



UNDERSTAND



APPLY



PRACTISE

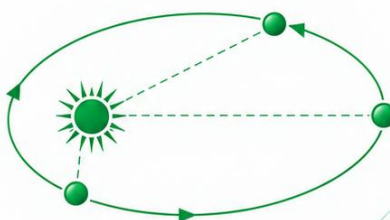


IMPROVE

$$F = \frac{GMm}{r^2}$$



Gravitational Fields and Planetary Motion



OWNER DETAILS



NAME:

GROUP:

TEACHER:



Using this workbook

Use the blank areas for class modelling, your own calculations and diagrams. Show substitutions, unit conversions and a final answer with units.

Data and conventions

Unless a question gives different data, use the values below. Treat orbits as circular when calculating orbital speed or period. Measure r from the centre of the source mass.

Quantity	Value
Gravitational constant G	$6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Mass of Earth	$6.0 \times 10^{24} \text{ kg}$
Radius of Earth	$6.40 \times 10^6 \text{ m}$
Surface field strength on Earth	9.81 N kg^{-1}
Speed of light c	$3.00 \times 10^8 \text{ m s}^{-1}$
Molar gas constant R	$8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
Mass of a hydrogen atom	$1.67 \times 10^{-27} \text{ kg}$
Mass of the Sun	$1.99 \times 10^{30} \text{ kg}$

Use positive magnitudes for force, field strength and speed unless a direction or sign convention is stated. Take gravitational potential and potential energy to be zero at infinity.

Optional videos

Crash Course: Newtonian gravity

<https://www.youtube.com/watch?v=7gf6YpdvtE0>

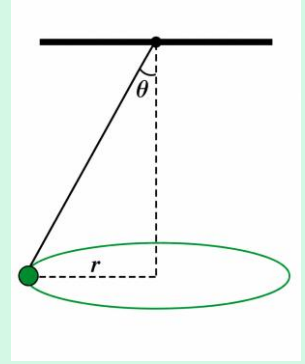
DrPhysicsA: gravitational fields

<https://www.youtube.com/watch?v=zdQ54siEfvc>

5-A-Day

A conical pendulum rotates steadily in a horizontal circle. Period = 1.2 s, radius = 0.20 m and bob mass = 0.150 kg. Use $g = 9.81 \text{ m s}^{-2}$.

1. Calculate the speed of the bob.
2. Show that $\tan \theta = v^2/(rg)$.
3. Calculate the angle of the string to the vertical.
4. Find the magnitude of the displacement from the starting point after 0.80 s (the straight-line chord).
5. Calculate the tension in the string.



5-A-Day

Use the constants on page 2.

1. State Newton's law of gravitation.
2. Calculate the gravitational force between two hydrogen atoms whose centres are 0.70 nm apart.
3. The Earth–Moon separation is 3.84×10^8 m. Their gravitational attraction is 2.0×10^{20} N. Calculate the mass of the Moon.
4. A mass M at distance R from Earth's centre experiences force F . A second mass $5M$ is at distance $2R$. Express its force in terms of F .
5. Calculate the acceleration due to gravity at a height of 2.0×10^4 km above Earth's surface.

5-A-Day

A satellite has a circular orbit 300 km above Mars's surface. Mars radius = 3400 km and orbital speed = 3.4 km s^{-1} . Use Earth data from page 2.

1. Calculate the gravitational field strength at the satellite. First find the radius measured from the centre of Mars.
2. Calculate the gravitational field strength on the surface of Mars.
3. Calculate the mass of Mars.
4. A radio signal takes 380 s to travel one way from Mars to Earth. Calculate their separation at this time.
5. Calculate the gravitational attraction between Earth and Mars using your results.

5-A-Day

Earth's mass is 6.0×10^{24} kg. Consider circular orbits outside Earth.

1. Define gravitational potential and state its units.
2. Sketch V against distance r from Earth's centre for $r \geq R$. Mark the surface and the value approached at infinity.
3. Calculate the potential at orbital radius 8000 km.
4. The satellite moves to a circular orbit of radius 7000 km. Explain how its orbital speed and period change.
5. Calculate the change in potential, $V_{\text{final}} - V_{\text{initial}}$.

5-A-Day

Earth's mass is 6.0×10^{24} kg and radius 6400 km. Ignore air resistance and Earth's rotation.

1. A payload is lifted from the surface to a height of 450 km. Calculate its change in gravitational potential.
2. The payload has mass 2.0×10^5 kg. Calculate its gain in gravitational potential energy. Ignore any change in kinetic energy.
3. Derive the escape-speed equation $v_{\text{esc}} = \sqrt{2GM/r}$, using conservation of energy.
4. Calculate the escape speed from Earth's surface.
5. Sketch how gravitational potential changes from the surface to infinity. Explain the sign of the potential and the work needed to escape.

5-A-Day

$M_{\text{Earth}}/M_{\text{Moon}} = 82$. Earth radius = 6400 km, Moon radius = 1730 km and $g_{\text{Earth}} = 9.81 \text{ N kg}^{-1}$.

1. Use the mass and radius ratios to calculate the field strength at the Moon's surface.
2. Use your answer to calculate the mass of the Moon.
3. Use the mass ratio to calculate the mass of Earth.
4. Calculate the escape speed from the Moon's surface.
5. Calculate the escape speed from Earth's surface.

5-A-Day

A 2500 kg payload is lifted slowly from 300 km to 370 km above Earth. Its kinetic energy change is negligible. Use Earth data from page 2.

1. Calculate the work done against gravity during the lift.
2. The fuel releases 19.5 MJ kg^{-1} . At 75% efficiency, calculate the mass of fuel needed for the lift.
3. All wasted energy heats a stationary 3750 kg engine ($c = 520 \text{ J kg}^{-1} \text{ K}^{-1}$). Treat c as constant. Calculate its temperature rise.
4. The payload is later released from rest at 370 km. If 80% of its loss of GPE becomes kinetic energy, calculate its speed at Earth's surface.
5. On impact, all this kinetic energy melts ice initially at 0°C . Use ice density 920 kg m^{-3} and latent heat 330 kJ kg^{-1} to calculate the volume melted.

5-A-Day

Model Earth's and Mars's orbits as circular around the same Sun. Use Earth's orbital period as 365 days.

1. State Kepler's first law.
2. State Kepler's second law.
3. State Kepler's third law.
4. Derive $T^2 = (4\pi^2/GM)r^3$ for a circular orbit.
5. Earth's mean orbital radius is 1.50×10^{11} m and Mars's is 2.28×10^{11} m. Calculate Mars's orbital period.

5-A-Day

Use $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ throughout.

Take the mass of the satellite as 2500 kg and ignore the change in mass as fuel is used.

1. A satellite is moved from an altitude of 300 km above the Earth's surface to an altitude of 370 km.

The Earth has a mass of $6.0 \times 10^{24} \text{ kg}$ and a radius of 6400 km. For this calculation, ignore any change in the satellite's kinetic energy. Calculate the work done by the satellite's thrusters.

2. The satellite uses hydrazine fuel, N_2H_4 , which releases 19.5 MJ of energy per kilogram. The thrusters are 75% efficient, meaning that 75% of the energy released by the fuel is transferred into useful work. Calculate the mass of fuel used during the manoeuvre in Question 1.

3. Some of the energy that is not transferred into useful work heats the engine. Assume that 3.0% of the wasted energy calculated in Question 2 is transferred to the metal of the engine. The engine has a mass equal to 15% of the total satellite mass and a specific heat capacity of $520 \text{ J kg}^{-1} \text{ K}^{-1}$. Calculate the increase in temperature of the engine.

4. The satellite is later decommissioned. For this calculation, assume that it starts from rest at an altitude of 370 km and falls towards the Earth's surface. During the fall, 80% of the decrease in gravitational potential energy is transferred to kinetic energy. The remaining energy is transferred to the surroundings due to resistive forces. Calculate the speed of the satellite immediately before it reaches the Earth's surface.

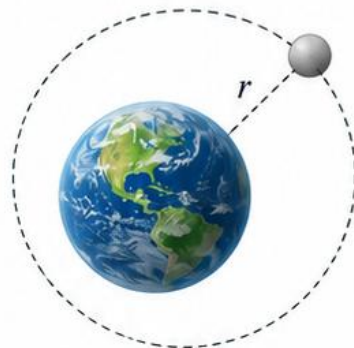
5. The satellite crashes into an ice cap. The ice is initially at 0°C . The density of ice is 920 kg m^{-3} and its specific latent heat of fusion is 330 kJ kg^{-1} . Calculate the volume of ice melted by the impact, in m^3 .

5.4.2 Newton's law of gravitation

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

- (a) Newton's law of gravitation; $F = -\frac{GMm}{r^2}$ for the force between two point masses
- (b) gravitational field strength $g = -\frac{GM}{r^2}$ for a point mass
- (c) gravitational field strength is uniform close to the surface of the Earth and numerically equal to the acceleration of free fall.



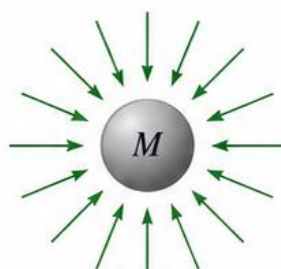
The gravitational force acts along the line joining two point masses and is always attractive.

5.4.4 Gravitational potential and energy

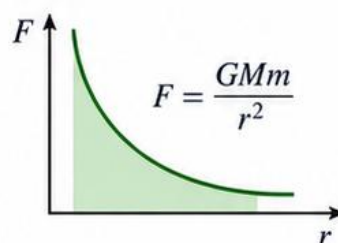
Learning outcomes

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- (a) gravitational potential at a point as the work done in bringing unit mass from infinity to the point; gravitational potential is zero at infinity
- (b) gravitational potential $V_g = -\frac{GM}{r}$ at a distance r from a point mass M ; changes in gravitational potential
- (c) force–distance graph for a point or spherical mass; work done is area under graph
- (d) gravitational potential energy $E = mV_g = -\frac{GMm}{r}$ at a distance r from a point mass M
- (e) escape velocity.



Gravitational field lines point towards the mass (attractive).



Work done is the area under the force–distance graph.

Gravitational fields

OCR A 5.4.1 and 5.4.2

A gravitational field is a region in which a mass experiences a force. Field lines show the direction of the force on a small test mass.



Teacher modelling: field patterns

Sketch the field around a spherical mass, then sketch the nearly uniform field close to Earth's surface. Add arrows and explain the spacing.

Field strength

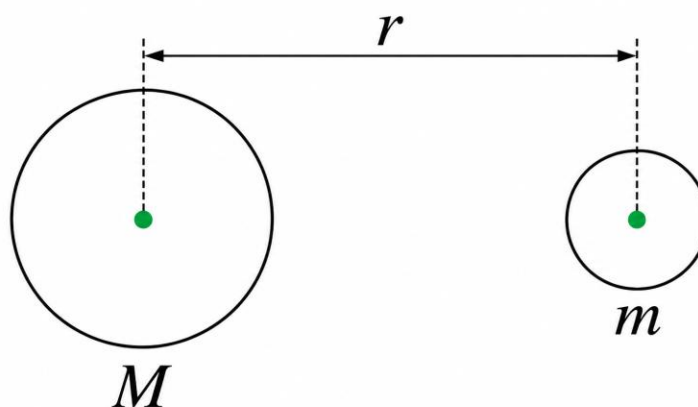
Define gravitational field strength. Write its equation and units, then explain why it is numerically equal to free-fall acceleration.

Newton's law of gravitation

What is the relationship between weight, mass and gravitational field strength?

For a point mass or a spherical mass, how does field strength depend on distance from its centre? Name the relationship.

State Newton's law of gravitation and define its symbols. On the diagram, add the equal and opposite gravitational force arrows.



Using Newton's law

Use $F = GMm/r^2$

1. A moon orbits Jupiter at a centre-to-centre distance of 422 000 km. Jupiter's mass is 1.9×10^{27} kg and the moon's mass is 8.9×10^{22} kg. Calculate their gravitational attraction.

2. A satellite of mass 2000 kg orbits Earth at radius 20 Mm. The gravitational force is 2000 N. Estimate Earth's mass.

3. A satellite travels at 3100 m s^{-1} in a circular orbit around Earth. Calculate its altitude above the surface. Use the Earth data on page 2.

Inverse-square relationships

Teacher modelling

Two point masses attract with force F when separated by R . Their separation increases to $3R$. Choose the new force and explain your reasoning.

- A $9F$ B $3F$ C $F/9$ D $F/3$

Your turn

The separation halves and one mass triples. Express the new gravitational force in terms of the original force. Show the two scaling factors separately.

Gravitational field strength

Teacher modelling: from $F = GMm/r^2$ to $g = GM/r^2$

Use the definition $g = F/m$ to derive an expression for the magnitude of g outside a spherical mass.

Your turn

Calculate the gravitational field strength at a satellite 100 km above Earth's surface. State the radius you substitute and the direction of the field.

Check for understanding 1

Fields and Newton's law

Choose one answer for each question. Explain why it is correct and why a tempting alternative is wrong.

1. At a fixed position outside an isolated planet, a probe of mass m experiences force F in a field of strength g . The probe's mass doubles. What are the new force and field strength?

- | | |
|------------------------|-----------------------|
| A F and $g/2$ | B $2F$ and g |
| C $2F$ and $2g$ | D F and g |

Reason: _____

2. An isolated spherical planet has radius R and surface field strength g_s . What is the field strength at an altitude $2R$ above its surface?

- | | |
|------------------|------------------|
| A $g_s/4$ | B $g_s/2$ |
| C $g_s/3$ | D $g_s/9$ |

Reason: _____

3. Which statements about the field outside an isolated spherical planet are correct?

1. The field points towards the planet's centre.
2. Doubling the planet's mass doubles g at fixed r .
3. Doubling r halves g .

- | | |
|-----------------------|-----------------------|
| A 1 and 2 only | B 1 and 3 only |
| C 2 and 3 only | D 1, 2 and 3 |

Reason: _____

Gravitational field practice

A satellite moves in a circular orbit of radius 15 300 km from the centre of the Earth.

i. State one of the main benefits satellites have on our lives.

.....
..... [1]

ii. Calculate the gravitational field strength g at the radius of 15 300 km.

$g = \text{.....} \text{ N kg}^{-1}$ [2]

Two point masses m_1 and m_2 are a distance r apart.

What is the magnitude of the gravitational field strength caused by m_1 at m_2 ?

A $\frac{Gm_1m_2}{r}$

B $\frac{Gm_1m_2}{r^2}$

C $\frac{Gm_1}{r^2}$

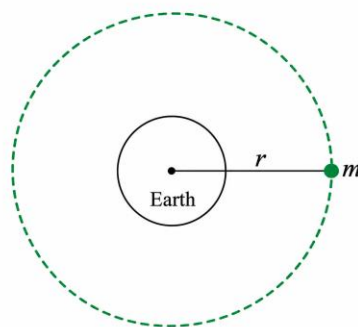
D $\frac{Gm_2}{r^2}$

A satellite in a circular orbit

Orbital radius = 13 500 km from Earth's centre. Satellite mass = 3500 kg. Use Earth's mass and radius from page 2.

Core

Explain why its speed can be constant while its velocity changes. State the direction of its acceleration and the force causing it.



Apply

State Newton's law of gravitation. Calculate the force on the satellite, then use circular motion to calculate its orbital speed.

Challenge

Calculate the field strength at the satellite. Use Newton's third law to find Earth's acceleration towards the satellite.

Force and field comparisons

Explain how Newton's law of gravitation is applied between two non-spherical asteroids.

A satellite is in a circular orbit around the Earth. The vertical **height** of the satellite above the surface of the Earth is 3200 km. The radius of the Earth is 6400 km.

What is the ratio

$$\frac{\text{weight of satellite in orbit}}{\text{weight of satellite on the Earth's surface}}?$$

- A 0.25
- B 0.44
- C 0.50
- D 0.67

The ratio of masses $M_{\text{Earth}} / M_{\text{Mars}} \approx 10$ and the ratio of radii $R_{\text{Earth}} / R_{\text{Mars}} \approx 2$.

What is the best estimate of the ratio of gravitational fields at the surface of the two bodies $g_{\text{Earth}} / g_{\text{Mars}}$?

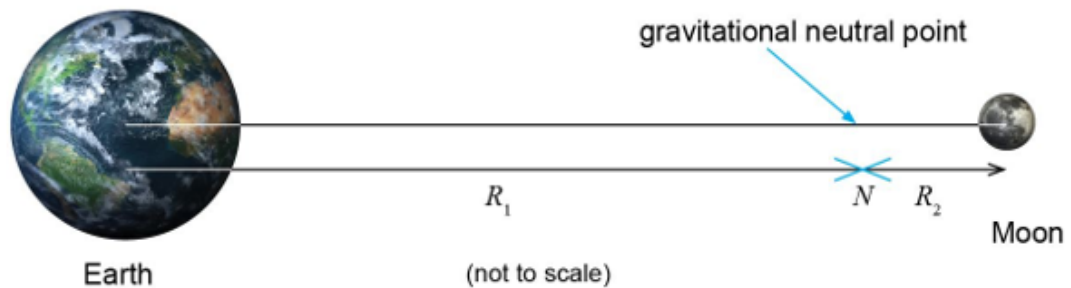
- A 1
- B 3
- C 4
- D 5

The Earth–Moon neutral point

The gravitational neutral point N between the Earth and the Moon is the position where g_{total} for the two bodies is zero.

N is a distance R_1 from centre of the Earth and R_2 from centre of the Moon.

The ratio of masses $M_{\text{Earth}} / M_{\text{Moon}} \approx 80$.



What is the correct value of the ratio R_1/R_2

- A $1/\sqrt{80}$
- B $\sqrt{80}$
- C 80
- D 80^2

Working and explanation

5.4.4 Gravitational potential and energy

Learning outcomes

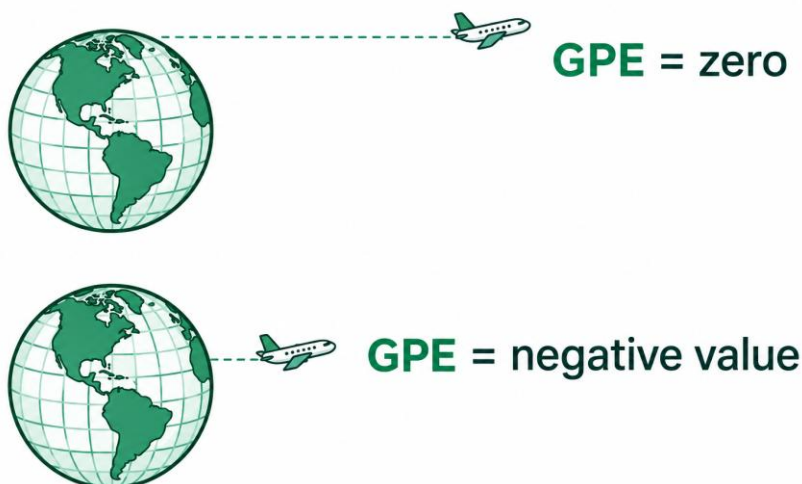
Learners should be able to demonstrate and apply their knowledge and understanding of:

- (a) gravitational potential at a point as the work done in bringing unit mass from infinity to the point; gravitational potential is zero at infinity
- (b) gravitational potential $V_g = -\frac{GM}{r}$ at a distance r from a point mass M ; changes in gravitational potential
- (c) force–distance graph for a point or spherical mass; work done is area under graph
- (d) gravitational potential energy $E = mV_g = -\frac{GMm}{r}$ at a distance r from a point mass M
- (e) escape velocity.

Potential and energy

OCR A 5.4.4(a–e)

This section covers gravitational potential, changes in potential and potential energy, work from a force–distance graph, and escape speed.



Before we start

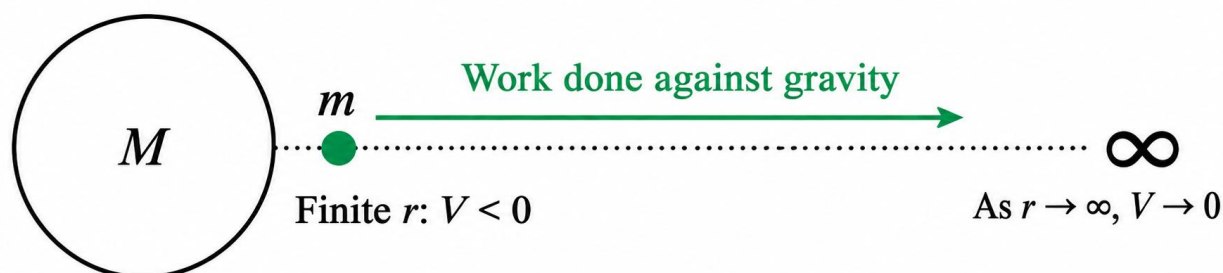
Explain how gravitational field strength and gravitational potential differ. Include their definitions, units and whether each is a vector or a scalar.

Teacher modelling: choosing a zero

Why do we take gravitational potential energy to be zero at infinity? Explain why its value is negative at a finite distance from an isolated mass.

Gravitational potential

Define gravitational potential at a point. State the reference position where potential is zero.



Explain why positive work is needed to move a mass away from Earth towards infinity. Distinguish potential V from potential energy mV .

The potential–distance graph

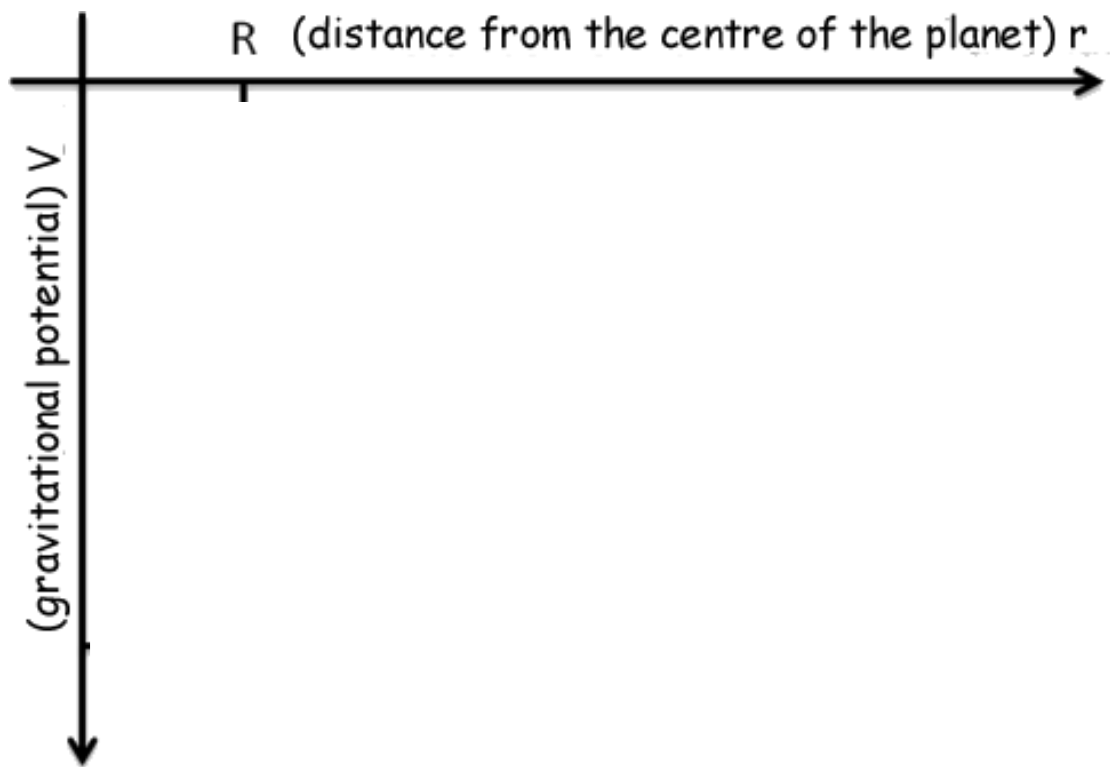


Investigate field graphs here

$$V = -GM/r$$

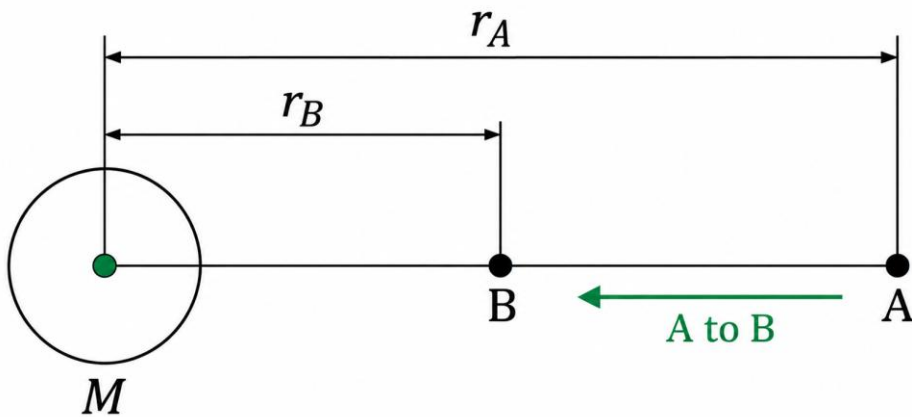
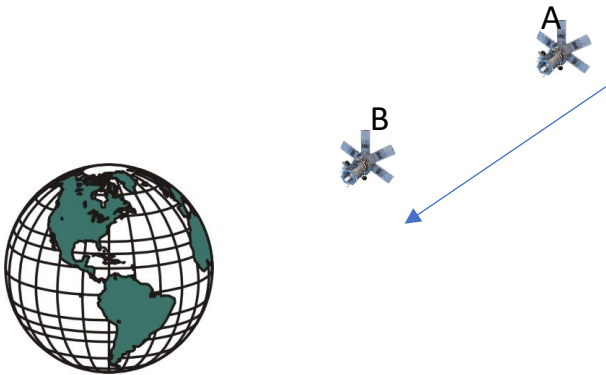
Sketch V against distance r from Earth’s centre for points at and outside its surface. Label both axes with units, mark $r = R$ and show the behaviour as r tends to infinity.

Teacher modelling and graph space



How does the gradient relate to the radial gravitational field strength? Take the outward radial direction as positive.

Changes in gravitational potential



$$\Delta V = -\frac{GM}{r_B} - \left(-\frac{GM}{r_A} \right)$$

Final position

Initial position

Check for understanding 2

Potential and potential energy

Choose one answer for each question. Explain why it is correct and why a tempting alternative is wrong.

1. At a fixed point outside an isolated planet, a probe of mass m has gravitational potential V and potential energy E . It is replaced by a probe of mass $3m$. What are the new values?

- | | |
|------------------------|-----------------------|
| A $3V$ and $3E$ | B V and E |
| C V and $3E$ | D $3V$ and E |

Reason:

2. A payload moves from point 1, where $V_1 = -12 \text{ MJ kg}^{-1}$, to point 2, where $V_2 = -4 \text{ MJ kg}^{-1}$. What is its change in gravitational potential, $V_2 - V_1$?

- | | |
|-----------------------------------|-----------------------------------|
| A $+8 \text{ MJ kg}^{-1}$ | B -8 MJ kg^{-1} |
| C $+16 \text{ MJ kg}^{-1}$ | D -16 MJ kg^{-1} |

Reason:

3. For an isolated positive mass, take $V = 0$ at infinity. Which statements are correct outside the mass?

1. V is negative at finite r .
2. V increases as r increases.
3. Gravity points towards increasing V .

- | | |
|---------------------|-----------------------|
| A 1 only | B 2 and 3 only |
| C 1, 2 and 3 | D 1 and 2 only |

Reason:

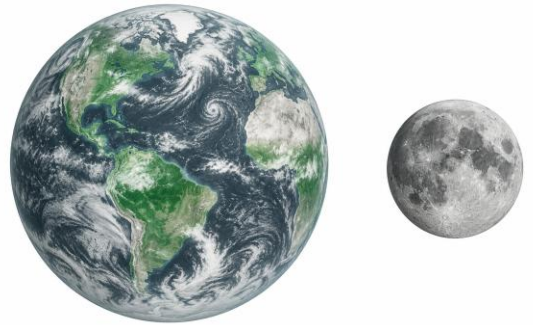
Work done per kilogram

A small payload is lifted slowly from 120 km to 140 km above Earth's surface. Calculate the work done against gravity per kilogram. Ignore kinetic energy changes.

Use $M = 6.0 \times 10^{24}$ kg and $R = 6400$ km. Begin by writing both distances from Earth's centre in metres.

Estimating Earth's mass

1. Model the Moon's orbit as circular. Its speed is 1.0 km s^{-1} and its centre-to-centre distance from Earth is $3.80 \times 10^8 \text{ m}$. Estimate Earth's mass.



Not to scale

2. Using your estimated mass, calculate the change in gravitational potential if the Moon's orbital radius decreases by 12%.

Potential and escape calculations

Use Earth's mass and radius from page 2. Take potential to be zero at infinity.

Core

Calculate the gravitational potential at 140 km above Earth's surface.

Apply

A satellite moves from altitude 300 km to altitude 130 km. Calculate $V_{\text{final}} - V_{\text{initial}}$. Explain the sign of your answer.

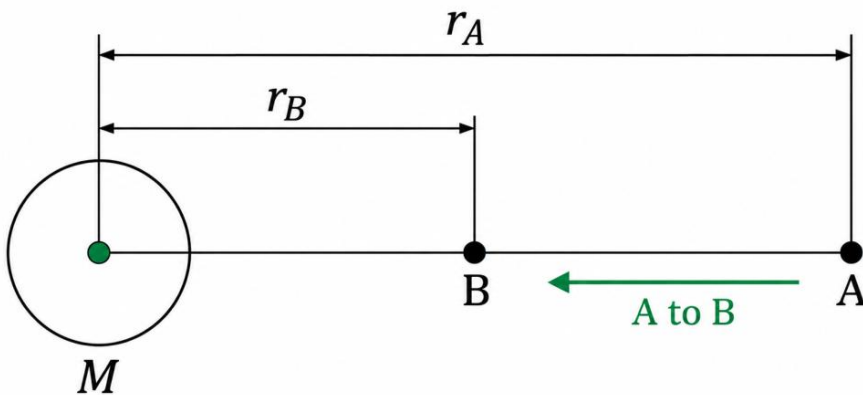
Challenge

A 13 000 kg object is launched from Earth's surface. Calculate the minimum escape speed. Ignore air resistance and Earth's rotation.

Gravitational potential energy

$$E = mV = -GMm/r$$

A change in gravitational potential gives a change in potential energy: $\Delta E = m\Delta V$. This is the work done against gravity when kinetic energy does not change.



Teacher modelling: signs and units

Write expressions for the change in GPE for A to B and for B to A. Explain which movement releases energy.

For a change between circular orbits, the kinetic energy changes too. Calculate the total energy change if a question asks for the energy supplied by an engine.

A satellite moves to a lower orbit

A satellite of mass 1.2×10^4 kg moves from a circular orbit 200 km above Earth to a circular orbit 120 km above Earth. Use the Earth data on page 2.

1. Calculate its change in gravitational potential energy. State whether GPE increases or decreases.

2. Explain why this is not, by itself, the work done by an engine during the change of orbit.

Potential and potential energy

$$V = -GM/r \qquad E = -GMm/r$$

Earth's mass is 5.97×10^{24} kg and its radius is 6370 km. Calculate the GPE of a 75 kg satellite 1200 km above Earth's surface. Use zero GPE at infinity.

Your working

Explain the physical meaning of the negative sign in your answer.

Escape speed

Escape speed is the minimum launch speed needed to reach infinity with zero speed remaining. Ignore drag, Earth's rotation and further propulsion.

$$\frac{1}{2}mv^2 - GMm/r = 0$$



Teacher modelling: the derivation

Rearrange the energy equation to obtain the escape speed. Explain why the mass of the escaping object cancels.

What would happen to the escape speed if the source mass doubled while the launch radius stayed the same?

Escape speed in context

Core

Calculate the escape speed of a golf ball from the Moon's surface. Use Moon mass 7.35×10^{22} kg and radius 1740 km.

Apply

Calculate the GPE of a 1420 kg point mass at a distance of 200 m from a 5320 kg point mass. Take zero GPE at infinity.

Challenge

Helium gas is at 1200 °C. Use molar mass $0.0040 \text{ kg mol}^{-1}$ to calculate its rms speed. Compare this with Earth's escape speed, using the Earth data on page 2.

Explain why a comparison using the rms speed alone cannot show that every helium atom escapes or that no helium atoms escape.

Work from a force–distance graph

Sketch the magnitude of the gravitational force on a small mass against distance r from an isolated spherical mass, for $r \geq R$.

Teacher modelling and graph space

Shade the area between two radii. Explain what this area represents and why the sign of work depends on the direction of motion.

Check for understanding 3

Work, graphs and escape speed

Choose one answer for each question. Explain why it is correct and why a tempting alternative is wrong.

1. The area under a gravitational force-magnitude versus r graph between r_1 and r_2 is 12 MJ, with $r_2 > r_1$. A mass moves outward from r_1 to r_2 . What is the work done by gravity?

- | | |
|-----------------|-----------------|
| A +12 MJ | B -12 MJ |
| C 0 MJ | D +6 MJ |

Reason:

2. Two isolated spherical planets have the same mass. One has radius R and surface escape speed v . The other has radius $2R$. What is its surface escape speed? Ignore drag and rotation.

- | | |
|------------------------------|-----------------------|
| A $\sqrt{2} \times v$ | B $2v$ |
| C $v/2$ | D $v/\sqrt{2}$ |

Reason:

3. Ignore drag, rotation and further propulsion. Which statements about escape from an isolated planet are correct?

1. Minimum escape speed gives zero speed at infinity.
2. A payload of mass $4m$ needs twice the escape speed of m .
3. Gravity can still act while the payload escapes.

- | | |
|-----------------------|-----------------------|
| A 1 and 3 only | B 1 and 2 only |
| C 2 and 3 only | D 1, 2 and 3 |

Reason:

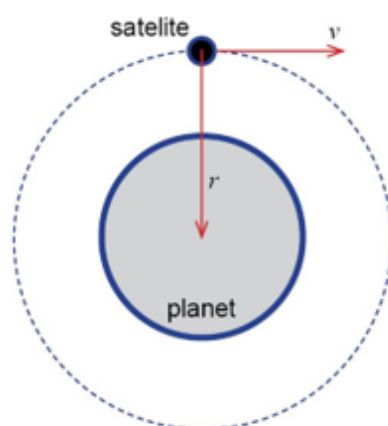
A satellite of mass 3000 kg moves from a parking orbit of radius 6800 km to a geostationary orbit of radius 42 000 km. The mass of the Earth is 6.0×10^{24} kg.

What is the magnitude of the change in gravitational potential?

- A. 8.4 J kg^{-1}
- B. $2.5 \times 10^4 \text{ J kg}^{-1}$
- C. $4.9 \times 10^7 \text{ J kg}^{-1}$
- D. $1.5 \times 10^{11} \text{ J kg}^{-1}$

Your answer

A satellite of mass m moves in a circular orbit at a speed v and a distance r from the centre of a planet of mass M .

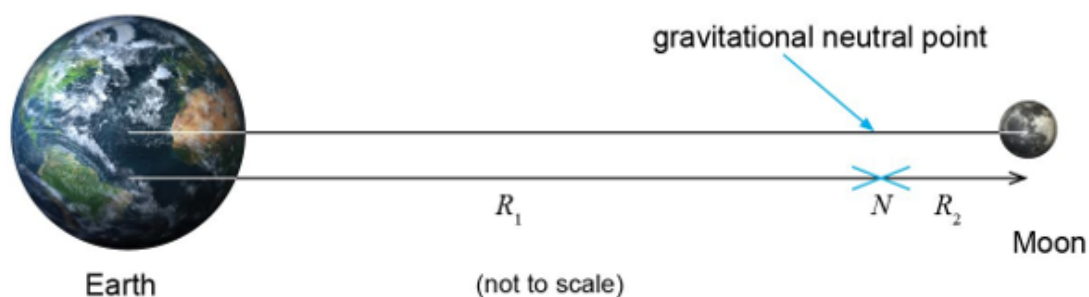


Which expression gives the total energy of the satellite?

- A $m\left(\frac{v^2}{r} - \frac{GM}{r}\right)$
- B $m\left(\frac{v^2}{2} - \frac{GM}{r}\right)$
- C $m\left(\frac{v^2}{r} + \frac{GM}{r}\right)$
- D $m\left(\frac{v^2}{2} + \frac{GM}{r}\right)$

The gravitational neutral point N between the Earth and the Moon is the position where g_{total} for the two bodies is zero.

N is a distance R_1 from centre of the Earth and R_2 from centre of the Moon.



Which of the following statements is/are true?

1 The ratio $R_1 / R_2 = \sqrt[3]{(M_{\text{Earth}} / M_{\text{Moon}})}$

2 At N two gravitational attractions act on a body but they are equal and opposite

3 At N, $\frac{\Delta V_{\text{gravity}}}{\Delta x} = 0$

A 1, 2 and 3 are correct

B Only 1 and 2 are correct

C Only 2 and 3 are correct

D Only 1 is correct

Scientists are planning to launch a rocket from the surface of the Earth into an orbit at a distance of 18000 km above the centre of the Earth. The radius of the Earth is 6400 km and it has mass 6.0×10^{24} kg.

What is the minimum work done to move the 150 kg mass of the rocket into this orbit?

A. -13×10^5 J

B. -6.0×10^5 J

C. $+6.0 \times 10^5$ J

D. $+13 \times 10^5$ J

Your answer

Error in OCR print – All powers should be 9.

Explain how Newton's law of gravitation is applied between two non-spherical asteroids.

.....

.....

..... [1]

A satellite moves in a circular orbit of radius 15 300 km from the centre of the Earth.

i. State one of the main benefits satellites have on our lives.

.....

..... [1]

ii. Calculate the gravitational field strength g at the radius of 15 300 km.

$g =$ N kg^{-1} [2]

Determine the average density of the Earth. The radius of the Earth is 6400 km.

average density = _____ kg m⁻³ [3]

A satellite of mass 3000 kg moves from a parking orbit of radius 6800 km to a geostationary orbit of radius 42 000 km. The mass of the Earth is 6.0×10^{24} kg.

What is the magnitude of the change in gravitational potential?

- A. 8.4 J kg^{-1}
- B. $2.5 \times 10^4 \text{ J kg}^{-1}$
- C. $4.9 \times 10^7 \text{ J kg}^{-1}$
- D. $1.5 \times 10^{11} \text{ J kg}^{-1}$

Your answer

This question is about Mars and its two moons, Phobos and Deimos.

Calculate

i. the mass of Mars

gravitational field strength on the surface of Mars is 3.7 N kg^{-1} radius of Mars is $3.4 \times 10^3 \text{ km}$

mass = kg [2]

ii. the gravitational field strength at a height of $3.4 \times 10^3 \text{ km}$ **above** the surface of Mars.

gravitational field strength = N kg^{-1} [1]

Planets **X** and **Y** each have a single moon.

Planet **X** has twice the mass of planet **Y**. The orbital radius of the moon around planet **X** is three times the orbital radius of the moon around planet **Y**.

The gravitational potential of the planet **X** is V_X at the position of its moon. The gravitational potential of the planet **Y** is V_Y at the position of its moon.

What is the value of the ratio $\frac{V_X}{V_Y}$?

- A 0.22
- B 0.67
- C 1.50
- D 6.00

Your answer

The ratio of masses $M_{\text{Earth}} / M_{\text{Mars}} \approx 10$ and the ratio of radii $R_{\text{Earth}} / R_{\text{Mars}} \approx 2$.

What is the best estimate of the ratio of gravitational potentials at the surface of the two bodies $V_{\text{Earth}} / V_{\text{Mars}}$?

- A 1
- B 3
- C 4
- D 5

A satellite of mass m is in a circular orbit around a planet of mass M . The radius of the orbit from the centre of the planet is r .

The gravitational potential V_g at a point a distance r from the centre of the planet is given by the equation

$$V_g = -\frac{GM}{r},$$

- i. By considering the cause of the centripetal force on the satellite, show that the kinetic energy of the satellite is equal to half the magnitude of its gravitational potential energy.

[2]

- ii. A tiny satellite of mass 1.0 kg is to be launched from rest from the surface of the Earth into a low Earth orbit. The gravitational potential at any point in this orbit is -56 MJ kg^{-1} .

The value of the gravitational potential at the Earth's surface is -63 MJ kg^{-1} .

Show that the satellite must gain more than 30 MJ of **total** energy to achieve and remain in orbit.

[2]

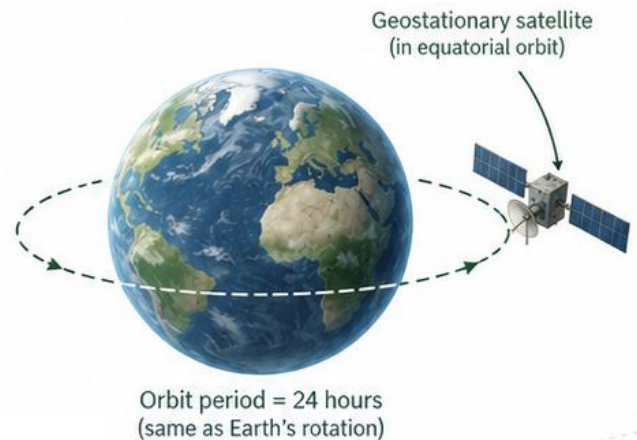
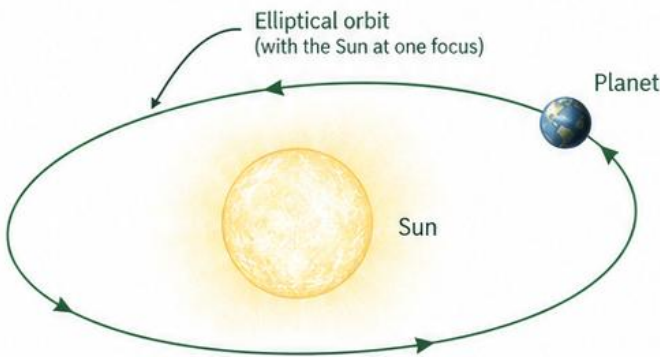
5.4.3 Planetary motion

LEARNING OUTCOMES

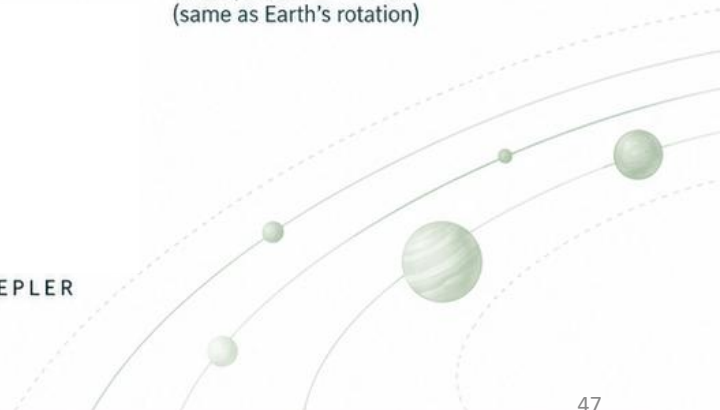


Learners should be able to demonstrate and apply their knowledge and understanding of:

- (a) Kepler's three laws of planetary motion
- (b) the centripetal force on a planet is provided by the gravitational force between it and the Sun
- (c) the equation $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$
- (d) the relationship for Kepler's third law $T^2 \propto r^3$ applied to systems other than our solar system
- (e) geostationary orbit; uses of geostationary satellites.



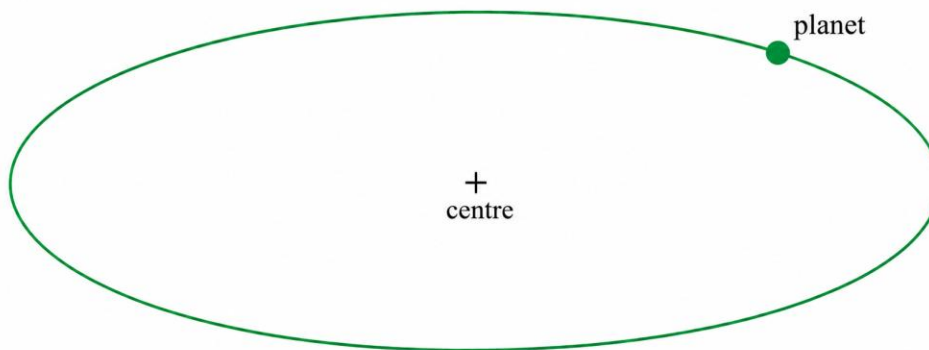
— JOHANNES KEPLER



Planetary motion

OCR A 5.4.3(a–e)

This section covers Kepler's laws, gravitational force as the centripetal force, the period–radius equation and geostationary satellites.



Teacher modelling: an elliptical orbit

Mark the two foci and place the Sun at one focus. Label perihelion and aphelion. Explain how the planet's speed changes around the orbit.

Kepler's three laws

First law

State the law and sketch a labelled orbit.

Second law

State the law. Use a diagram to show what happens during equal time intervals.

Third law

State the relationship and define the quantities. For an ellipse, use the semi-major axis a .

Deriving Kepler's third law

For a planet of mass m in a circular orbit of radius r around a star of mass M , use gravity as the centripetal force to derive the period–radius equation.

Teacher modelling

Explain why the planet's mass cancels. Identify the central mass in the final equation.

Check for understanding 4

Kepler's laws and circular orbits

Choose one answer for each question. Explain why it is correct and why a tempting alternative is wrong.

1. Two small moons follow circular orbits around the same planet. Moon A has radius r and period T . Moon B has radius $4r$. What is Moon B's period?

A $2T$

B $4T$

C $8T$

D $64T$

Reason:

2. Small satellites follow circular orbits outside two spherical planets at the same orbital radius. Planet B has four times the mass of planet A. If A's satellite has period T , what is B's satellite's period?

A T

B $T/2$

C $T/4$

D $2T$

Reason:

3. A planet follows an elliptical orbit around an isolated star. Which statements are correct?

1. The star lies at the centre of the ellipse.

2. Equal areas are swept out in equal times.

3. The planet moves fastest when closest to the star.

A 1 and 2 only

B 1 and 3 only

C 1, 2 and 3

D 2 and 3 only

Reason:

Comparing planetary orbits

Use Earth’s mean orbital radius of 1.50×10^{11} m and period of 365.26 days. The periods below are supplied data for the calculations.

Planet	Period / Earth days
Mercury	87.97
Venus	224.70
Earth	365.26
Mars	686.98
Jupiter	4332.82
Saturn	10 755.7
Uranus	30 687.15
Neptune	60 190.03

Core

State Kepler’s third law and write a ratio equation for two planets orbiting the same star.

Apply

Calculate Saturn’s mean orbital radius. Compare it with 1.4×10^{12} m. Continue your working on page 44.

Kepler's law: working space

Complete the Saturn calculation from page 43.

Challenge

Use the supplied periods to calculate the mean orbital radii of Venus, Jupiter and Neptune. Give each answer in metres.

Satellites in daily life

For three uses of satellites, explain what information or service they provide. Suggest why a geostationary orbit or another type of orbit would suit each use.



Use 1

Use 2

Use 3

Planetary motion review

1. State Kepler's first law.

2. State Kepler's second law.

3. Derive Kepler's third law for a circular orbit.

4. Describe a geostationary orbit. Include the plane, direction and period of the motion.

A geostationary satellite

A geostationary satellite has orbital radius 4.20×10^7 m. Use an orbital period of 8.616×10^4 s, equal to Earth's sidereal rotation period.

5. Calculate the satellite's orbital speed.

6. Use the radius and period to estimate Earth's mass.

7. Calculate the gravitational field strength at this orbital radius.

8. Suggest two uses of a geostationary satellite and explain the benefit of its fixed position in the sky.

Check for understanding 5

Satellites and atmospheric escape

Choose one answer for each question. Explain why it is correct and why a tempting alternative is wrong.

1. Each proposed Earth orbit has a period equal to Earth's sidereal rotation period. Which orbit could be geostationary?

- | | |
|---|---|
| A Circular, equatorial, eastward | B Circular, equatorial, westward |
| C Circular, polar | D Elliptical, equatorial, eastward |

Reason: _____

2. A geostationary satellite of mass m has orbital radius r and gravitational force magnitude F . What radius and force apply to a geostationary satellite of mass $4m$ around Earth?

- | | |
|-----------------------|-------------------------|
| A $2r$ and F | B r and $4F$ |
| C r and F | D $4r$ and $F/4$ |

Reason: _____

3. A hot atmosphere has an rms atomic speed below the planet's escape speed. Which statement is correct?

- | | |
|--|---|
| A No atom can exceed the rms speed. | B Half the atoms must exceed the escape speed. |
| C Some atoms may escape from the fast tail. | D Every atom travels at the rms speed. |

Reason: _____

Applying Kepler's laws

Core

Sketch two planetary orbits around the same star. Use them to illustrate Kepler's first and third laws. Add an equal-time area diagram to illustrate the second law.

Apply

For circular orbits around the same star, state how the orbital period changes if the radius: (a) doubles, (b) triples, (c) becomes one ninth of its original value.

Challenge

A planet orbits a star in 322 Earth days. Its mean orbital radius is 5.0 light-minutes. Calculate the mass of the star. Treat the planet's mass as negligible.

Satellite calculations

A satellite moves in a circular orbit of radius 15 300 km from the centre of the Earth.

- i. State one of the main benefits satellites have on our lives.

..... [1]

- ii. Calculate the gravitational field strength g at the radius of 15 300 km.

$g =$ N kg^{-1} [2]

- iii. Calculate the period of the orbiting satellite.

period = s [3]

A satellite of mass 3000 kg moves from a parking orbit of radius 6800 km to a geostationary orbit of radius 42 000 km. The mass of the Earth is 6.0×10^{24} kg.

What is the magnitude of the change in gravitational potential?

- A. 8.4 J kg^{-1}
B. $2.5 \times 10^4 \text{ J kg}^{-1}$
C. $4.9 \times 10^7 \text{ J kg}^{-1}$
D. $1.5 \times 10^{11} \text{ J kg}^{-1}$

Your answer

Kepler's laws in diagrams

Observations of the planets led to Kepler's three empirical laws:

1. The orbit of a planet is an ellipse with the Sun at one focus.
2. A line joining a planet and the Sun sweeps out equal areas during equal time intervals.
3. The square of the orbital period T is proportional to the cube of the average orbital radius r .

With the help of a labelled diagram, illustrate Kepler's second law for the planets in our Solar System.

[2]

Fig. 22 shows the elliptical orbit of a planet around the Sun.

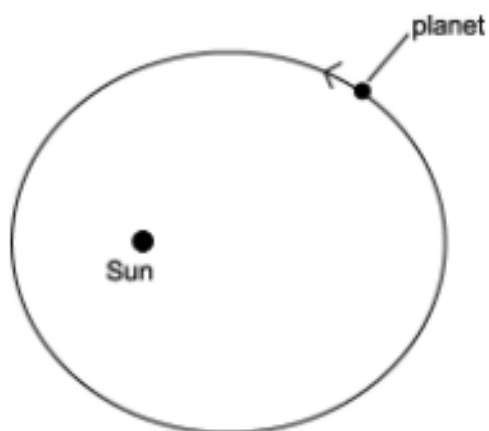


Fig. 22

Draw the gravitational force acting on the planet at the position shown in Fig. 22.

Testing Kepler’s third law

Observations of the planets led to Kepler’s three empirical laws:

- 1. The orbit of a planet is an ellipse with the Sun at one focus.
- 2. A line joining a planet and the Sun sweeps out equal areas during equal time intervals.
- 3. The square of the orbital period T is proportional to the cube of the average orbital radius r .

Three exoplanets orbit the star KIC 11442793. Measurements of average orbital radius r and period T for the exoplanets are shown in the table.

r / AU	T / days
0.0881	8.719
0.520	124.9
0.996	331.6

It is suggested that the relationship between T and r is given by Kepler’s third law:

$$T^2 \propto r^3$$

Propose and carry out a test to check if the relationship is true for the three exoplanets.

Test proposed:

Working:

Conclusion:

[3]

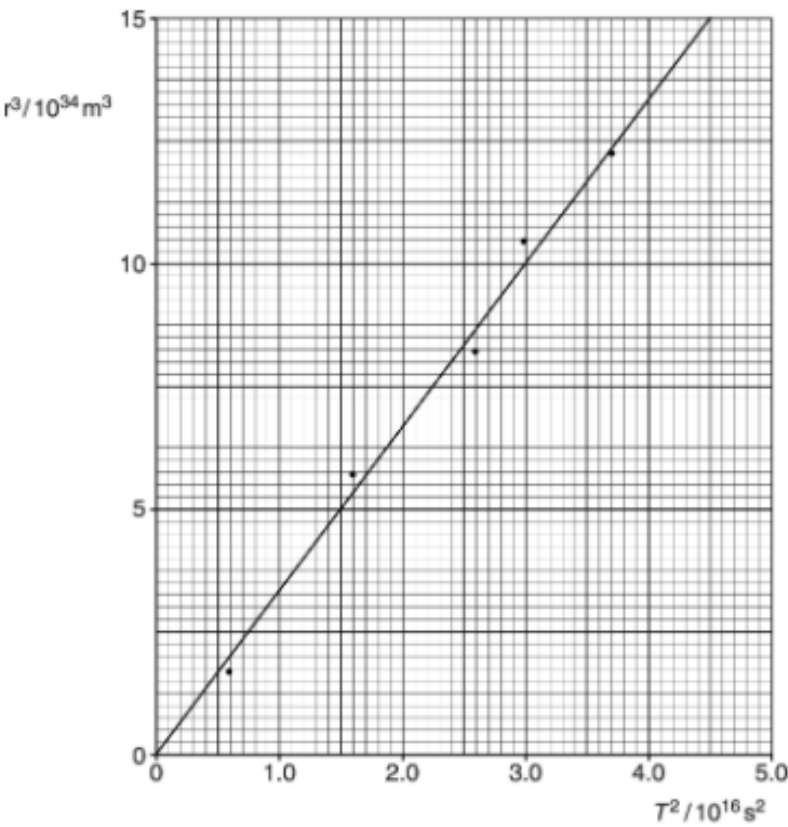
Orbits and graphical data

A planet of mass m moves in a circular orbit of radius r about a star of mass M . The planet has an orbital period T .

Use your knowledge of circular motion and Newton's law of gravitation to derive Kepler's third law.

[4]

The star HD10180 in the constellation Hydrus is notable for its large planetary system. The period T and the mean orbital radius r for HD10180's planets have been deduced from recent observations. Fig. 4.1 has been constructed using these data.



Using orbital data

- i. State what features of Fig. 4.1 support the view that Kepler's third law may be applied to this system.

.....
.....
..... [1]

- ii. Use Fig. 4.1 to determine the mass of the star HD10180.

mass = kg [3]

This question is about Mars and its two moons, Phobos and Deimos.

- i. State Kepler's third law.

.....
..... [1]

- ii. Phobos completes a circular orbit of mean radius 9.4×10^3 km in 7.7 hours. Deimos completes its orbit in 30 hours.
Calculate the mean radius of the orbit of Deimos.

mean radius = km [2]

Helium and escape speed

Model Earth's atmosphere as a thin layer whose height is negligible compared with Earth's radius R . Use $g = 9.81 \text{ N kg}^{-1}$ and the gas constant on page 2.

1. Show that the minimum speed required to escape Earth's gravitational field is $v = \sqrt{2gR}$. Ignore air resistance and further propulsion. [3]

2. Earth's radius is $6.4 \times 10^6 \text{ m}$. Calculate this escape speed. [1]

3. Calculate the temperature in kelvin at which the rms speed of helium atoms equals this escape speed. Use molar mass $0.0040 \text{ kg mol}^{-1}$. [3]

Helium and the speed distribution

iv. Fig. 1 shows the distribution of the speeds of the atoms of an ideal gas.

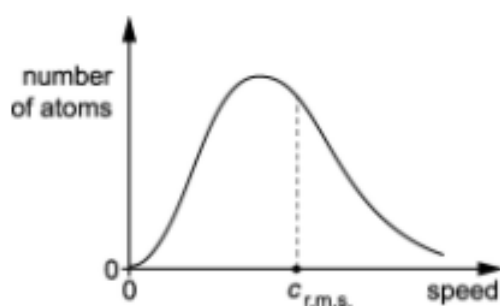


Fig. 1

Use your knowledge of the kinetic theory of gases to describe the shape of this distribution and explain why some helium is able to escape from the Earth.

[4]

v. Over a very long period of time all of the helium should have escaped from the Earth. Suggest why there is still a small amount of helium, about 0.0001%, in the Earth's atmosphere.

[2]

Checking your understanding

Gravitational force and field

Explain the difference between force F and field strength g . Show how their equations connect, and explain the meaning of r .



Circular orbits

Explain why gravity can change a satellite's velocity without changing its speed. Write the equations that connect force, speed, radius and period.

Identify one mistake you made in a calculation and write how you will avoid it next time.

Potential, energy and escape review

Explain why V and GPE are negative when their zero is at infinity. Write their equations and explain the units.

A satellite moves to a higher circular orbit. Explain what happens to its GPE, kinetic energy and total energy.

Derive the escape-speed equation. State the assumptions needed to use it for a launch from Earth's surface.

Additional notes

Definitions, explanations and corrections

Additional working

Calculations, algebra and diagrams

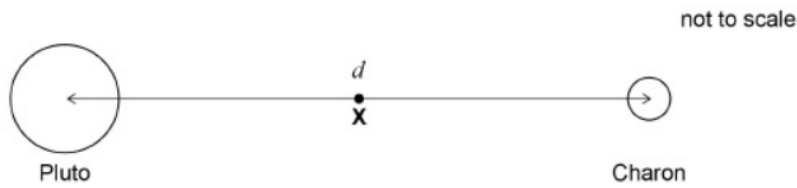
Teacher modelling and notes

Extension questions

Charon is a moon of Pluto that has a mass equal to $\frac{1}{9}$ that of Pluto.

The distance between the centre of Pluto and the centre of Charon is d .

X is the point at which the resultant gravitational field due to Pluto and Charon is zero.



What is the distance of **X** from the centre of Pluto?

- A

$\frac{2}{9}d$
- B

$\frac{2}{3}d$
- C

$\frac{3}{4}d$
- D

$\frac{8}{9}d$

(Total 1 mark)

The distance between the centres of the Earth and the Moon is 3.8×10^8 m. The mass of the Earth is 6.0×10^{24} kg and the mass of the Moon is 7.4×10^{22} kg.

A spacecraft of mass 10×10^3 kg is moving along a line joining their centres.

At what distance from the centre of the Earth would the spacecraft experience no resultant force due to the Earth and the Moon?

- A

3.8×10^7 m
- B

4.8×10^7 m
- C

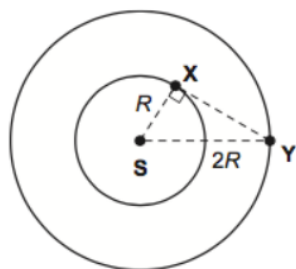
3.4×10^8 m
- D

3.8×10^8 m

(Total 1 mark)

Extension questions

Two planets **X** and **Y** are in concentric circular orbits about a star **S**. The radius of the orbit of **X** is R and the radius of orbit of **Y** is $2R$.



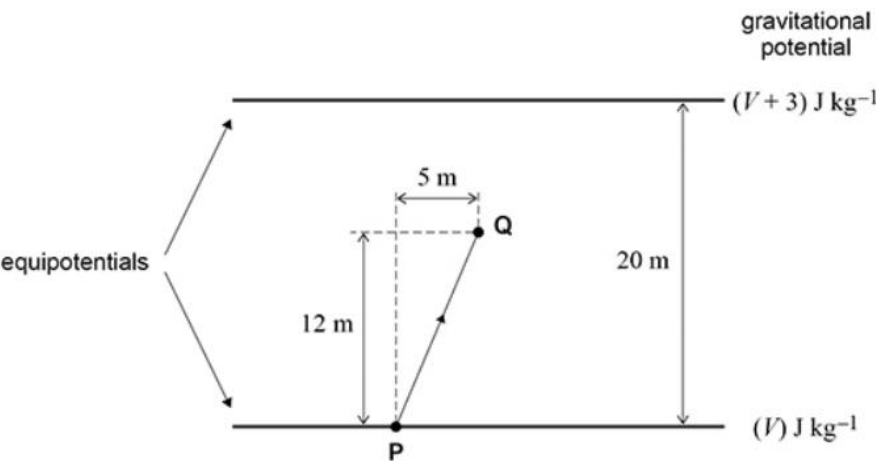
The gravitational force between **X** and **Y** is F when angle **SXY** is 90° , as shown in the diagram.

What is the gravitational force between **X** and **Y** when they are nearest to each other?

- A $2F$
- B $3F$
- C $4F$
- D $5F$

(Total 1 mark)

The diagram shows equipotential lines for a uniform gravitational field. The lines are separated by 20 m.



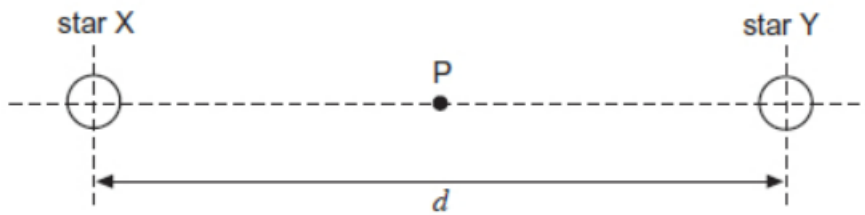
An object of mass 4 kg is moved from **P** to **Q**.

What is the work done against gravity to move the object?

- A 7.2 J ☐
- B 7.8 J ☐
- C 10.2 J ☐
- D 36 J ☐

Extension questions

X and Y are two stars of equal mass M . The distance between their centres is d .



What is the gravitational potential at the mid-point P between them?

- A $\frac{GM}{2d}$
- B $-\frac{GM}{d}$
- C $-\frac{4GM}{d}$
- D $-\frac{8GM}{d}$

A spacecraft of mass 1.0×10^6 kg is in orbit around the Sun at a radius of 1.1×10^{11} m. The spacecraft moves into a new orbit of radius 2.5×10^{11} m around the Sun.

What is the total change in gravitational potential energy of the spacecraft?

- A -6.76×10^{14} J ☐
- B -3.38×10^{14} J ☐
- C 3.38×10^{14} J ☐
- D 6.76×10^{14} J ☐

(Total 1 mark)

Additional working

Calculations, graphs and diagrams

Consolidation 1: force and field

OCR A 5.4.1(a-e) and 5.4.2(a-c)

1. State what produces a gravitational field. Define g , give its SI units and name one other type of force field.

2. Sketch the field outside a spherical planet and the field close to its surface. Add arrows. Explain when the planet can be modelled as a point mass.

3. State Newton's law of gravitation. A 720 kg probe is at radius 8.00×10^6 m from Earth's centre. Use $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ and $M = 6.0 \times 10^{24} \text{ kg}$. Calculate (a) g at the probe and (b) the force on it. State the force's direction.

4. The probe moves to radius $2r$. Find the factors by which g and F change. Explain why g near Earth's surface is nearly uniform and equals free-fall acceleration.

Consolidation 2: potential and energy

OCR A 5.4.4(a-d)

Use $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ and Earth's mass $M = 6.0 \times 10^{24} \text{ kg}$.

1. Define gravitational potential and state its units. Explain why it is negative at a finite distance when its zero is at infinity. Give the equation linking potential to potential energy.

2. A 900 kg spacecraft moves slowly from radius $7.00 \times 10^6 \text{ m}$ to $1.20 \times 10^7 \text{ m}$, measured from Earth's centre. Its kinetic energy change is negligible. Calculate the change in potential and its gain in gravitational potential energy.

3. Sketch F against r (using force magnitude) and V against r for $r \geq R$, where R is Earth's radius. Label axes with units. On the F graph, shade the area between the two radii in question 2. Explain its meaning and the sign of work done against gravity for the outward journey.

Consolidation 3: planetary motion

OCR A 5.4.3(a-d)

1. State Kepler's three laws. Add labelled sketches for the first and second laws. For an elliptical orbit, use the semi-major axis a in the third law.

2. A planet of mass m follows a circular orbit of radius r around a star of mass M . State the force that provides its centripetal acceleration and its direction. Use Newton's law and $v = 2\pi r/T$ to derive $T^2 = (4\pi^2/GM)r^3$. Explain why m cancels.

3. Two small moons orbit the same exoplanet in circular orbits. Moon A has radius 2.00×10^8 m and period 10.0 days. Moon B has radius 3.20×10^8 m. Calculate B's period. Explain why the same period–radius relationship applies beyond the Solar System.

Consolidation 4: satellites and escape

OCR A 5.4.3(e) and 5.4.4(e), using 5.4.4(b,d)

Use $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$, Earth mass $6.0 \times 10^{24} \text{ kg}$ and Earth radius $6.40 \times 10^6 \text{ m}$.

1. State the conditions for a geostationary orbit. Give one use and explain why the satellite's fixed apparent position is helpful.

2. Calculate the orbital radius and altitude of a geostationary satellite. Use Earth's rotation period of $8.616 \times 10^4 \text{ s}$.

3. Define escape speed. Use conservation of energy to derive an expression for the minimum launch speed from a spherical planet of mass M and radius R . State your assumptions, then calculate Earth's surface escape speed.

4. A hot atmosphere contains helium. Explain why some helium can escape even when its rms speed is below the planet's escape speed.



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