 320g bird flies into the wind. The bird provides a forward force of 1.2N against a drag force of 0.9N. Calculate the acceleration of the bird.
4. The bird is travelling at a velocity of 6m/s and drops a stick from 13m high. Calculate horizontal distance travelled by the stick before it reaches the ground. Assume air resistance is negligible.
5. Calculate the speed of the stick when it hits the ground.

5-A-Day



Calculate the acceleration experienced by the mass.



Calculate the angle required to cause this system to be in equilibrium. (Top-down view)

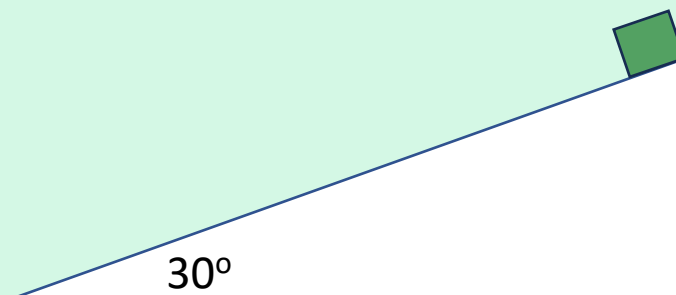
Define the term velocity

Define the term acceleration

A particle takes 40ns to complete an orbit of diameter 12mm at constant speed. Calculate the speed of the particle.

5-A-Day

1. State Newton's second law of motion.



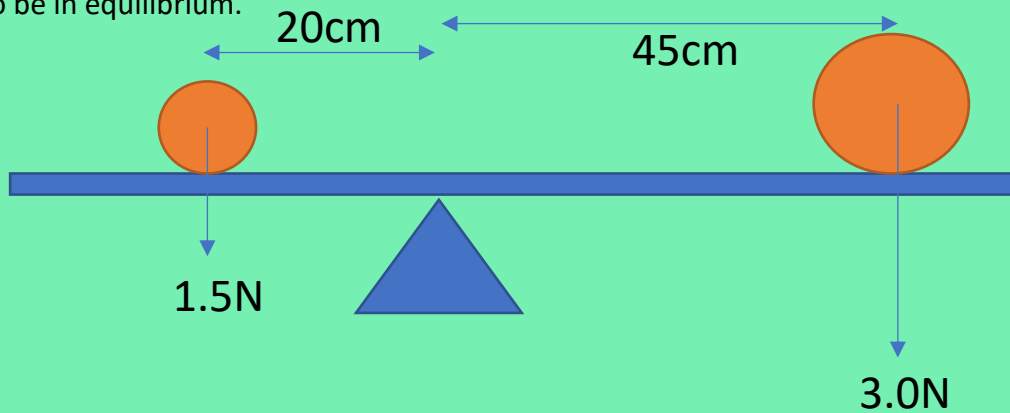
2. The block above has a mass of 'm'. Give an expression for the force acting down the ramp (F_w). (Assume no friction)
3. A frictional force of ' F_r ' is measured where $F_r < F_w$. Give an expression for the acceleration for the block down the ramp.
4. Using your answer to question 3, calculate the acceleration when the mass of the block is 80g and the frictional force is measured to be 0.2N.
5. The ramp has a length of 65cm. Calculate the time taken to reach the bottom of the ramp. (Assume the block begins from rest)

5-A-Day

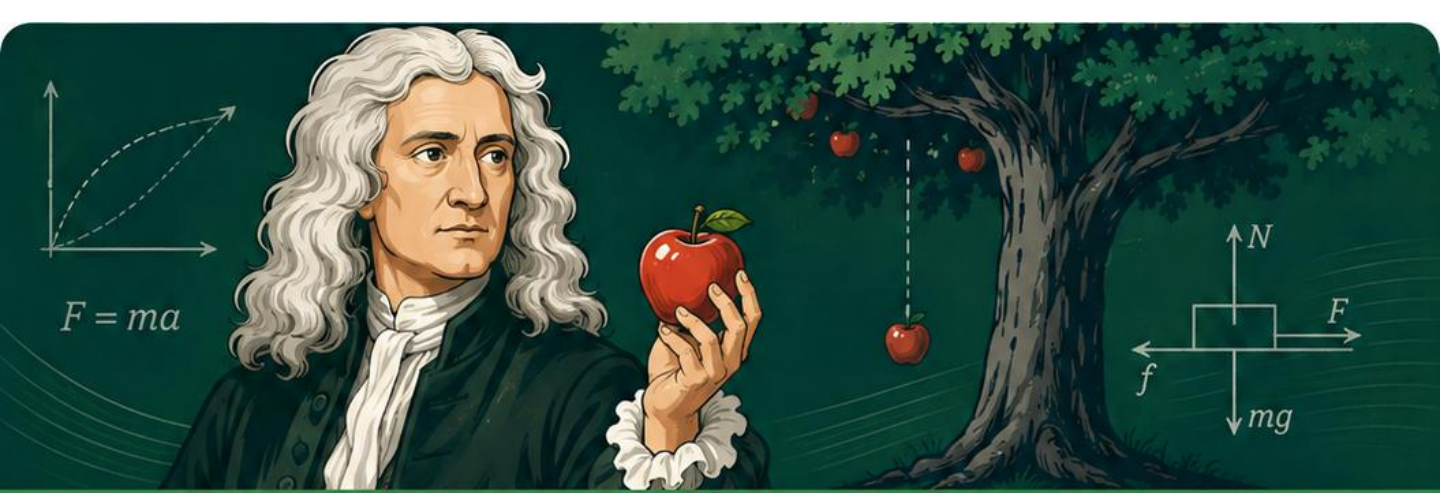
1. Sketch a labelled diagram showing the forces acting on a mass 'm' falling through a fluid.
2. Write an expression for the acceleration of the object.
3. Sketch a velocity–time graph from the instant the object is released from rest in the fluid.
4. Explain the shape of your graph in terms of the motion of the object.
5. A second object of mass $3m$, with the same size and shape, is released into the same fluid. **Explain** how the motion of this mass is different to the first mass.

5-A-Day

1. Define the term 'moment'
2. Define the term 'principle of moments'
3. A force of 12 N is applied perpendicular to the end of a handle of length 14 cm. Calculate the moment.
4. Calculate the additional mass required on top of the 1.5N weight to cause the system below to be in equilibrium.



5. The two spheres above have the same density. The 1.5N sphere has a radius of 2cm. Calculate the radius of the 3.0N sphere.



3.2 Forces in action

This section provides knowledge and understanding of the motion of an object when it experiences several forces and also the equilibrium of an object. Learners will also learn how pressure differences give rise to an *upthrust* on an object in a fluid.

3.2.1 Dynamics

Learning outcomes



Learners should be able to demonstrate and apply their knowledge and understanding of:

- a** net force = mass $\times$ acceleration; $F = ma$
- b** the newton as the unit of force
- c** weight of an object; $W = mg$
- d** the terms tension, normal contact force, upthrust and friction
- e** free-body diagrams
- f** one- and two-dimensional motion under constant force.



Net force

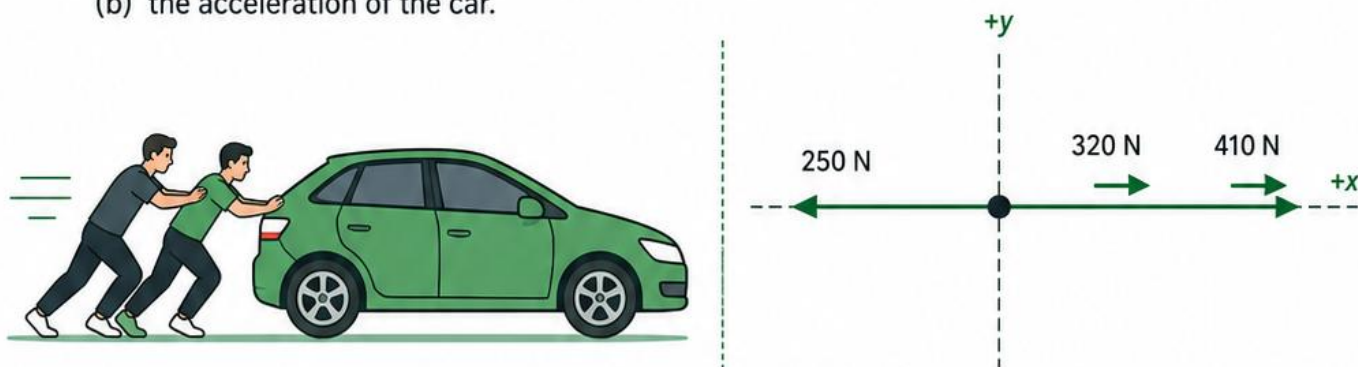
- 1 State Newton's second law of motion.
- 2 How does this relate to the equation $F = ma$?
- 3 What are the base units of the newton?
- 4 Define the newton.

GCSE

- 1 How much resultant force is needed to accelerate a 72 kg skier at 1.8 m/s^2 ?
.....
- 2 A 1200 kg lift is in free fall and accelerates downward at 9.8 m/s^2 . Calculate the resultant force acting on it.
.....
- 3 What is the acceleration of a 40 kg box pushed with a resultant force of 360 N?
.....
- 4 The mass of a car is 950 kg. What resultant force is required to accelerate it at 2.5 m/s^2 ?
.....
- 5 A 60 kg skateboarder is pushed and accelerates at 4.0 m/s^2 . How much force was applied?
.....

AS

1. Two people are pushing a stalled car. The mass of the car is 1600 kg. One person applies a force of 320 N and the other applies a force of 410 N, both in the same direction. A resistive force of 250 N acts in the opposite direction. Calculate:
(a) the resultant force on the car
(b) the acceleration of the car.



2. The car starts from rest and continues with the same acceleration for 8.0 s. Calculate the distance travelled.
.....

The average mass of a human is 70 kg.

The mass of a lift is 500 kg.

A lift starts from rest and carries 8 people vertically upwards with an acceleration of 1.5 m s^{-2} .



1 Write an equation for the resultant vertical force acting on the lift and passengers.

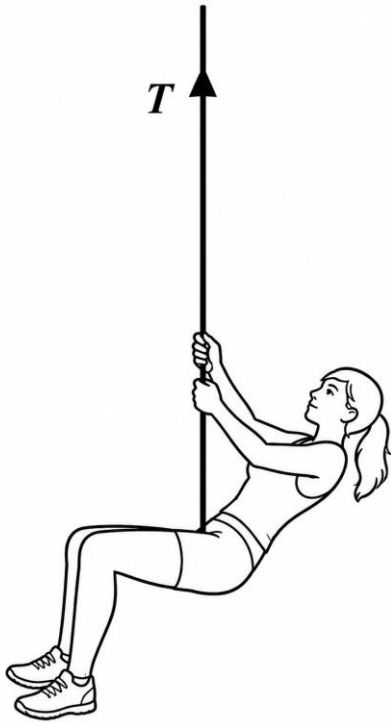
2 Calculate the tension in the lift cable.

3 Calculate the distance travelled by the lift after 5.0 seconds.

4 After 5.0 seconds, the lift travels at a constant speed.
What is the resultant force acting on the lift?

5 At constant speed, the lift takes 2.0 seconds to move from floor 6 to floor 9.
Calculate the distance between each floor.





A rescue mission takes place to pull a lady of mass 63kg from a well.

The rope pulls the lady from the well with an acceleration of 2.3m/s^2 .

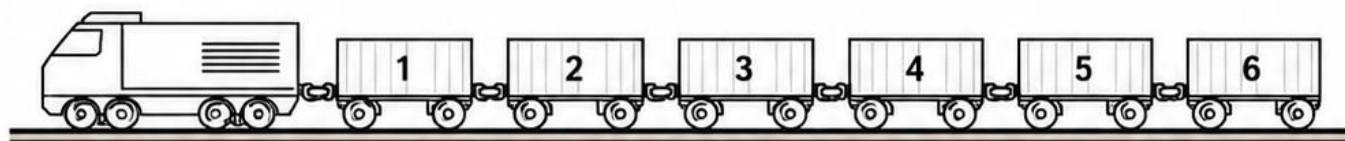
Calculate the tension in the rope.

What distance will she be pulled up in 4 seconds?

Challenge

Calculate the distance travelled by the lady between the 3rd and 4th second.

Scaffolded train coupling problem



A locomotive of mass 24×10^4 kg pulls six trucks, each of mass 6.0×10^4 kg, along a straight horizontal track at constant speed. The horizontal resistive force on each truck is 4000 N. The coupling between trucks 2 and 3 snaps. Assume the locomotive's driving force is unchanged at the instant the coupling snaps, and neglect any resistive force on the locomotive.

- 1 Calculate the total resistive force acting on all six trucks while the train moves at constant speed.

- 2 Hence determine the driving force provided by the locomotive.

- 3 After the coupling snaps, calculate the total resistive force acting on the two trucks still attached to the locomotive.

- 4 Calculate the resultant forward force on the locomotive and the two attached trucks immediately after the coupling snaps.

- 5 Calculate the total mass of the locomotive and the two attached trucks.

- 6 Use Newton's second law to calculate the initial acceleration.



Hint: compare the total resistance before the coupling snaps with the resistance after only two trucks remain attached.

Multiple-choice check

What is the initial acceleration of the locomotive and the two attached trucks immediately after the coupling snaps?

- A. 0.022 m s^{-2}
C. 0.067 m s^{-2}

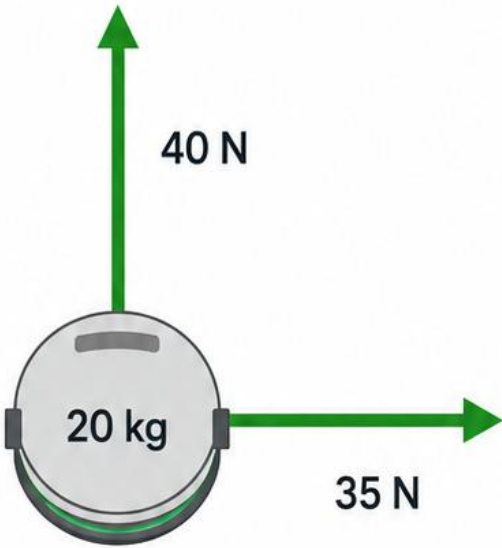
- B. 0.044 m s^{-2}
D. 0.133 m s^{-2}

2D vectors

Example

A 20 kg autonomous warehouse cart moves on a smooth floor. Two perpendicular forces act on it: 35 N horizontally to the right and 40 N vertically upward.

Calculate the magnitude of the acceleration and the angle of motion from the horizontal.



1 Find the magnitude of the resultant force.

.....

.....

.....

.....

2 Find the angle of the resultant force to the horizontal.

.....

.....

.....

.....

3 Use Newton’s second law to calculate the acceleration.

.....

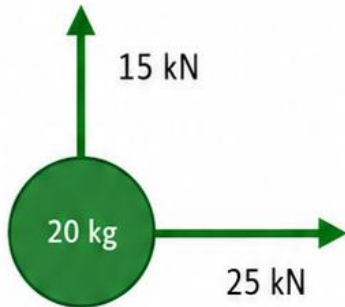
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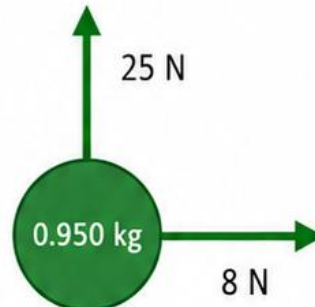
2D vectors

- 1 Multiple choice.** A 20 kg object is acted on by two perpendicular forces: 15 kN upward and 25 kN to the right. What is the magnitude of the resultant force?



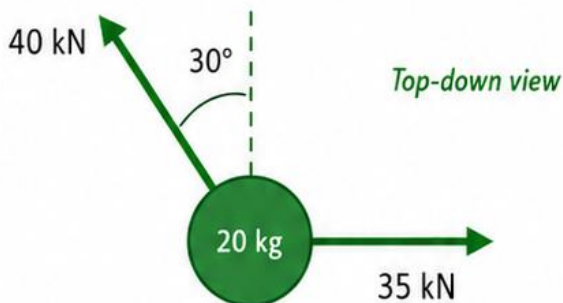
- A. 15 kN
- B. 5800 kN
- C. 29 kN
- D. 650 kN

- 2 Multiple choice.** A 0.950 kg object is acted on by two perpendicular forces: 25 N upward and 8 N to the right. What is the acceleration of the object?

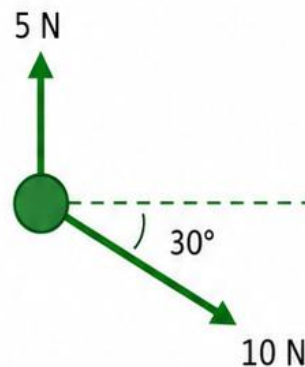


- A. 60 m s^{-2}
- B. 28 m s^{-2}
- C. 15 m s^{-2}
- D. 8 m s^{-2}

- 3 Short answer.** Top-down view. A 20 kg object is acted on by a force of 35 kN to the right and a force of 40 kN acting 30 degrees to the vertical, directed up and to the left. Calculate the acceleration of the object.

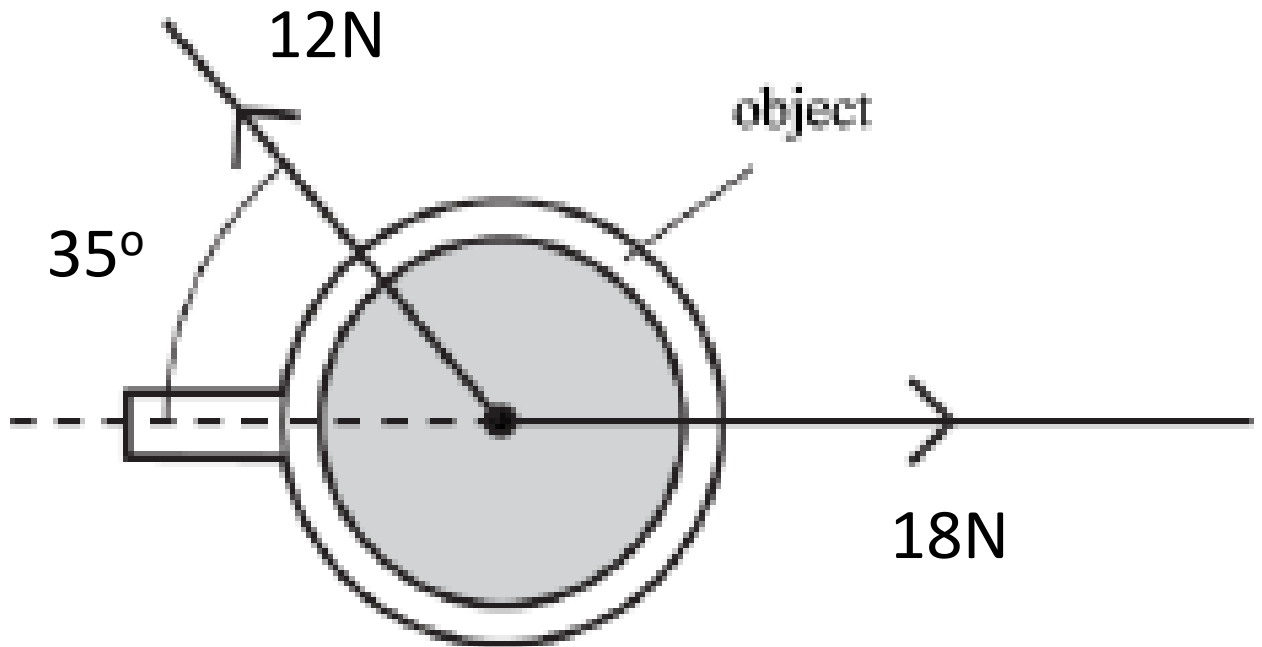


- 4 Multiple choice.** A 0.200 kg object is acted on by a 5 N force vertically upward and a 10 N force acting 30 degrees below the horizontal to the right. What is the acceleration of the object?



- A. 21 m s^{-2}
- B. 43 m s^{-2}
- C. 65 m s^{-2}
- D. 86 m s^{-2}

Calculate the mass of the object if it experiences an acceleration of 3m/s^2



An object is falling.

The weight of the object is 4.5 N.

The wind provides a horizontal force of magnitude F on the object.

The **resultant** force on the object is 5.8 N.

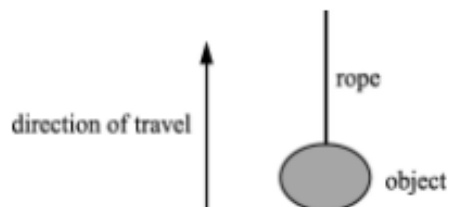
Air resistance and upthrust on the object are negligible.

What is the value of F ?

- A** 1.3 N
- B** 3.7 N
- C** 7.3 N
- D** 13 N

Your answer

An object of mass 7.0 kg is pulled vertically upwards by a rope. The acceleration of the object is 2.0 m s^{-2} .



What is the tension in the rope?

- A. 14 N
- B. 55 N
- C. 69 N
- D. 83 N

A balloon is travelling vertically downwards at a constant acceleration. The upthrust on the balloon is U , its weight is W and it experiences air resistance F .

Which statement is correct?

- A. $F + W > U$
- B. $W + U > F$
- C. $F > W + U$
- D. $W > U + F$

An object is placed on a smooth horizontal surface. Two horizontal forces act on this object. Fig. 21.3 shows the magnitudes and the directions of these two forces.

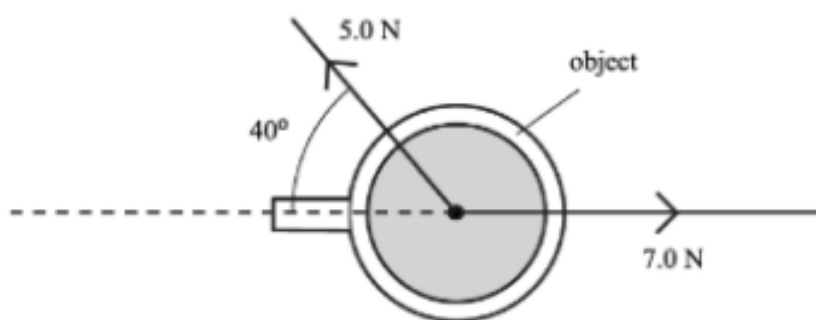


Fig. 21.3

The mass of the object is 320 g.

Calculate the magnitude of the acceleration of the object.

Fig. 19 shows a crane lifting a car of mass 850 kg at constant velocity through a height of 12 m in a time of 40 s. The crane has a working efficiency of 60 %.

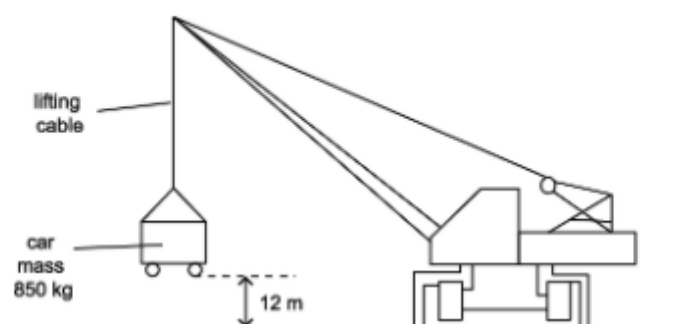


Fig. 19

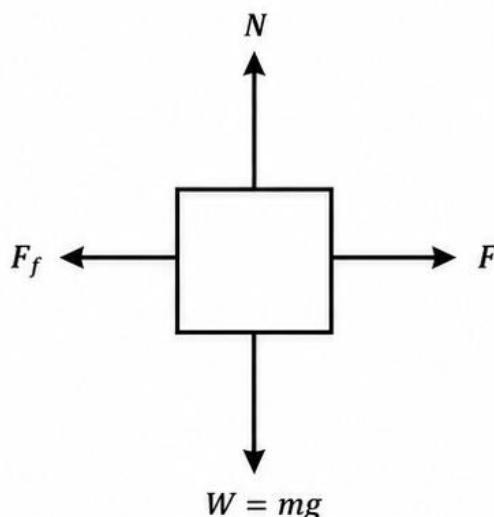
i. Calculate the tension in the lifting cable.

tension = N [1]

Free-body diagrams and vector addition

1 Free-body diagrams

A free-body diagram shows all the external forces acting on an isolated object. The direction of each arrow shows the direction of the force, and its length can represent the force magnitude.

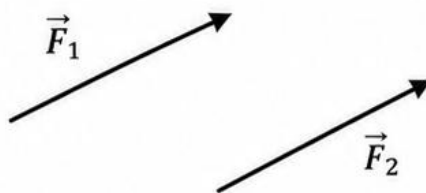


Conventions

- represent the object as a simple shape or dot
- draw only forces acting on the object
- label forces clearly
- arrow direction shows force direction
- arrow length may show relative size

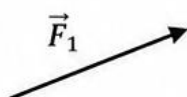
2 From forces to a resultant

Once the forces have been identified on a free-body diagram, they can be treated as vectors and combined to find the resultant force.

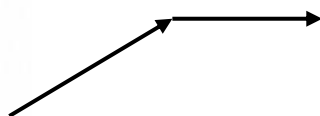


Vectors can be added graphically.

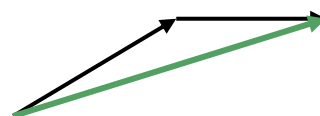
3 Tip-to-tail method



Step 1: draw the first vector.

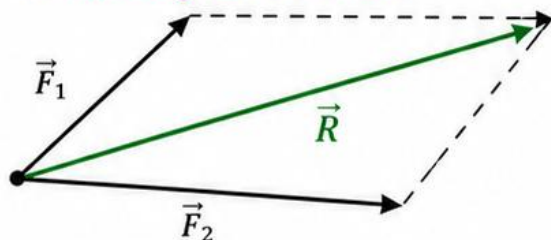


Step 2: place the tail of the second vector at the head of the first.



Step 3: draw the resultant vector from the tail of the first vector to the head of the second.

4 Parallelogram method



The diagonal from the common starting point gives the resultant vector.



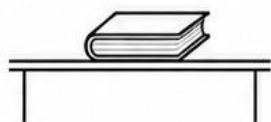
Key idea

Only draw forces acting on the object — do not draw motion, velocity or acceleration arrows on a free-body diagram.

Free-body diagrams and vector addition – Practice

Draw the diagrams carefully and label all forces or vectors clearly.

- 1** A book is resting on a horizontal table. Draw a free-body diagram for the book.



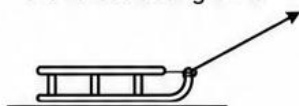
Your diagram

- 2** A climber is hanging motionless from a vertical rope. Draw a free-body diagram for the climber.



Your diagram

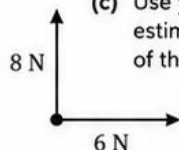
- 3** A sledge is being pulled across rough horizontal ground by a rope at an angle above the horizontal. Draw a free-body diagram for the sledge and label the forces acting on it.



Your diagram

- 4** An object is acted on by two vectors: 6 N to the right and 8 N upward.

- (a) Draw the vectors using the tip-to-tail method.
(b) Draw and label the resultant vector, $\vec{R}$.
(c) Use your diagram to estimate the magnitude of the resultant.



Working / diagram



Key reminders

- Only draw forces acting on the object.
- Arrow direction shows direction of force or vector.
- Resultant vectors can be found using tip-to-tail or the parallelogram method.

Slopes

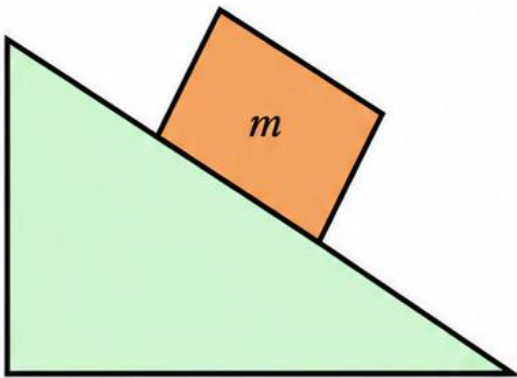
Teacher demo example



A block rests on a slope. Assuming the block is stationary, draw and label all the forces acting on the block.

Model solution

Situation



A block of mass m rests on a slope.

Free-body diagram (block only)

$W = mg$	weight, acts vertically downward
N	normal contact force, perpendicular to the slope
F_f	friction force, acts up the slope

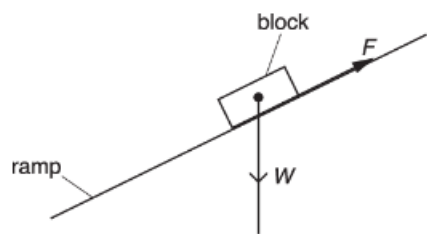
Key points for teachers

- Only draw forces acting on the block.
- Weight acts vertically downward.
- The normal contact force is perpendicular to the surface.
- Because the block would tend to slide down the slope, friction acts up the slope.
- Do not draw components on the free-body diagram unless asked.



If the slope were smooth, there would be no friction force.

A block of wood is at rest on a ramp.
The weight of the block is W and the frictional force on the block is F .



A triangle of forces diagram can be used to determine the magnitude and the direction of the normal contact force N .

Which is the correct diagram for this triangle?

- A**

Diagram A shows a triangle of forces. The left side is a vertical vector pointing downwards labeled W . The top side is a vector pointing up and to the right labeled N . The bottom side is a vector pointing up and to the right labeled F .
- B**

Diagram B shows a triangle of forces. The left side is a vertical vector pointing downwards labeled W . The top side is a vector pointing up and to the right labeled N . The bottom side is a vector pointing up and to the left labeled F .
- C**

Diagram C shows a triangle of forces. The left side is a vertical vector pointing downwards labeled W . The top side is a vector pointing up and to the right labeled F . The bottom side is a vector pointing up and to the right labeled N .
- D**

Diagram D shows a triangle of forces. The left side is a vertical vector pointing downwards labeled W . The top side is a vector pointing up and to the right labeled F . The bottom side is a vector pointing up and to the left labeled N .

Slopes

Teacher modelling examples

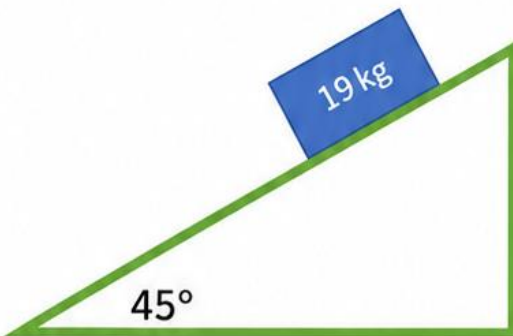


For each example, resolve forces parallel to the slope, find the resultant force, then use $F = ma$ to calculate the acceleration.

Assume both slopes are smooth.

Example 1

A 19 kg block rests on a smooth 45° slope. Calculate the acceleration of the block.



Teacher model

1. Force parallel to slope: $mg \sin \theta$

2. Resultant force

3. Acceleration

$a =$

Example 2

A 13 kg block is on a smooth 20° slope. A force of 20 N acts up the slope. Calculate the acceleration of the block.



Teacher model

1. Force down slope: $mg \sin \theta$

2. Resultant force along slope

3. Acceleration

$a =$

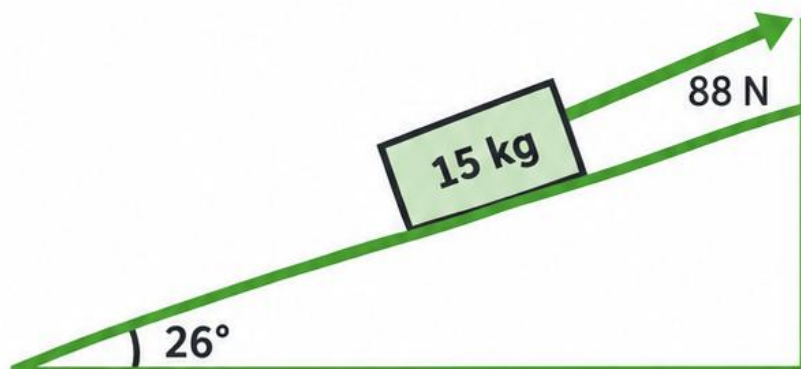
Slopes

Multiple-choice practice

Assume the slopes are smooth and take $g = 9.8 \text{ m s}^{-2}$.

1

A 15 kg block is on a smooth slope of angle 26° . A force of 88 N acts up the slope as shown. What is the acceleration of the block?



- A) 1.6 m s^{-2} down the slope B) 5.9 m s^{-2} up the slope
C) 1.6 m s^{-2} up the slope D) No acceleration – stationary

2

A 13 kg block is on a smooth slope of angle 20° . A force of 20 N acts up the slope as shown. What is the acceleration of the block?

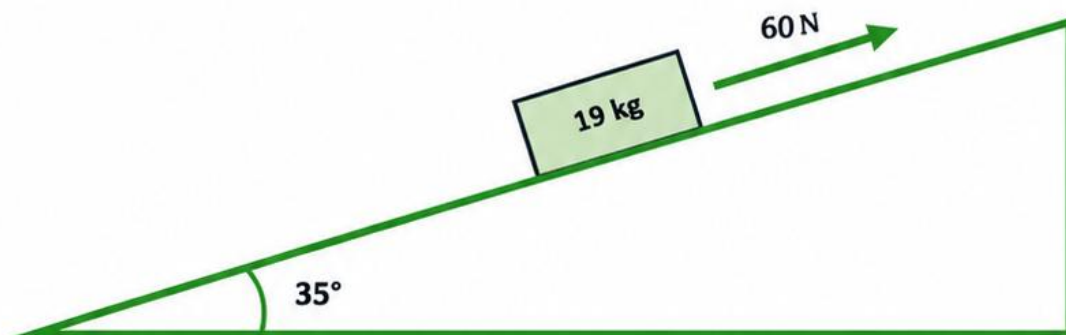


- A) 0.54 m s^{-2} up the slope B) 1.8 m s^{-2} up the slope
C) 1.8 m s^{-2} down the slope D) 4.3 m s^{-2} down the slope

Slopes

Worksheet

Assume the slope is smooth and take $g = 9.8 \text{ m s}^{-2}$.



- 1** Label the diagram with all the forces acting on the block and state the component of the weight acting down the slope.

*Identify forces first.
Resolve weight parallel
to the slope.*

- 2**
- a) Calculate the resultant force acting down the slope.
 - b) Hence calculate the acceleration of the 19 kg block.

Use $F = ma$.

- 3** The force up the slope remains 60 N. Calculate the angle of the slope required to give the block an acceleration of 4.0 m s^{-2} down the slope.

*Use $F = ma$.
You may need to
rearrange for the angle.*

Two metal spheres, A and B, are positively charged. Sphere A is fixed in position. Sphere B is suspended by an insulating thread from a point directly above the centre of sphere A as shown in the diagram:

Sphere B has a weight of 0.50 m N . The size of the electrostatic repulsive force is 0.29 m N .

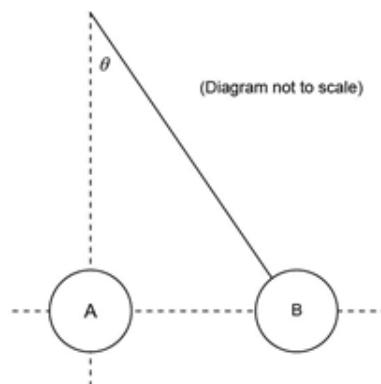
What angle, θ , does the thread suspending sphere B make to the vertical ?

A 0.66°

B 1.0°

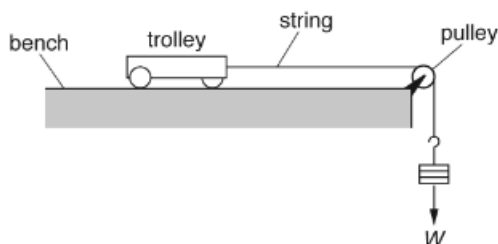
C 30°

D 60°



Your answer

A trolley of mass M is pulled along a horizontal table by a force W provided by a mass hanging from the end of a string as shown.



Frictional forces are negligible. The acceleration of free fall is g .

What is the correct equation for the acceleration a of the trolley?

A $a = \frac{W}{M}$

B $a = g$
 $a = \frac{W}{2M}$

D $a = \frac{W}{M + \frac{W}{g}}$

1. The engine of a car of mass 800kg, which is travelling along a straight horizontal road, is producing a driving force of 1200N. Assuming that there are no forces resisting the motion, calculate the acceleration of the car.
2. A van is pulling a broken-down car of mass 1200kg along a straight horizontal road. The only force acting which affects the motion of the car is the tension in the horizontal tow bar. Calculate the acceleration of the car when the tension is 750N.
3. For the first stage of its motion on the runway, before take-off, an aircraft of mass 2200kg has a constant acceleration 4.2m/s^2 . Calculate the magnitude of the force necessary to provide this acceleration.
4. A novice skier is being pulled along a horizontal section of a nursery slope. Given that her acceleration of 0.8m/s^2 is provided by a force of 52N, calculate her mass.

5. A wooden block of mass m kg is at rest on a table, 1.6 metres from the edge. The block is pulled directly towards the edge by a horizontal string. The tension in the string has magnitude $0.2mN$. Calculate the time taken for the block to reach the edge of the table.
6. Two children are sliding a puck to each other on a frozen lake. The puck, of mass 0.4kg , leaves one child with a speed of 5m/s and reaches the other one, who is 8m away after 2.5 seconds. Calculate the deceleration of the puck and find the frictional force resisting the motion of the puck.
7. A particle P of mass m kg is moving in a straight line with deceleration. It passes point A with speed 6m/s and point B with speed 3.6m/s . Given that the distance between A and B is 12m , calculate, in terms of m , the magnitude of the force resisting the motion of P .
8. A man pushes a car with a force of 127.5N along a straight horizontal road. He manages to increase the speed of the car from 1m/s to 2.8m/s in 12 seconds. Find the mass of the car.
9. A runaway sledge of mass 10kg travelling at 15m/s reaches a horizontal snow field. It travels in a straight line before it comes to rest. Given that the force of friction slowing the sledge has magnitude 60N , calculate how far the sledge travels in the snow field.

10. A hockey player hits a stationary ball of mass 0.2kg . The contact time between the stick and the ball is 0.3secs and the force exerted on the ball by the stick is 60N . Find the speed with which the ball leaves the stick.

11. A boy slides a box of mass 2kg across the floor of the stage in the school theatre. The initial speed of the box is 8m/s and it comes to rest in 5m . Calculate the deceleration of the box and find the frictional force between the box and the floor.

12. A boat of mass 3000kg , travelling at a speed of $u\text{ m/s}$ is brought to rest in 20 seconds by water resistance of 370N . Find the value of u .

13. A car of mass 1000kg runs out of petrol and comes to rest just 30m from a garage. The car is pushed with a force of 120N , along a horizontal road towards the garage. Calculate the acceleration of the car and find the time it takes to reach the garage.

14. A bullet of mass 0.12kg is travelling horizontally at 150m/s when it enters a fixed block of wood. Assuming that the bullet's motion remains horizontal and that the force resisting the motion has constant magnitude 10kN , calculate how far the bullet penetrates the block.

15. A jet plane of mass 30 tonnes touches down with a speed of 55m/s and comes to rest after moving for 560m in a straight line on the runway. Assuming that the only forces stopping the plane are provided by the reverse thrust of its two engines, and that these are equal and directed in the opposite direction of motion, calculate the magnitude of the thrust in each engine.

ANSWERS

- 1) 1.5m/s^2
- 2) 0.625m/s^2
- 3) 9240N
- 4) 65kg
- 5) 4 seconds
- 6) $1.44\text{m/s}^2, 0.576\text{N}$
- 7) 0.96m N
- 8) 850kg
- 9) 18.75m
- 10) 90m/s
- 11) $6.4\text{m/s}, 12.8\text{N}$
- 12) 2.47
- 13) $0.12\text{m/s}^2, 22.4\text{ secs}$
- 14) 13.5cm
- 15) 40.5kN

3.2.2 Motion with non-uniform acceleration

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

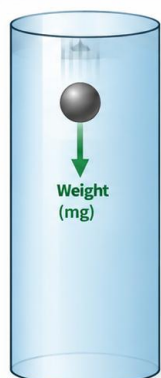
- (a) drag as the frictional force experienced by an object travelling through a fluid
- (b) factors affecting drag for an object travelling through air
- (c) motion of objects falling in a uniform gravitational field in the presence of drag
- (d)
 - (i) terminal velocity
 - (ii) techniques and procedures used to determine terminal velocity in fluids.

Falling through a fluid

When an object falls through a fluid (such as air or oil), it experiences a weight downwards and a resistance force (drag) upwards. The speed changes until it reaches a constant value called the terminal velocity.

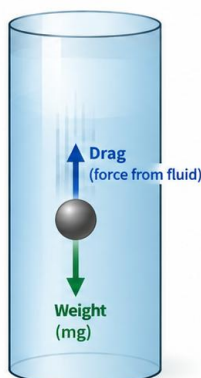
1 Just released

The object starts from rest. Weight is greater than drag, so it accelerates downwards.



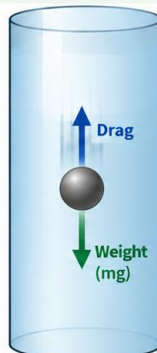
2 Speed increasing

As the object speeds up, the drag force increases (opposes the motion).



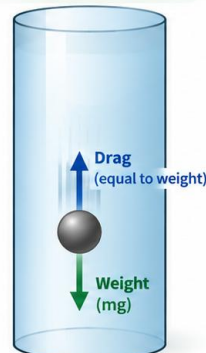
3 Acceleration decreases

The drag force grows larger, so the resultant force becomes smaller. The object still accelerates downwards, but less and less.



4 Terminal velocity

Eventually, the drag force equals the weight. The resultant force is zero, so the object moves at a constant speed (terminal velocity).



5 Constant speed

The object continues at this constant speed unless the fluid or object changes.

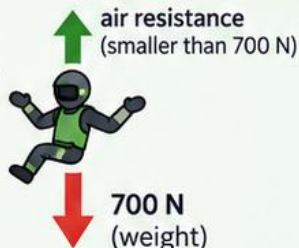
At terminal velocity:
Drag force = Weight
Resultant force = 0
Acceleration = 0
Speed = constant

Parachutist motion

Speed–time graph and changing forces

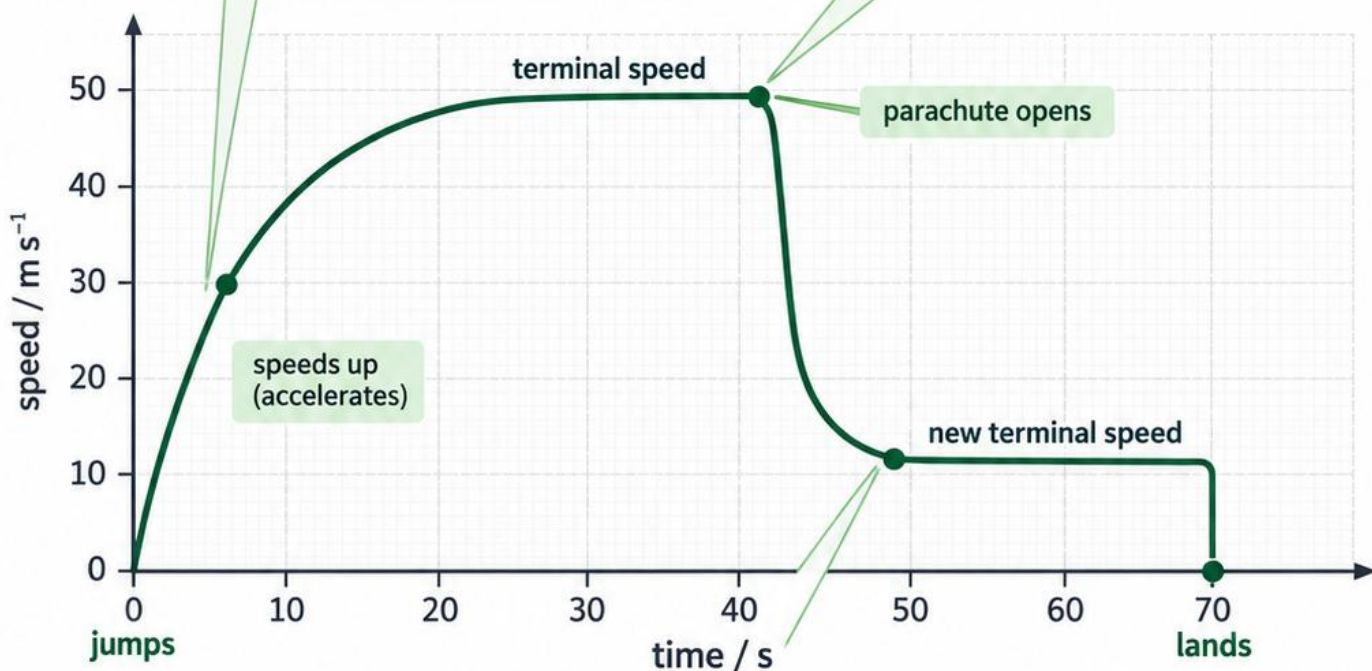
1 Just after the jump

At first, weight is greater than air resistance, so the parachutist **accelerates**.



2 Terminal speed (parachute closed)

As speed increases, air resistance increases until it equals weight. The speed becomes **constant**.



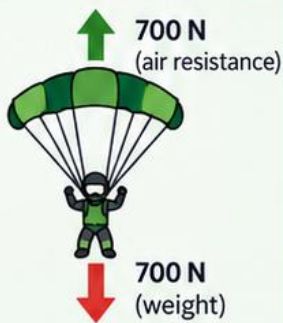
3 Parachute opens

When the parachute opens, air resistance becomes much larger than weight, so the parachutist **decelerates**.



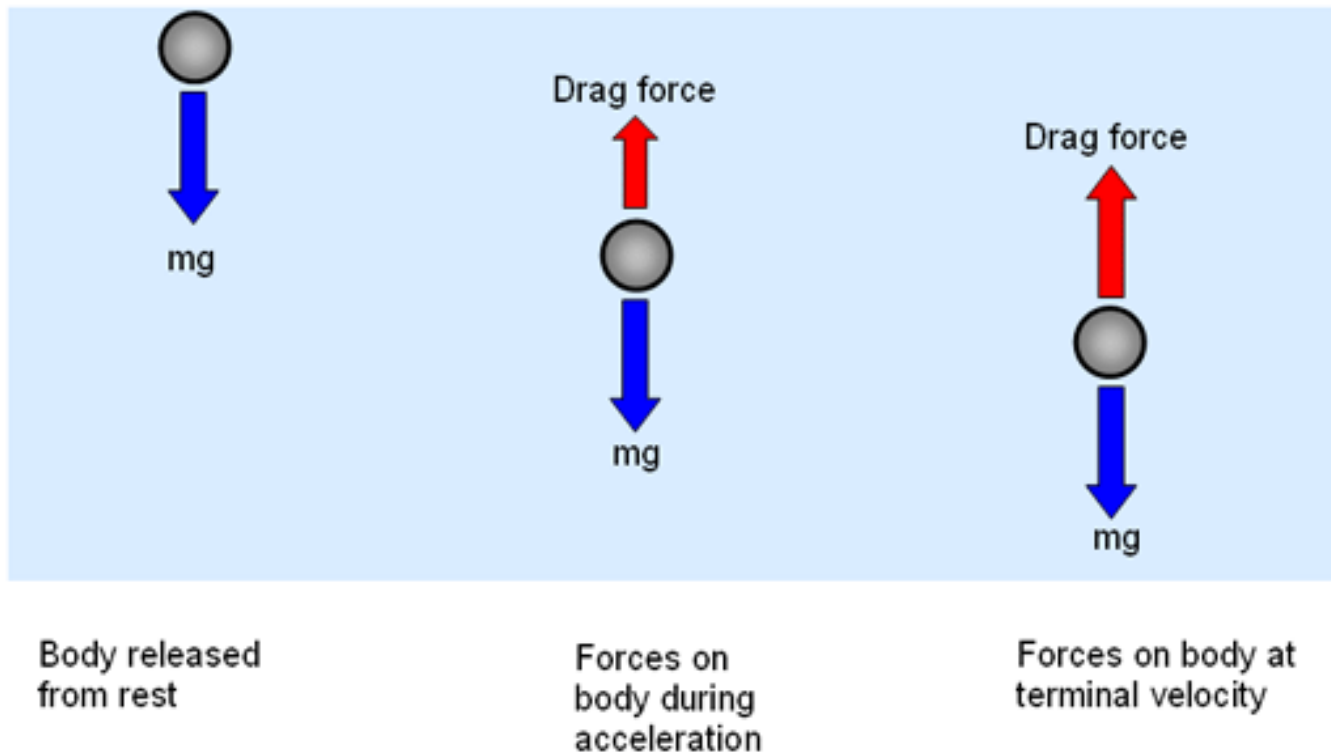
4 New terminal speed (parachute open)

As the parachutist slows down, air resistance decreases until it again equals weight. A **lower terminal speed** is reached.



Key ideas

- **Resultant force = 0** at terminal speed.
- Acceleration occurs only when forces are **unbalanced**.



Explain what is meant by the term 'drag'.

How does the surface area of an object affect drag?

How does the velocity of an object affect drag?

A ball bearing is dropped into a viscous fluid. After some time 't' the ball bearing stops accelerating and remains at a constant velocity.

Using ideas of Newton's second law, drag and a graph of velocity-time, explain the motion of the ball bearing as it passes through the liquid.

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Two balls **X** and **Y** are dropped from a very tall building. Both balls reach terminal velocity before hitting the ground. The balls have the same diameter. The mass of **X** is greater than the mass of **Y**.

Which statement is correct?

- A. The balls hit the ground at the same time.
- B. The terminal velocity of **Y** is greater than that of **X**.
- C. The initial acceleration of both balls is the same.
- D. The balls have the same kinetic energy just before hitting the ground.

Your answer

☐

A piece of flat A4 paper is dropped and falls to the floor. The same piece of paper is then collapsed into a ball and dropped again.

Which of the following will change in the second situation?

- A. the maximum magnitude of the air resistance
- B. the weight of the paper
- C. the time taken to reach terminal velocity
- D. the initial acceleration when dropped

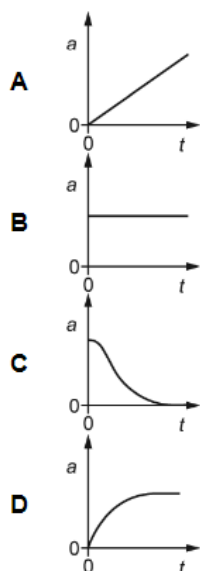
Your answer

☐

[1]

A ball, made from scrunched-up paper, is dropped from rest at time $t = 0$. It reaches terminal velocity before it hits the ground.

Which acceleration a against time t graph is correct for the ball in flight?



Additional notes:

3.2.3 Equilibrium

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

- (a) moment of force
- (b) couple; torque of a couple
- (c) the principle of moments
- (d) centre of mass; centre of gravity; experimental determination of centre of gravity
- (e) equilibrium of an object under the action of forces and torques
- (f) condition for equilibrium of three coplanar forces; triangle of forces.

Moments in action

A crane stays balanced when the clockwise and anticlockwise moments about the pivot are equal.

Counterweight

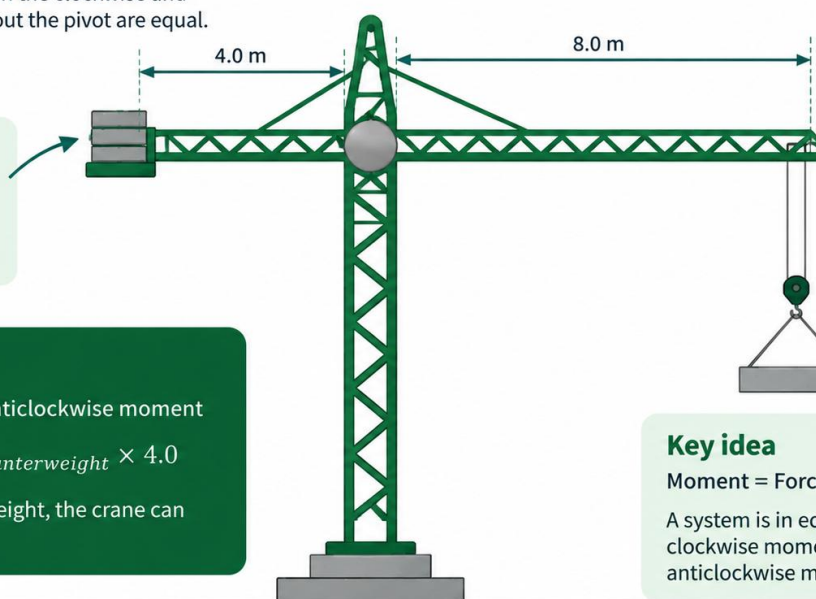
A movable mass that can be adjusted to provide an anticlockwise moment.

Balanced crane

Clockwise moment = Anticlockwise moment

$$W_{load} \times 8.0 = W_{counterweight} \times 4.0$$

By moving the counterweight, the crane can balance different loads.



Load

The weight of the load produces a clockwise moment about the pivot.

Key idea

Moment = Force $\times$ perpendicular distance

A system is in equilibrium when the total clockwise moment equals the total anticlockwise moment about the pivot.

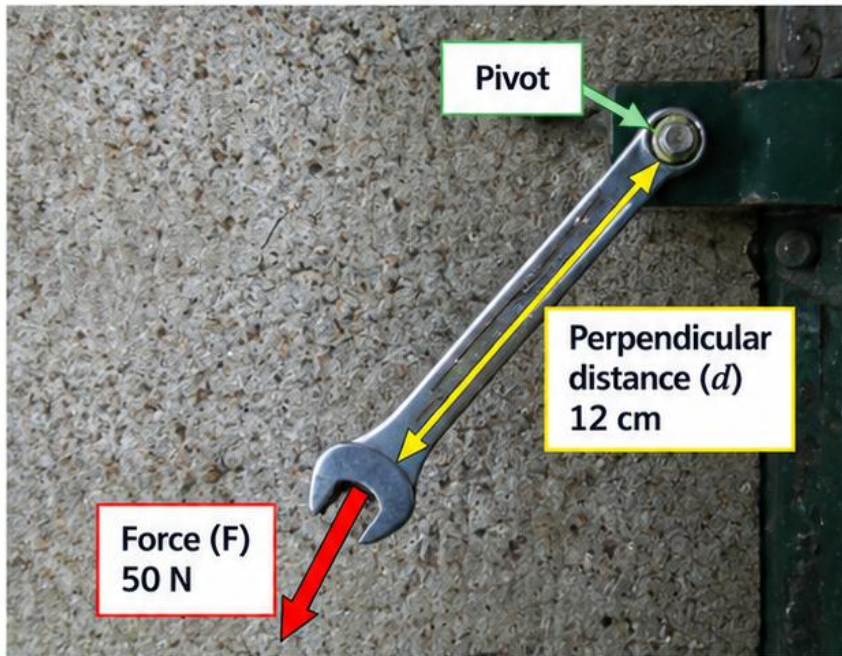
What is a moment?

A moment is a term used to quantify a turning effect about a pivot (or a point).

Moment, $M = \text{Force, } F \times \text{perpendicular distance, } d \text{ to the pivot}$

$$M = F \times d$$

(units: N m)



Example

Force applied: 50 N

Perpendicular distance: 12 cm

Moment:

$$M = F \times d$$

$$M = 50 \times 0.12$$

$$M = 6.0 \text{ N m}$$

? Think about it

- 1 How would increasing the length of the spanner change the moment?

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- 2 A different spanner is used in the scenario to the left. Half the force is applied, and the same moment is achieved. What is the length of the new spanner?

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The principle of moments

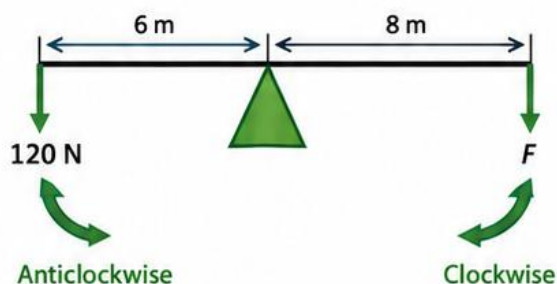
State the principle of moments.

Moment = Force $\times$ perpendicular distance from the pivot

$$M = Fd$$

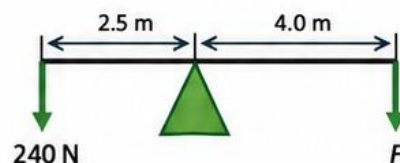
Example 1

1. Calculate the force F needed to keep the beam in equilibrium.



Question 2

2. A 240 N force acts 2.5 m to the left of a pivot. A second force acts 4.0 m to the right of the pivot. Calculate the force required to keep the beam in equilibrium.



For a body in rotational equilibrium, total clockwise moment = total anticlockwise moment.

Moments

Multiple-choice check

Answer the following questions about moments. Choose the correct answer (A–D).

1 Which is the correct definition for the term moment?

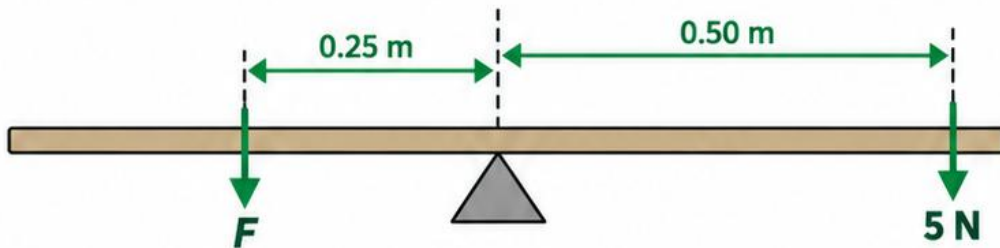
A Distance $\div$ force perpendicular to pivot.

B Force $\times$ distance to pivot.

C Force $\div$ distance to pivot.

D Force $\times$ perpendicular distance from the pivot.

2 What is the correct value for F ?



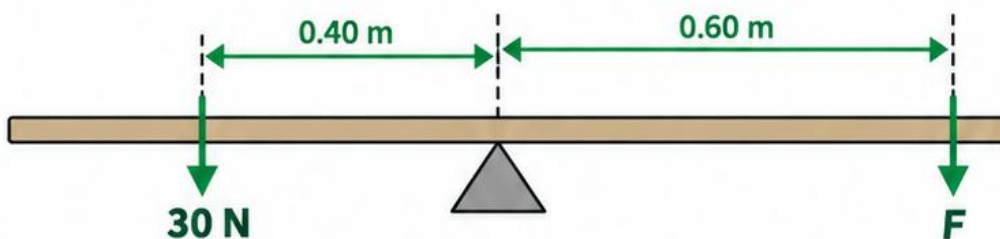
A 2.5 N

B 12 N

C 15 N

D 10 N

3 The beam is in equilibrium. What is the value of F ?



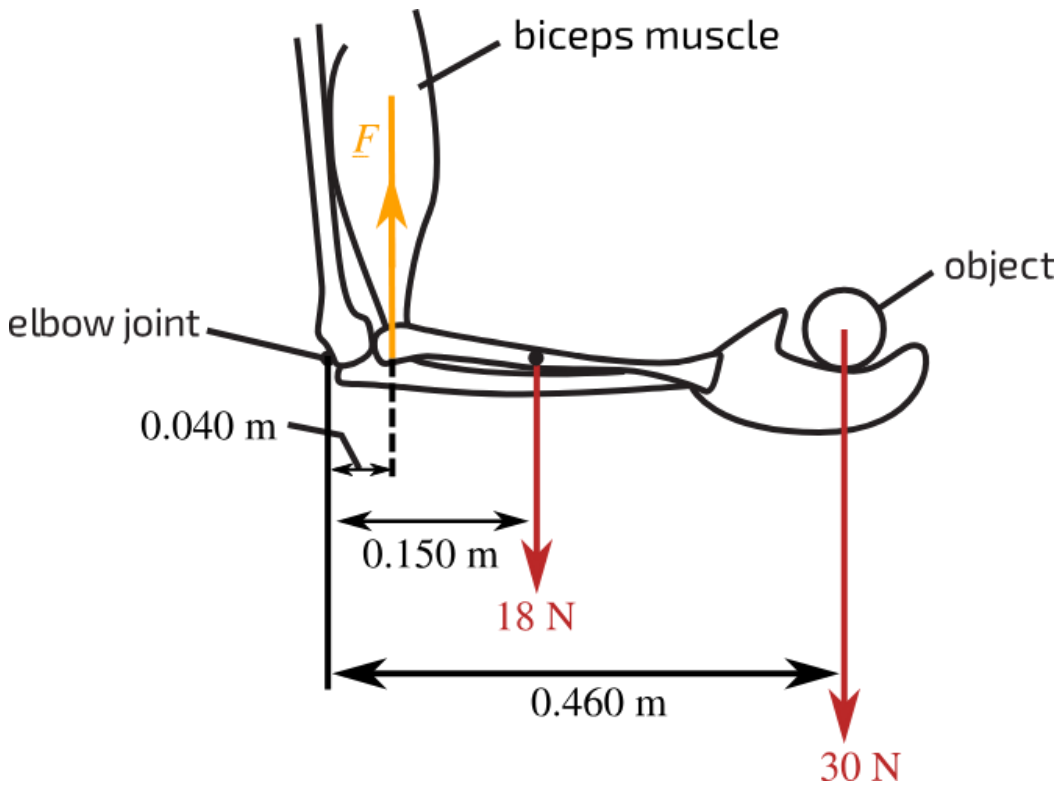
A 10 N

B 15 N

C 20 N

D 45 N

Multiple forces



$$\sum \text{Clockwise moments} = \sum \text{Anticlockwise moments}$$

Calculate the force required by the bicep to keep the arm level

Resolving forces in moments

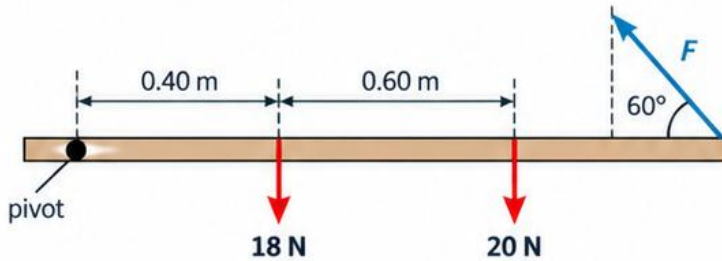
Key idea

When a force acts at an angle, only the component perpendicular to the beam produces a moment.

$$M = F \times d \times \sin \theta$$

where d is the perpendicular distance from the pivot to the point where the force acts, and θ is the angle between the force and the beam.

Teacher demo

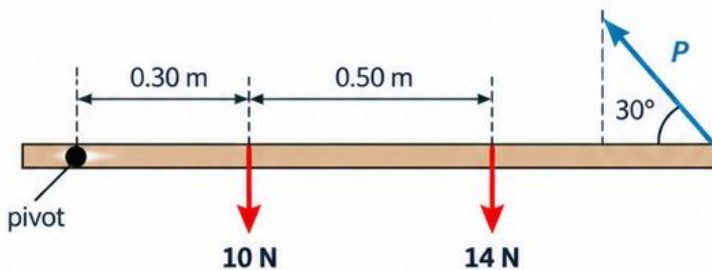


A uniform beam of length 1.00 m is pivoted at the left-hand end. Two downward forces of 18 N and 20 N act at the positions shown. A force F acts at 60° to the beam at the right-hand end. The beam is in equilibrium.

Calculate the magnitude of F .

- 1 Calculate the total clockwise moment from the two downward forces.
- 2 Write an expression for the anticlockwise moment due to the angled force F .
- 3 Use the principle of moments to find F .

Now try this



A uniform beam of length 1.00 m is pivoted at the left-hand end. Two downward forces of 10 N and 14 N act at the positions shown. A force P acts at 30° to the beam at the right-hand end. The beam is in equilibrium.

Calculate the magnitude of P .

Moments and angled forces

Use the perpendicular component of the force, or $\text{moment} = \text{force} \times \text{perpendicular distance from the pivot}$.

1 Multiple choice

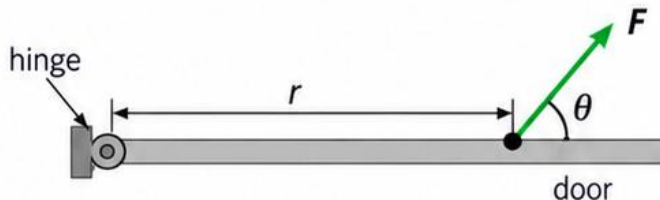
A force F is applied to a door at a distance r from the hinge. The force makes an angle θ with the door. Which expression gives the moment about the hinge?

A. $Fr \cos \theta$

B. $Fr \sin \theta$

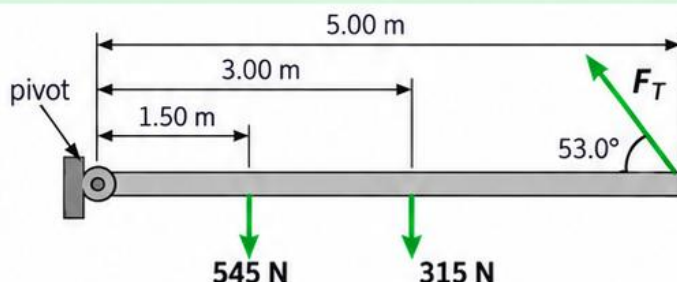
C. $F \sin \theta$

D. $\frac{F}{r}$



2 Question

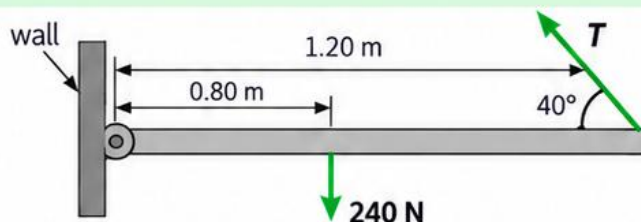
A uniform beam of length 5.00 m is pivoted at the left-hand end. A 545 N force acts downward 1.50 m from the pivot and a 315 N force acts downward 3.00 m from the pivot. A force F_T acts at the right-hand end at 53.0° to the beam, as shown. Calculate the value of F_T required for equilibrium.



Working space

3 Question

A horizontal sign is supported by a bar of length 1.20 m pivoted at the wall. A 240 N weight acts downward 0.80 m from the pivot. A cable is attached at the far end of the bar and pulls at 40° to the bar. Calculate the tension in the cable needed to keep the bar in equilibrium.



Working space

4 Extension

For the sign in Question 3, explain whether the required cable tension would increase or decrease if the cable angle increased from 40° to 60° . Justify your answer using moments.

Torque of a couple

Key idea

A couple is a pair of equal and opposite parallel forces separated by a perpendicular distance.

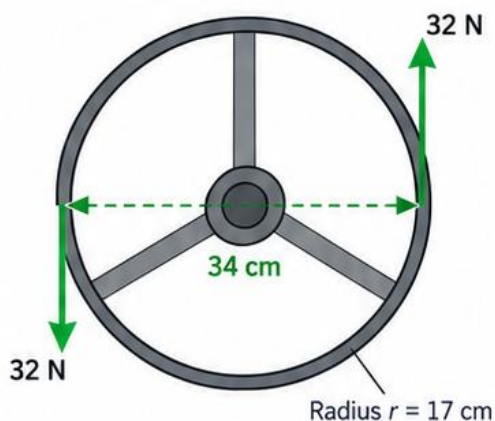
Formula

Torque of a couple = force $\times$ perpendicular distance between the lines of action

Teacher model

A worked example with clear steps

Steering wheel (top view)



Example

A steering wheel has radius 17 cm. Two equal and opposite tangential forces of 32 N act on opposite sides of the wheel. Calculate the torque of the couple.

Working

distance between the forces = diameter = $2r = 34 \text{ cm} = 0.34 \text{ m}$

torque = $F \times d$

torque = 32×0.34

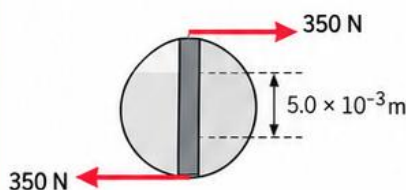
torque = 10.88 N m

torque $\approx 10.9 \text{ N m}$

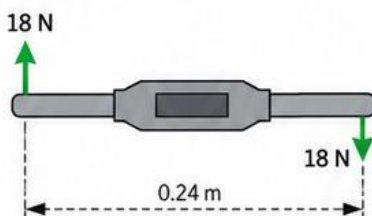
Now you try

Apply your knowledge to the questions below

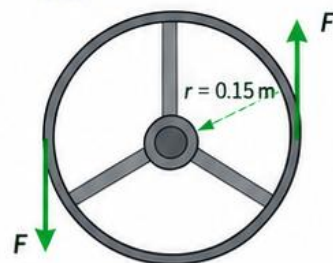
- 1** A screwdriver blade is $5.0 \times 10^{-3} \text{ m}$ wide. Equal and opposite forces of 350 N act at each end of the blade on the screw. Calculate the torque of the couple.



- 2** A tap wrench is turned by a couple. Two equal and opposite forces of 18 N act on the handles. The perpendicular distance between the lines of action of the forces is 0.24 m. Calculate the torque of the couple.



- 3** A couple produces a torque of 12 N m on a steering wheel of radius 0.15 m. Two equal and opposite tangential forces are applied at opposite sides of the wheel. Calculate the size of each force.



Moments and Equilibrium – Consolidation Questions

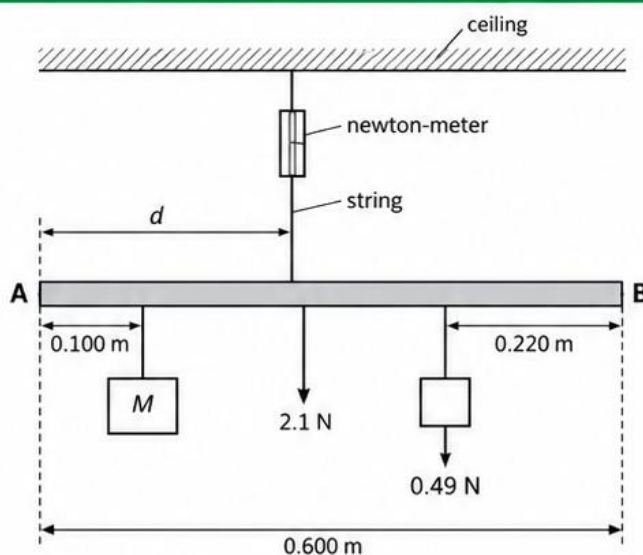
Answer in your exercise book. Use $g = 9.8 \text{ N kg}^{-1}$.

Section 1: Quick recall

1. State the two conditions required for a body to be in equilibrium.
2. Define the term moment.
3. State the principle of moments.
4. A turning effect of 2.3 N m is produced by applying a force perpendicular to the end of a 30 cm spanner. Calculate the force.

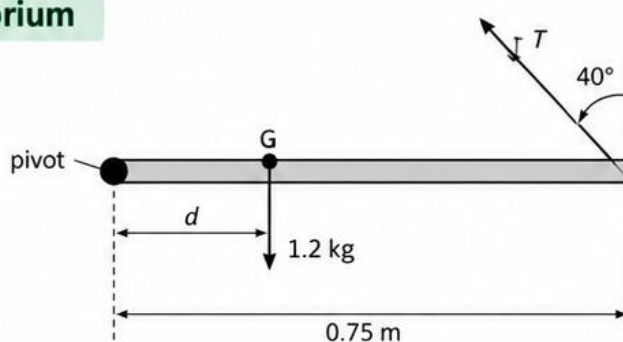
Section 2: Uniform rod in equilibrium

5. Show that the mass M is about 0.13 kg .
6. By taking moments about point A , determine d .



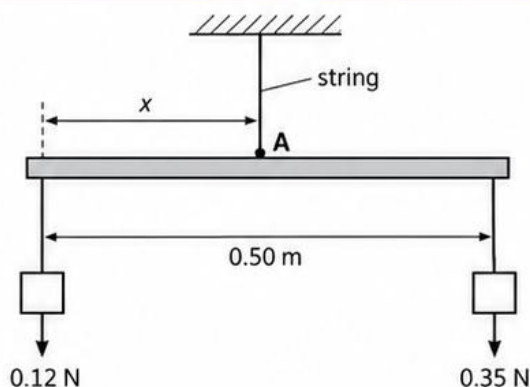
Section 3: Angled tension and equilibrium

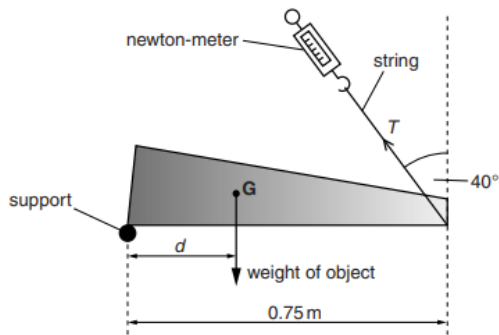
7. The object has mass 1.2 kg . Calculate the distance d from the pivot if the tension in the string is 6.4 N .
8. What angle to the vertical would be required for the tension to be 7.2 N when $d = 0.40 \text{ m}$?



Section 4: Challenge

9. Calculate the distance x required for the system to be in equilibrium.
10. Calculate the tension in the string.





The mass of the object to the left is 1.2 kg.

Calculate the distance 'd' from the pivot if the tension in the string is 6.4 N.

What angle would be required to cause a tension of 7.2 N where 'd' is 0.4 m.

Calculate the distance 'x' required for this system to be in equilibrium.
Calculate the tension in the sting.

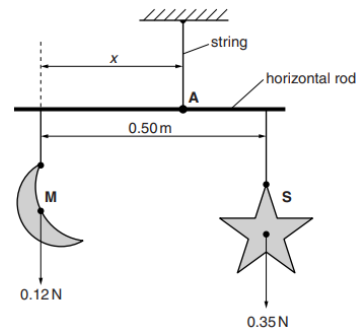
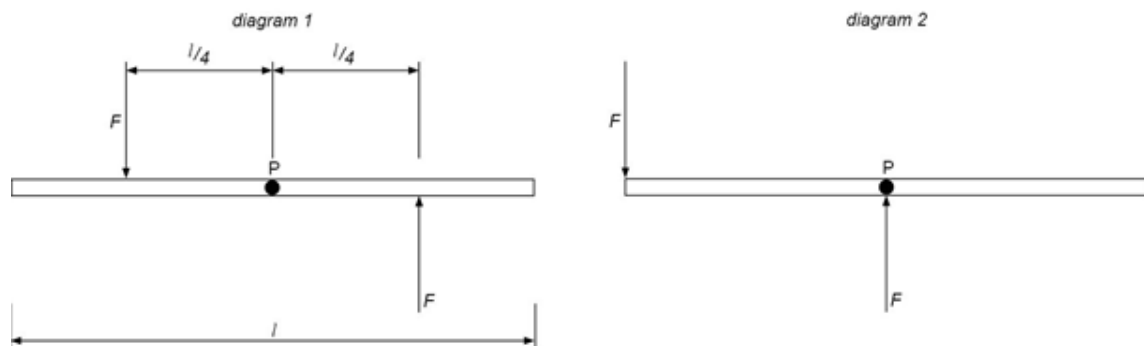


Diagram 1 shows two parallel forces F acting on a bar of length l pivoted at **P**. The forces create a couple of torque M .

In diagram 2, the two parallel forces have each moved $l/4$ to the left.



What is the torque of the couple in diagram 2?

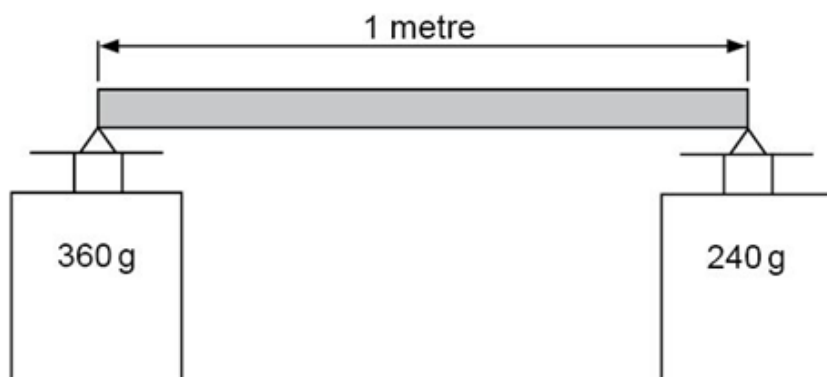
- A 0
- B $M/2$
- C M
- D $2M$

Your answer

A rod of length 1 metre has a non-uniform composition, so that its centre of gravity is not at the geometrical centre of the rod, which would be half way along its length.

The rod is laid on supports across two electronic balances, as shown in the diagram.

The balances, which have been previously set to zero, give readings of 360 g and 240 g.



Where is the centre of gravity of the rod relative to its geometric centre?

- A $\frac{1}{10}$ m to the left
- B $\frac{1}{10}$ m to the right
- C $\frac{1}{6}$ m to the left
- D $\frac{1}{6}$ m to the right

Your answer

Define the term 'principle of moments'

..... [2]

1(a). Fig. 4.1 shows a cyclist.

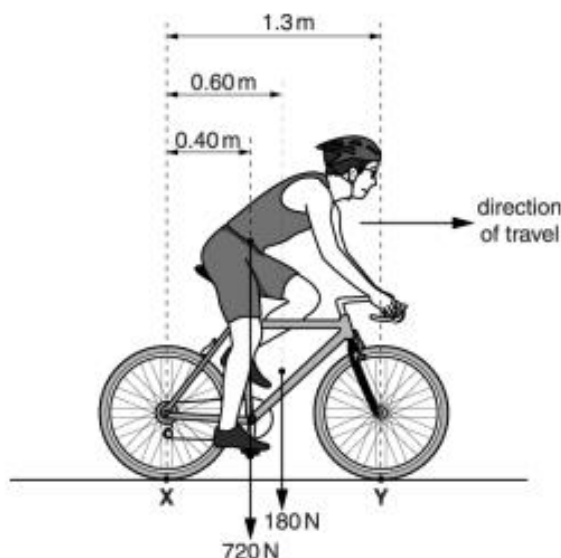


Fig. 4.1

The bicycle tyres are in contact with the road at X and Y. The cyclist is travelling at **constant** velocity on a level road. The weight of the bicycle is 180 N and the weight of the cyclist is 720 N.

Take moments about X to determine the size of the vertical force F acting on the tyre at Y.

$F =$ N [3]

(b). The cyclist leans further forward. How does this affect the force on the tyre at X? Explain your answer.

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..... [2]

END OF QUESTION PAPER

Question		Answer / Indicative content	Marks
1	(a)	The sum of clockwise moments equals the sum of anticlockwise moments for a system in equilibrium .	C1
		$398 = 1.3F$	
		$F = 300 \text{ N}$	A1
	(b)	The force at X decreases	B1
		The force at Y increases / greater clockwise moment / $F_x + F_y = 900 \text{ N}$	B1

Why does centre of gravity matter?

Every object has a centre of gravity (CoG) – the point at which its weight can be considered to act. The position of the CoG helps to determine whether an object is stable, balanced or likely to topple. Understanding the CoG is important in design, sport, engineering and everyday life.

Key idea

The centre of gravity is the point at which the weight of an object acts. If the line of action of the weight passes through the base of support, the object is stable.



Keeping the centre of gravity over the chair base helps him to stay balanced.



The bottle's centre of gravity stays above the small base, so it balances.



Gymnasts carefully control their centre of gravity to maintain balance.



The toy balances when its centre of gravity is directly above your finger.



Cranes are designed so that the combined centre of gravity remains within a stable base.



Ships have a low centre of gravity, which makes them more stable in waves.

Think about it...

- 1 Look at the examples above. How does the position of the centre of gravity affect whether the object is stable or likely to fall over?
- 2 Which everyday objects do you think have a high centre of gravity? Which have a low centre of gravity?
- 3 How might engineers change the position of the centre of gravity to make an object more stable?
- 4 Can you think of any sporting examples where a low centre of gravity gives an advantage?

Real-world links

- Buildings and bridges
- Vehicles (cars, lorries, ships, aircraft)
- Sports (gymnastics, cycling, motor racing)
- Furniture and product design
- Robotics
- Even keeping your balance!

Finding the centre of gravity of an object

Every object has a centre of gravity (CoG) – the point at which its weight can be considered to act. For a uniform object, the CoG may not be at the geometric centre, especially for irregular shapes. You can find the CoG of a flat object experimentally using a simple plumb line.

Key points

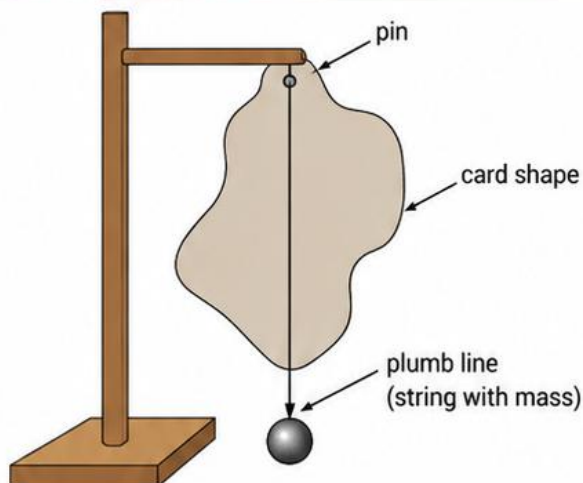
- The weight acts vertically through the centre of gravity.
- If the vertical line from the CoG falls within the base, the object is stable.
- This method works for any flat shape.

1 Method – try this at home!

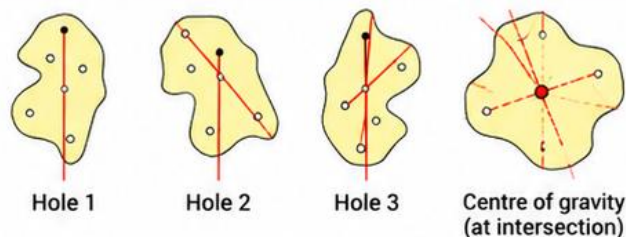
- 1 Cut out an irregular shape from card.
- 2 Make a small hole near the edge and hang the card from a pin or pencil.
- 3 Attach a plumb line (string with a mass) and allow it to hang freely.
- 4 Draw a straight line down the card along the plumb line.
- 5 Repeat for at least two more holes in different positions.
- 6 The position where the lines intersect is the centre of gravity.
- 7 Test your result by balancing the card on the tip of a pencil at this point.

You don't need a retort stand – you can improvise using a pencil, a piece of wood, a door frame, or anything the card can hang from.

Experimental setup



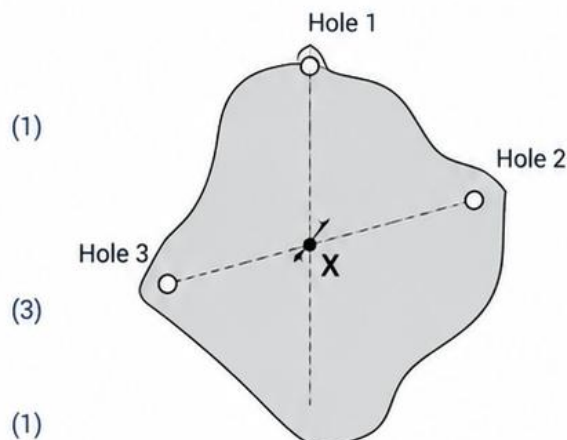
Example results



2 Exam-style question

A student investigates the centre of gravity of a thin, irregular laminate shape using the method described above.

- (a) Explain why a plumb line always hangs vertically.
- (b) The student hangs the shape from three different holes and draws the three lines as shown in the diagram. State how the centre of gravity of the shape is found and explain why this method works.
- (c) The shape is now balanced on the tip of a pencil at the point marked X. Explain what this shows.



A cylinder head gasket is a thin sheet of material used in a car engine.
A car manufacturer wants to locate the centre of gravity of the gasket shown in Fig. 18.1.



Fig. 18.1

Describe how the centre of gravity of the gasket can be determined using equipment from a laboratory.

Fig. 2.1 shows an object held horizontally by a string.

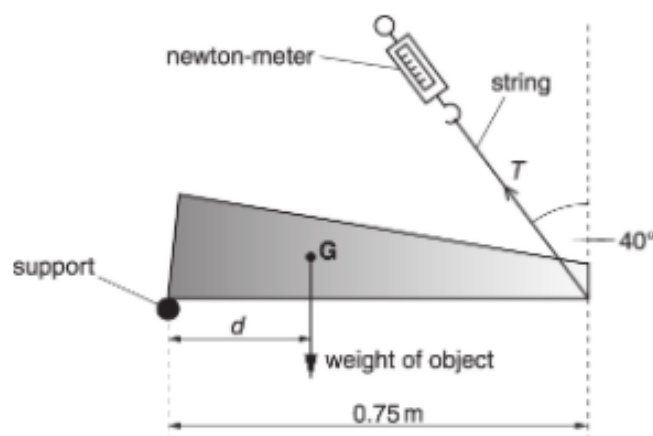


Fig. 2.1

One end of the object rests on a support and the other end is held by the string. The object is in equilibrium. Point G is the centre of gravity of the object. The mass of the object is 1.2 kg. The tension T in the string is 5.1 N. The string makes an angle of 40° with the vertical.

Take moments about the support and calculate the distance d .

$d =$ m [3]

Fig. 2.1 shows an object held horizontally by a string.

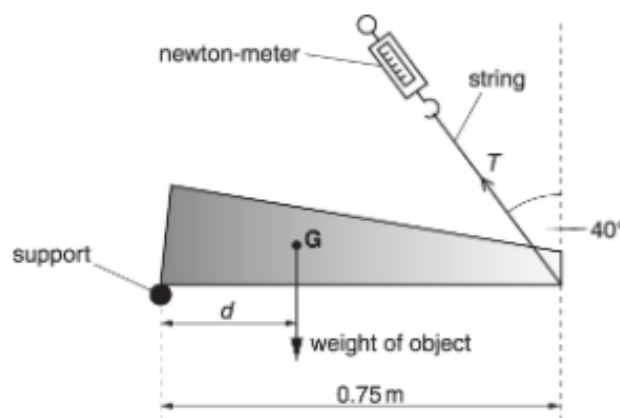


Fig. 2.1

One end of the object rests on a support and the other end is held by the string. The object is in equilibrium. Point G is the centre of gravity of the object. The mass of the object is 1.2 kg. The tension T in the string is 5.1 N. The string makes an angle of 40° with the vertical.

Explain why the force at the support cannot be vertically upwards.

.....

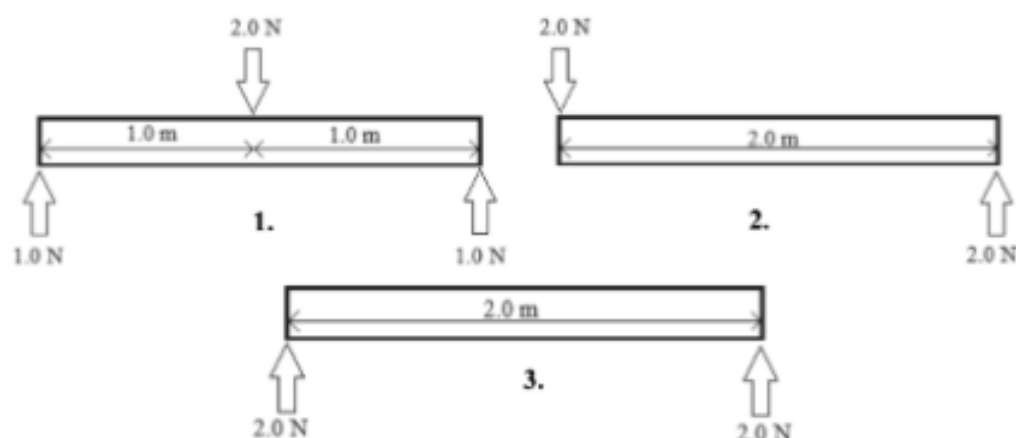
.....

.....

.....

[1]

A 2.0 m rigid rod with negligible weight is subject to forces in three different ways as shown in diagrams 1-3 below.



For the rod to be in equilibrium which of the diagrams above is / are correct?

- A. 1, 2 and 3
- B. Only 1 and 2
- C. Only 2 and 3
- D. Only 1

State the *principle of moments*.

.....

.....

[1]

i. Define the moment of a force about a point.

.....

[1]

ii. State the principle of moments as a condition for equilibrium.

.....

[1]

A gymnast hangs from the rings apparatus shown in Fig. 17.1. He raises his legs from a vertical to a horizontal position. Fig. 17.2 shows a simple model to demonstrate the forces exerted on his legs in the horizontal position. The total weight W of the legs is 260 N and acts at a point 40 cm from the pivot P in the hip. A force T is supplied by his hip flexor muscles which are attached to a point in the bone 3.0 cm from the pivot P . When horizontal, force T makes an angle of 50° with his legs.



Fig. 17.1

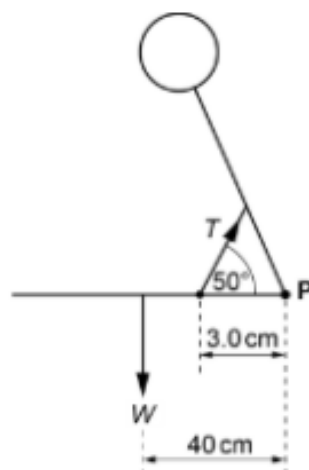


Fig. 17.2

- i. Take moments about P to calculate the force T needed to keep his legs horizontal.

$T = \dots\dots\dots$ N [3]

- ii. Describe and explain the change in the force T , if any, as his legs are lowered from the horizontal position to vertical position.

.....

.....

.....

..... [2]

Fig. 1 shows the top of a crane which has an arm AC of mass 1800 kg. The centre of mass of the arm is at D.

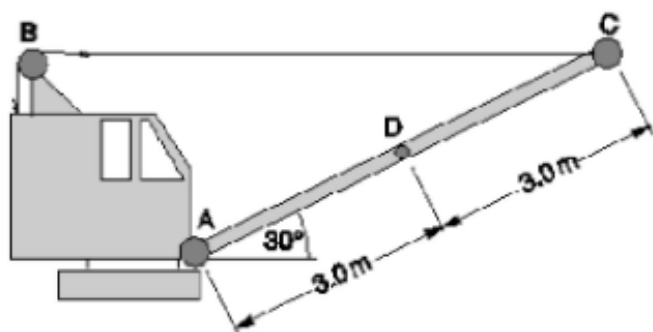


Fig. 1

The arm can rotate about the pivot at A. It is supported by a horizontal cable BC. The dimensions are as shown on the diagram.

- i. The arm is in equilibrium. Draw and label three arrows on Fig. 1 to represent the three forces acting on the arm.

[3]

- ii. Calculate the tension in the cable BC.

tension = kN [3]

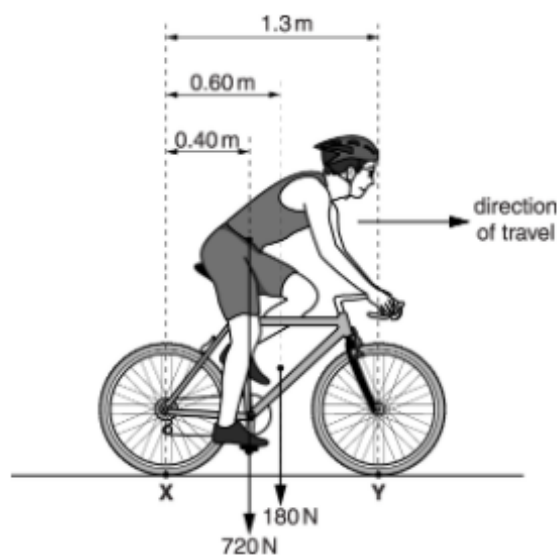
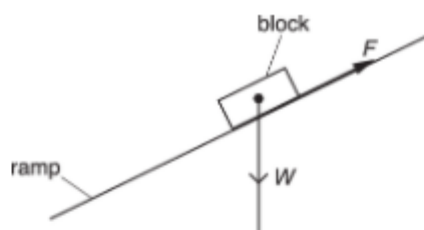


Fig. 4.1

The bicycle tyres are in contact with the road at X and Y. The cyclist is travelling at constant velocity on a level road. The weight of the bicycle is 180 N and the weight of the cyclist is 720 N.

Take moments about X to determine the size of the vertical force F acting on the tyre at Y.

$F = \dots\dots\dots$ N [3]



A triangle of forces diagram can be used to determine the magnitude and the direction of the normal contact force N .

Which is the correct diagram for this triangle?

- A
- B
- C
- D

Two forces act on an object in the same plane.

Which diagram shows a couple?

- A
- B
- C
- D

Consolidation 1: Forces and motion

Connect force, mass, weight and motion. Use $g = 9.81 \text{ m s}^{-2}$ throughout these pages.

Specification confidence check

After answering: R = revisit; A = nearly there; G = confident.

OCR A point and skill	RAG	Next step
3.2.1(a), (f) Use resultant force = mass $\times$ acceleration in one and two dimensions.	R A G	
3.2.1(b) Define the newton as a unit of force.	R A G	
3.2.1(c) Distinguish mass and weight, and use $W = mg$.	R A G	
3.2.1(d) Identify tension, normal contact force, upthrust and friction.	R A G	
3.2.1(e) Draw a free-body diagram containing only forces on the chosen object.	R A G	

Mixed questions

1

(a) Define one newton. (b) Name the force in each case: a stretched rope pulls; fluid pressure produces an upward resultant; a surface pushes perpendicularly; rough surfaces oppose sliding.

2

A 4.0 kg crate is pulled horizontally across a level floor by a 12 N force against 4.0 N friction. It has no vertical acceleration. (a) Draw and label its four forces. (b) Calculate its weight, normal contact force and horizontal acceleration.

Diagram

Calculations

3

A 2.0 kg puck on a smooth horizontal surface initially moves east at 4.0 m s^{-1} . Constant horizontal forces of 6.0 N east and 8.0 N north act on it. Calculate the magnitude and direction of its acceleration, then its east and north velocity components after 3.0 s.

My next step

Weakest point or skill: _____ Practice task: _____

Consolidation 2: Drag and terminal speed

Use changing forces to explain motion, then connect the model to a practical measurement.

Specification confidence check				
After answering: R = revisit; A = nearly there; G = confident.				
OCR A point and skill	RAG			Next step
3.2.2(a) Define drag as a resistive force on an object moving through a fluid.	☐	☐	☐	
3.2.2(b) Explain how speed, frontal area and shape affect drag in air.	☐	☐	☐	
3.2.2(c) Explain a falling object's changing acceleration when drag acts.	☐	☐	☐	
3.2.2(d)(i) Link terminal velocity to zero resultant force and zero acceleration.	☐	☐	☐	
3.2.2(d)(ii) Describe a method to measure and verify terminal velocity in a fluid.	☐	☐	☐	

Mixed questions

- 1

A small sphere is released from rest in a viscous liquid. Its weight is 0.060 N and its constant upthrust is 0.010 N. (a) Sketch its velocity-time graph until terminal speed is reached. (b) Explain the changing gradient and calculate the drag at terminal speed.

Graph

Explanation and working
- 2

A 0.020 kg sphere is falling vertically in air. At one instant its upward drag is 0.150 N; ignore upthrust. (a) Calculate its acceleration. (b) Define drag, name two factors besides speed that affect it, and explain the effect of streamlining.
- 3

Describe an experiment to determine the terminal speed of a ball bearing in a viscous liquid. Include the measurements, how you confirm terminal speed, repeat readings, and one way to reduce relative uncertainty.

My next step

Weakest point or skill: Practice task:

Consolidation 3: Moments and couples

Use perpendicular distances and test both conditions for equilibrium.

Specification confidence check				
After answering: R = revisit; A = nearly there; G = confident.				
OCR A point and skill	RAG			Next step
3.2.3(a) Define moment and use force $\times$ perpendicular distance from the pivot.	☐ R	☐ A	☐ G	
3.2.3(b) Identify a couple and calculate its torque using force separation.	☐ R	☐ A	☐ G	
3.2.3(c) State and apply the principle of moments.	☐ R	☐ A	☐ G	
3.2.3(e) Use zero resultant force as a condition for equilibrium.	☐ R	☐ A	☐ G	
3.2.3(e) Use zero resultant torque as a condition for equilibrium.	☐ R	☐ A	☐ G	

Mixed questions

- 1

Define the moment of a force about a point. A 25 N force acts 0.80 m from a door hinge, at 35° to the line from the hinge to the point of application. The force and this line are in a horizontal plane. Calculate the moment about the hinge.
- 2

A uniform horizontal beam of length 2.4 m and weight 60 N is hinged at its left end. A vertical cable supports its right end. A 90 N load hangs 0.80 m from the hinge. (a) Calculate the cable tension. (b) Find the vertical hinge force and its direction. (c) Explain why balancing moments alone does not establish equilibrium.
- 3

Two equal, opposite, parallel 30 N forces act on a handle, with their lines of action separated by 0.18 m. (a) Explain why they form a couple with zero resultant force but a turning effect. (b) Calculate the torque and state its new value if the separation is halved while each force stays unchanged.

My next step

Weakest point or skill: Practice task:

Consolidation 4: Centres and equilibrium

Explain centre of gravity, plan a practical method and use a triangle of forces.

Specification confidence check				
After answering: R = revisit; A = nearly there; G = confident.				
OCR A point and skill	RAG			Next step
3.2.3(d) Define centre of mass and centre of gravity, including a uniform field.	☐ R	☐ A	☐ G	
3.2.3(d) Determine the centre of gravity of an irregular lamina experimentally.	☐ R	☐ A	☐ G	
3.2.3(e) Explain equilibrium using resultant force and resultant torque.	☐ R	☐ A	☐ G	
3.2.3(f) Draw a closed triangle for three coplanar forces in equilibrium.	☐ R	☐ A	☐ G	
3.2.3(f) Resolve forces or use a force triangle to find unknown forces.	☐ R	☐ A	☐ G	

Mixed questions

1

Define centre of mass and centre of gravity. Explain why they coincide in a uniform gravitational field. A uniform ring has its centre of mass in its central hole: explain why the centre of mass does not need to be inside the material.

2

Describe how to find the centre of gravity of an irregular card lamina using suspension and a plumb line. Sketch the apparatus. Include the procedure, a check on reliability, and an explanation of why the method works using forces and moments.

Diagram

Explanation and working

3

A massless junction is in equilibrium supporting a 40.0 N downward load. Two strings run upwards, one left at 30° to the vertical and one right at 60° to the vertical. Draw a labelled closed triangle of forces and calculate both tensions.

Diagram

Calculations

My next step

Weakest point or skill: Practice task:

Consolidation 5: Mixed Dynamics check

Complete the questions without notes first, then choose a precise skill to revisit.

Specification confidence check				
After answering: R = revisit; A = nearly there; G = confident.				
OCR A point and skill	RAG			Next step
3.2.1(a), (c) Choose the resultant force and calculate weight or acceleration.	☐	☐	☐	
3.2.1(d)-(f) Draw forces and resolve them into useful directions.	☐	☐	☐	
3.2.2(c), (d) Use measured speeds to test whether terminal velocity is reached.	☐	☐	☐	
3.2.3(a)-(c), (e) Apply moments and force balance to an object in equilibrium.	☐	☐	☐	
3.2.3(d), (f) Explain centres of gravity and use a triangle of forces.	☐	☐	☐	

Mixed questions

1

A 0.80 kg cart moves down a rough slope at 20° to the horizontal. Friction is 1.2 N up the slope. (a) Draw a labelled free-body diagram. (b) Calculate the normal contact force and the acceleration down the slope. (c) Explain why the normal force is not equal to the weight.

Diagram

Calculations

2

A student times a ball bearing through three successive 0.150 m sections of a liquid column. The times are 0.62 s, 0.51 s and 0.45 s as it falls. Calculate the mean speed in each section. Do these data establish terminal speed? Explain and suggest a better measurement region.

3

A uniform horizontal shelf is 1.20 m long and weighs 30 N. Its left end is hinged to a wall. A cable at its right end pulls towards the wall at 35° above the horizontal. A 60 N box rests 0.90 m from the hinge. Calculate the cable tension, then both components of the force exerted by the hinge on the shelf. State their directions.

My next step

Weakest point or skill: Practice task:

71

Additional notes



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