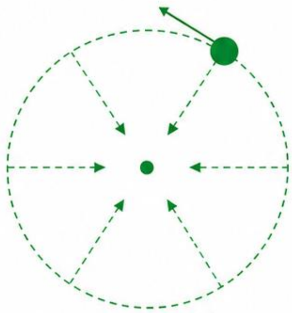
physicsUK

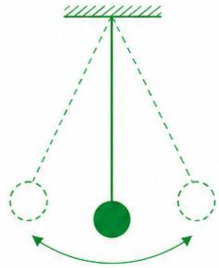
Smarter practice. Stronger understanding

WORKBOOKS

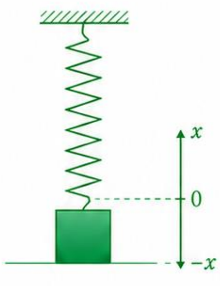
UNDERSTAND | APPLY | PRACTISE | IMPROVE



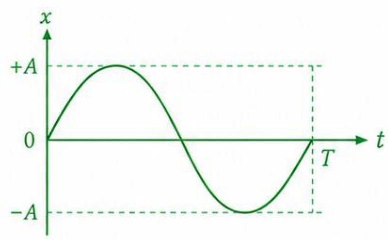
$$a = \frac{v^2}{r}$$

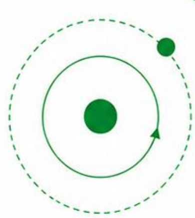


Circular Motion and Oscillations



$$T = 2\pi\sqrt{\frac{m}{k}}$$





OWNER DETAILS

NAME: | GROUP: | TEACHER:

physicsUK

FREE WORKBOOKS. A POWERFUL NEXT STEP.

Turn practice into progress.

These workbooks are free for teachers to share. PhysicsUK gives pupils a useful next step: exam-style questions, clear feedback and a better idea of what to practise next.

ATTEMPT

Answer real AQA and OCR exam-style questions.

UNDERSTAND

See what is correct, missing and what to do next.

IMPROVE

Practise with purpose between lessons.

Scan to try a marked question free

No card needed. Like it? Sign up at PhysicsUK to keep practising.

physicsuk.co.uk

Built by a UK physics teacher and examiner.

5-A-Day

1. Define the term internal energy of a gas.
2. A fixed amount of gas in a container at temperature 30°C has a pressure of 2.1×10^5Pa. The temperature of the gas is increased to 36°C, calculate the new pressure of the gas and state the law implemented.
3. A gas with pressure p, temperature T and volume V is heated so that the temperature becomes $\frac{3T}{2}$ and the volume increased to 2V. Give an expression for the new pressure.
4. A 12V heater is used to bring 1.2kg of water from 26°C to boiling point. What current is required to pass through the heater for this to take place in 1 hour? (c=4200)
5. Calculate the rms speed of a hydrogen atom at 30°C.

5-A-Day

1. Convert 500 degrees into radians.
2. A nucleus travels a circular path of radius 20cm in a period of 60ns. Calculate the angular velocity of the nucleus.
3. Calculate the frequency of the orbit.
4. An engine turns a wheel at a rate of 300 revolutions per minute. Calculate the angular velocity of the wheel.
5. The engine above is set so the output angular velocity remains constant. A generator is attached to the wheel so that each revolution outputs 130J of energy. Calculate the power output of the generator.

5-A-Day

1. Define the term centripetal force.
2. Explain why the centripetal force acting on an object does not cause the object to speed up.
3. A mass travels in a circular path with a frequency of 12Hz and radius of 8cm. Calculate the centripetal acceleration of the oscillator.
4. The mass above is measured to be 40g. Calculate the centripetal force acting on the mass.
5. Two identical strings are used to rotate a mass in a horizontal circle. The first-string snaps when a speed of ‘v’ is used to rotate the mass ‘m’ about a circle of radius ‘r’. The second string uses a mass of 3m. At what speed will the string snap if kept at the same radius?

5-A-Day

1. Define the term simple harmonic motion.
2. Sketch a displacement-time graph for an oscillator which is released from amplitude A with time period T.
3. Label your graph with the point of maximum velocity.
4. Sketch a second graph showing the velocity of the oscillator.
5. Show the points of maximum acceleration on both graphs and give an equation for the maximum acceleration of the oscillator.

5-A-Day

1. Sketch a graph showing the relationship between acceleration and displacement for a simple harmonic oscillator.
2. Sketch a graph showing the changes in energies of a simple harmonic oscillator and label the points of maximum displacement and maximum speed.
3. An oscillator of mass 12g oscillates with a frequency of 250Hz and amplitude 1.8cm. Calculate the maximum speed of the oscillator.
4. Calculate the maximum KE of the oscillator.
5. Describe the position of the oscillator when it has maximum KE.

5-A-Day

1. Define simple harmonic motion.
2. An oscillator with amplitude 3cm oscillates with angular velocity 15rad/s. Calculate the speed of the oscillator when it is 15% the way through a cycle. (Assume oscillator starts from the position of amplitude)
3. Sketch a graph showing the relationship between displacement and time for a simple harmonic oscillator.
4. Sketch a second graph showing how the shape of the graph changes for a lightly damped oscillator.
5. True/False – The time-period for a damped oscillator is greater than for a non-damped oscillator.

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) the radian as a measure of angle

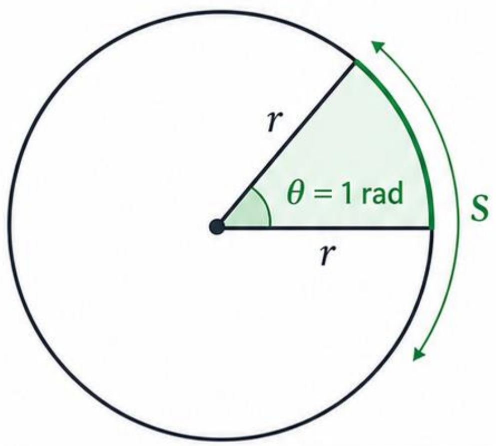
(b) period and frequency of an object in circular motion

(c) angular velocity ω, $\omega = \frac{2\pi}{T}$ or $\omega = 2\pi f$

Radians

Angles are often measured in degrees, but in physics we usually use radians.

- The **radian** is the **SI unit** of plane angle.

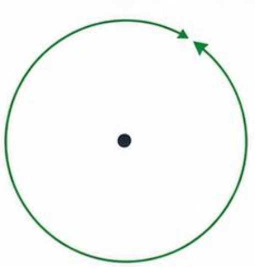


1 radian is the angle subtended when the arc length equals the radius.

For 1 radian: $s = r$

$$\theta = \frac{s}{r}$$

angle in radians = $\frac{\textit{arc length}}{\textit{radius}}$



For one complete revolution, the arc length is the whole circumference:

$$\boxed{s = 2\pi r}$$

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

360° | $360° = 2\pi$ rad

A spinning top completes 8 full revolutions before it comes to rest. Express the angle it moves through in degrees and radians.

Express 720 degrees in radians

Express 90 degrees in radians

Find 45 radians in degrees

By timing how long it takes for an object to complete one revolution, it is possible to calculate the frequency of revolutions.

Frequency, f, is the reciprocal of the period

$$\boldsymbol{f = \frac{1}{T}}$$

The frequency of an oscillator is 50Hz, how long does it take to make 1 oscillation?

Angular velocity

Angular velocity is the rate of change of angular displacement.

DEFINITION

Average angular velocity: $\omega = \dfrac{\Delta\theta}{\Delta t}$

Instantaneous angular velocity: $\omega = \dfrac{d\theta}{dt}$

KEY POINT

Angle θ must be measured in radians.

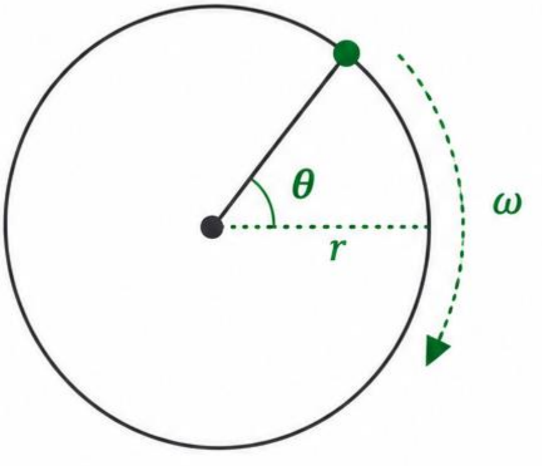
1 revolution = 2π radians

$360° = 2\pi$ radians

Uniform circular motion

For one complete revolution:

- Angular displacement: $\Delta\theta = 2\pi$ rad
- Time taken: $\Delta t = T$ (the period)



Therefore,

$$\boxed{\omega = \frac{2\pi}{T}}$$

YOUR TURN

Use $f = \dfrac{1}{T}$ to derive an expression for the angular velocity ω in terms of the frequency f of oscillations.

....

HINT

Start with

$\omega = \dfrac{2\pi}{T}$ and

$f = \dfrac{1}{T}$

Then substitute.

Angular velocity
Practice

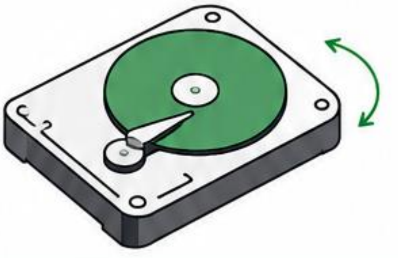
REMEMBER:
Angular velocity is the rate of change of angle.

$\omega = \dfrac{\Delta\theta}{\Delta t}$ For one full revolution: $\omega = \dfrac{2\pi}{T}$ $f = \dfrac{1}{T}$ $\omega = 2\pi f$

1 **A hard disk takes 8.33 ms to complete one full revolution. Calculate:**

(a) its angular velocity in rad s^{-1}

(b) the number of revolutions completed per second.



2 **A hard disk drive takes 0.30 s to complete one revolution. Calculate:**

(a) its angular velocity in rad s^{-1}

(b) the number of revolutions completed per second.

3 **A spinning top completes 30 revolutions per second. Calculate:**

(a) the period of rotation

(b) its angular velocity in rad s^{-1}

4 **A hard disk rotates at 4500 rpm. Calculate:**

(a) its angular velocity in rad s^{-1}

(b) the time taken to complete 50 revolutions.

5 **An analogue clock has a second hand, minute hand and hour hand. Assuming each moves at a constant rate, calculate the magnitude of the angular velocity of each hand in rad s^{-1}.**

USEFUL FORMULAS

$\omega = \dfrac{\Delta\theta}{\Delta t}$	$\omega = \dfrac{2\pi}{T}$	$f = \dfrac{1}{T}$	$\omega = 2\pi f$
average angular velocity	uniform circular motion	frequency	angular velocity in terms of frequency

Centripetal acceleration and forces

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) a constant net force perpendicular to the velocity of an object causes it to travel in a circular path

(b) constant speed in a circle; $v = \omega r$

(c) centripetal acceleration; $a = \dfrac{v^2}{r}$; $a = \omega^2 r$

(d) (i) centripetal force; $F = \dfrac{mv^2}{r}$; $F = m\omega^2 r$

(ii) techniques and procedures used to investigate circular motion using a whirling bung.

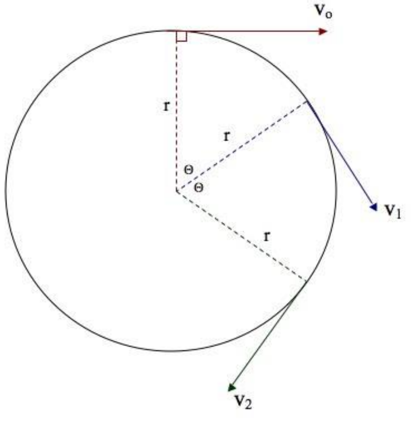
State Newton's 3 laws of motion

1.

2.

3.

Centripetal acceleration



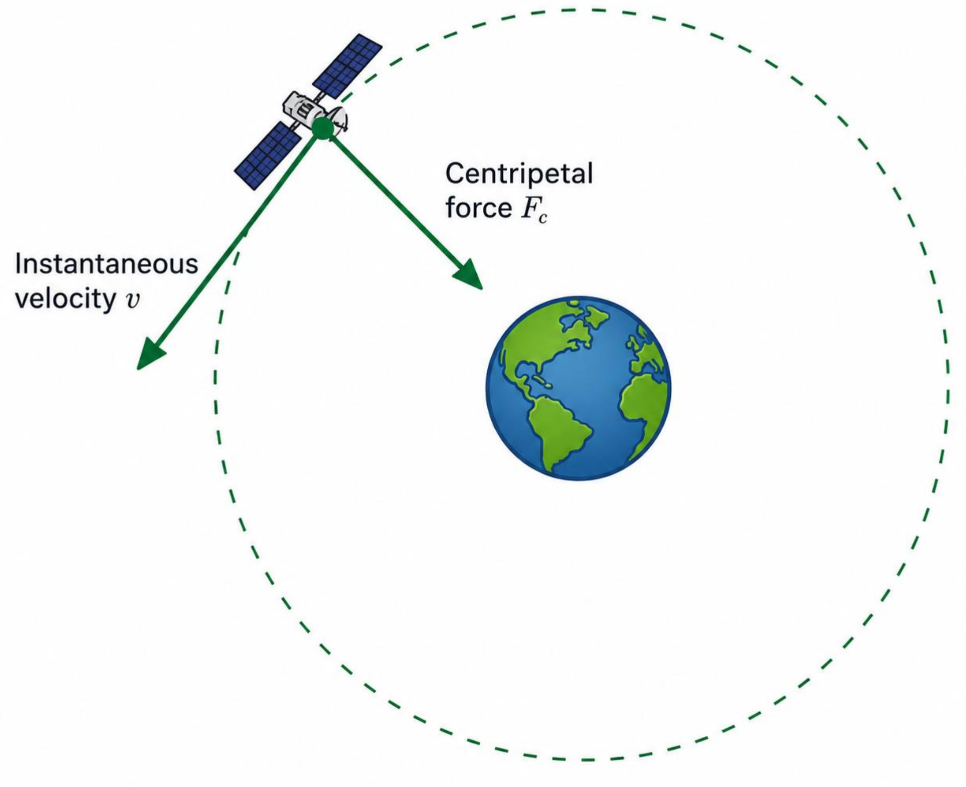
The direction of the velocities in the image to the left is changing.

Define the term acceleration.

Give an equation for centripetal acceleration.

If there is an acceleration present, there must be a force.

Any **<u>NET</u>** force that keeps a body moving with uniform acceleration in a circular path is called a centripetal force.



Centripetal force is always inwards keeping an object in a circular path. It is always **perpendicular** to the velocity (no work is done)

<u>This is the resultant force</u>.

1. Which best describes the term centripetal acceleration?
    A. A changing velocity per unit time.
    B. Acceleration perpendicular to the direction of motion.
    C. An acceleration keeping an object moving in a circular path.
    D. Acceleration caused by a force opposing the motion of an object.

2. Which of the following statements is incorrect?
    A. Work is done on an object causing it to change direction and keep a circular path.
    B. The speed of an object which experiences a centripetal force remains constant.
    C. The centripetal force is directed perpendicular to the direction of motion.
    D. A centripetal force is the resultant force experienced by an object moving in a circular path.

3. A mass of 320g on a string is being rotated in the air tracing out a horizontal circle of radius 40cm. The maximum tension the string can experience before breaking is 22N. Calculate the time period of the swing which causes the string to snap.
    A. 0.90s
    B. 5.2s
    C. 0.48s
    D. 0.23s

Centripetal acceleration and force Practice

FORMULAS REMINDER

$$a_c = \frac{v^2}{r} \qquad v = \frac{2\pi r}{T} \qquad F_c = ma_c$$

1

Calculate the centripetal acceleration of a person standing on the equator as the Earth rotates. The radius of the Earth is 6370 km.

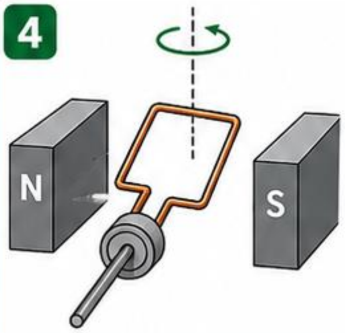
2

A car travels in a circle of radius 10 m and completes a full circle in 15 s. Calculate the centripetal acceleration of the car. If the car has a mass of 1000 kg, calculate the resultant centripetal force acting on the car.

3

A man swings a toy cat in a horizontal circle of radius 0.70 m. The toy cat takes 2.0 s to complete one full cycle. Find the centripetal acceleration of the toy cat. If the toy cat has a mass of 12 kg, calculate the centripetal force acting on it.

4



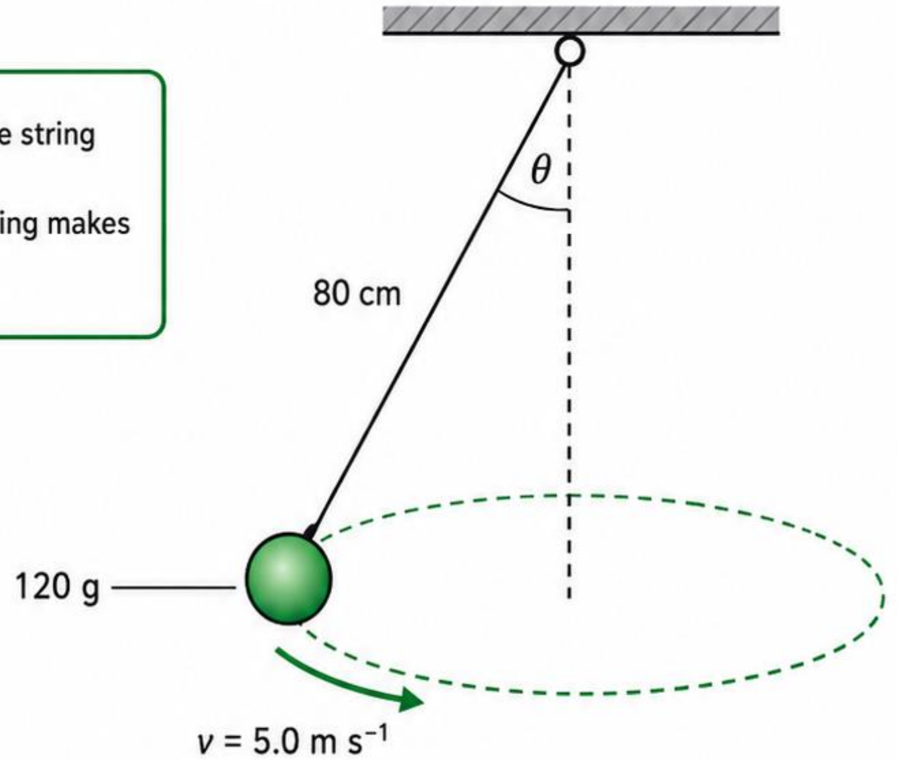
A dynamo coil rotates with a frequency of 80 Hz. A 0.20 kg section of the coil is 4.0 cm from the axis of rotation. Calculate the centripetal force acting on this section of the coil.

Conical pendulum

A 120 g mass is attached to a string of length 80 cm and moves as a conical pendulum. The mass travels at a speed of 5.0 m s^{-1}.

Calculate:

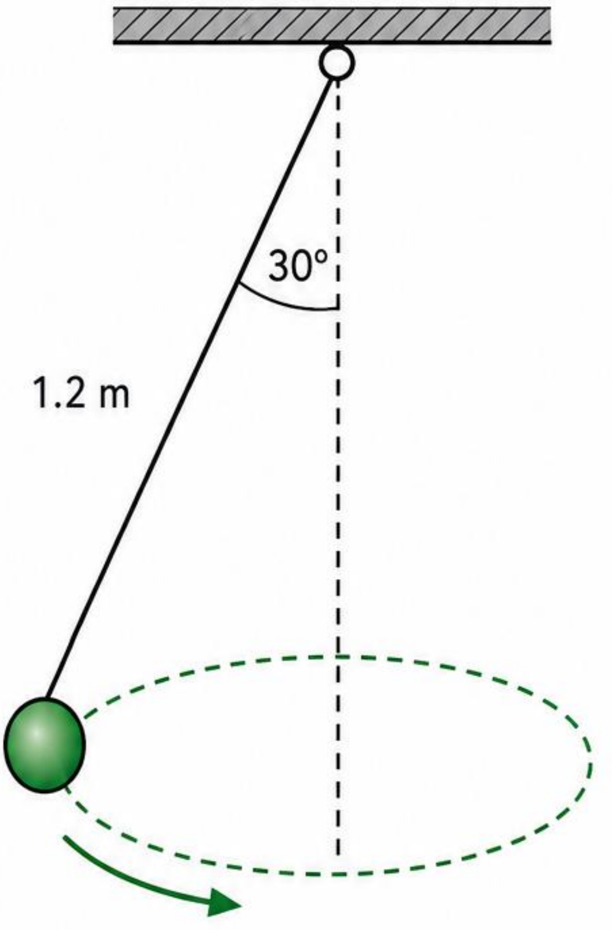
1. the tension in the string
2. the angle the string makes with the vertical



Your working

Conical pendulum MCQs

A conical pendulum consists of a 0.50 kg mass attached to a string of length 1.2 m. The string makes an angle of 30° to the vertical.



1

The tension in the string is:

A 0.67 N

B 11 N

C 5.7 N

D 3.2 N

2

The mass completes one revolution in 2.0 s.

The speed of the mass is:

A $0.60\ \text{m s}^{-1}$

B $1.9\ \text{m s}^{-1}$

C $5.9\ \text{m s}^{-1}$

D $1.3\ \text{m s}^{-1}$

HELPFUL REMINDERS

- $T\cos\theta = mg$
- $r = L\sin\theta$
- $v = \dfrac{2\pi r}{T}$

Centripetal force

A simple apparatus can be used to investigate uniform circular motion.

A small rubber bung is attached to one end of a glass tube of length L.

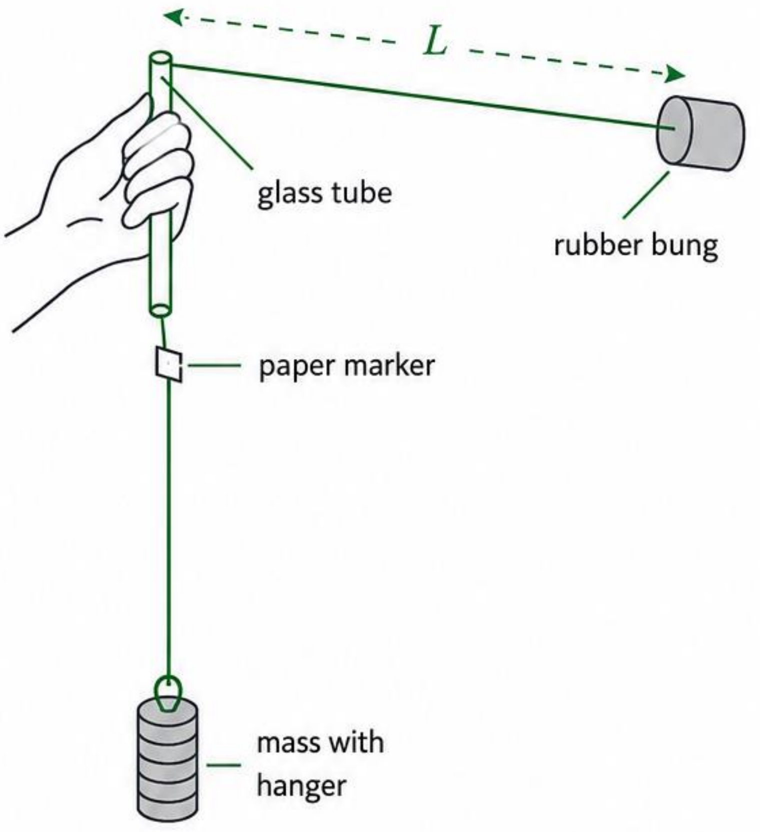
The other end of the tube is held in a hand.

A string passes through the tube and is tied to the bung.

A small paper marker is attached to the string below the tube.

A mass with a hanger is attached to the lower end of the string.

When the system is swung in a horizontal circle, the mass provides the centripetal force for the circular motion.



1 The mass with hanger has a mass of $m = 50\,\mathrm{g}$.
The length of the glass tube is $L = 0.80\,\mathrm{m}$.
The apparatus is swung in a horizontal circle so that the rubber bung moves in a circle of radius $r = 0.60\,\mathrm{m}$ at a constant speed.
Calculate the tension in the string, T.

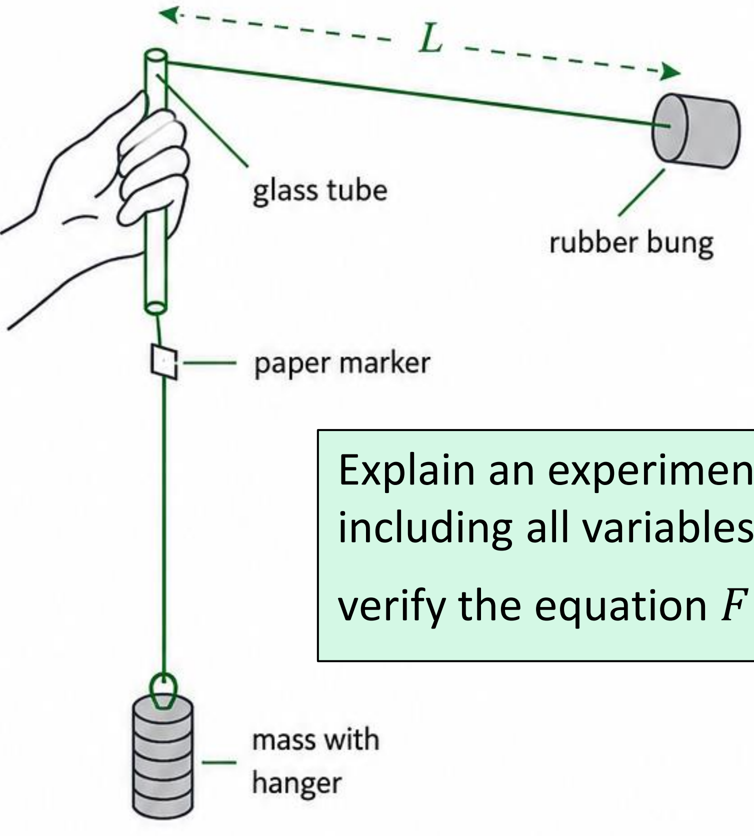
A 0.25 N **B** 0.49 N **C** 0.82 N **D** 1.6 N

2 The mass with hanger in Question 1 is swung in a horizontal circle.
The rubber bung completes one revolution every 1.50 s.
Calculate the speed of the rubber bung.

A 0.25 m s^{-1} **B** 0.63 m s^{-1} **C** 2.5 m s^{-1} **D** 4.0 m s^{-1}

3 In an investigation, the radius of the circle is kept constant but the mass with hanger is increased.
What happens to the centripetal force and the speed of the rubber bung?

	Centripetal force	Speed
A	increases	increases
B	increases	remains the same
C	remains the same	increases
D	remains the same	remains the same



Explain an experiment that could be conducted, including all variables and how they're measured to verify the equation $F = \frac{mv^2}{r}$

....

....

....

....

....

....

....

....

....

....

....

Which of the following are the appropriate units for angular velocity, ω?

A s^{-1}

B $m\ s^{-1}$

C $kg\ m\ s^{-1}$

D $rad\ m^{-1}\ s^{-1}$

If the Sun takes 250 million years to complete one orbit around the centre of the milky way, what is the angular velocity of the Sun?

A 8×10^{-16} rad s^{-1}

B 3×10^{-13} rad s^{-1}

C 3×10^{-12} rad s^{-1}

D 5×10^{16} rad s^{-1}

What is the centripetal acceleration of a bike travelling at a speed of 12 m s^{-1} around a track of radius 50m?

A 0.24 m s^{-2}

B 2.9 m s^{-2}

C 4.5 m s^{-2}

D 7200 m s^{-2}

OSCILLATIONS

5.3 Oscillations

Oscillatory motion is all around us, with examples including atoms vibrating in a solid, a bridge swaying in the wind, the motion of pistons of a car and the motion of tides. (HSW1, 2, 3, 5, 6, 8, 9, 10, 12)

This section provides knowledge and understanding of simple harmonic motion, forced oscillations and resonance.

5.3.1 Simple harmonic oscillations

	Learning outcomes	Additional guidance
	Learners should be able to demonstrate and apply their knowledge and understanding of:	
(a)	displacement, amplitude, period, frequency, angular frequency and phase difference	*M4.7* HSW8
(b)	angular frequency ω; $\omega = \frac{2\pi}{T}$ or $\omega = 2\pi f$	
(c)	**(i)** simple harmonic motion; defining equation $a = -\omega^2 x$	HSW5
	(ii) techniques and procedures used to determine the period/frequency of simple harmonic oscillations	**PAG10** e.g. mass on a spring, pendulum
(d)	solutions to the equation $a = -\omega^2 x$ e.g. $x = A\cos\omega t$ or $x = A\sin\omega t$	*M3.9, M3.12*
(e)	velocity $v = \pm\omega\sqrt{A^2 - x^2}$ hence $v_{max} = \omega A$	*M2.2*
(f)	the period of a simple harmonic oscillator is independent of its amplitude (isochronous oscillator)	
(g)	graphical methods to relate the changes in displacement, velocity and acceleration during simple harmonic motion.	HSW1

Simple harmonic motion

Use the revision notes and the SHM Explorer simulation before answering the questions.

Revision notes

physicsuk.co.uk/ocr-a-level-physics/module-5/simple-harmonic-motion

SHM Explorer simulation

physicsuk.co.uk/tools/simple-harmonic-motion-simulator

Helpful equations

$a = -\omega^2 x$ $\quad T = \dfrac{1}{f}$ $\quad \omega = 2\pi f$ $\quad \omega = \dfrac{2\pi}{T}$

1. Definition

Complete the sentence: In simple harmonic motion, the acceleration is ________________ to the displacement from equilibrium and is always directed _____________ the equilibrium position.

In SHM, acceleration is ...

____

____

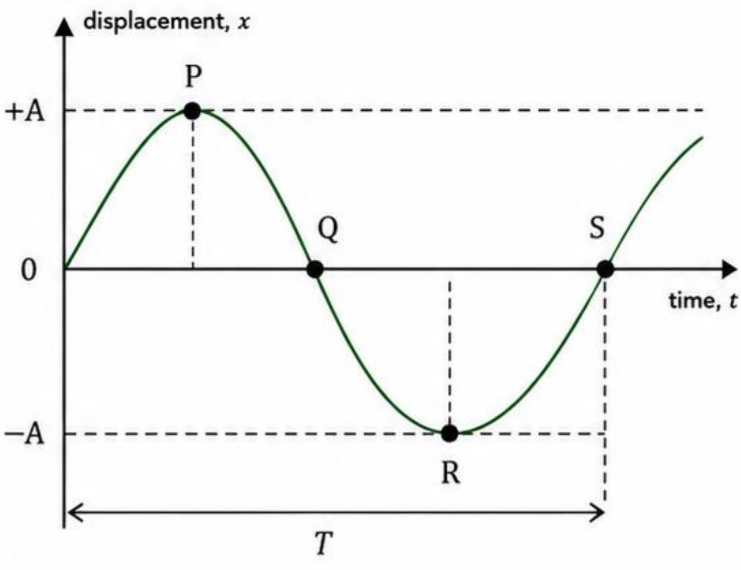
____

2. Key features

State what is meant by the following terms:

equilibrium position	
amplitude	
period	
frequency	

3. Using the graph



(a) What is the amplitude of the motion?

____

(b) What is the period of the motion?

____

(c) At which points is the speed greatest?

____

(d) At which points is the magnitude of the acceleration greatest?

____

(e) At which points is the acceleration zero?

____

Helpful equations

$a = -\omega^2 x$ $\quad T = \frac{1}{f}$ $\quad \omega = 2\pi f$ $\quad \omega = \frac{2\pi}{T}$ $\quad a_{\max} = \omega^2 A$

4. Calculations

4a. A particle in SHM has a period of 0.40 s.
Calculate: (i) the frequency, (ii) the angular frequency.

(i)

(ii)

4b. The amplitude of an oscillation is 0.12 m and the angular frequency is 8.0 rad s^{-1}.
Calculate the maximum acceleration.

Use $a_{max} = \omega^2 A$.

5. Energy in SHM

Complete the table using the words **maximum** or **zero**.

Position	Kinetic energy	Potential energy
equilibrium position		
maximum displacement		

Explain why the total energy stays constant in ideal SHM.

6. Challenge

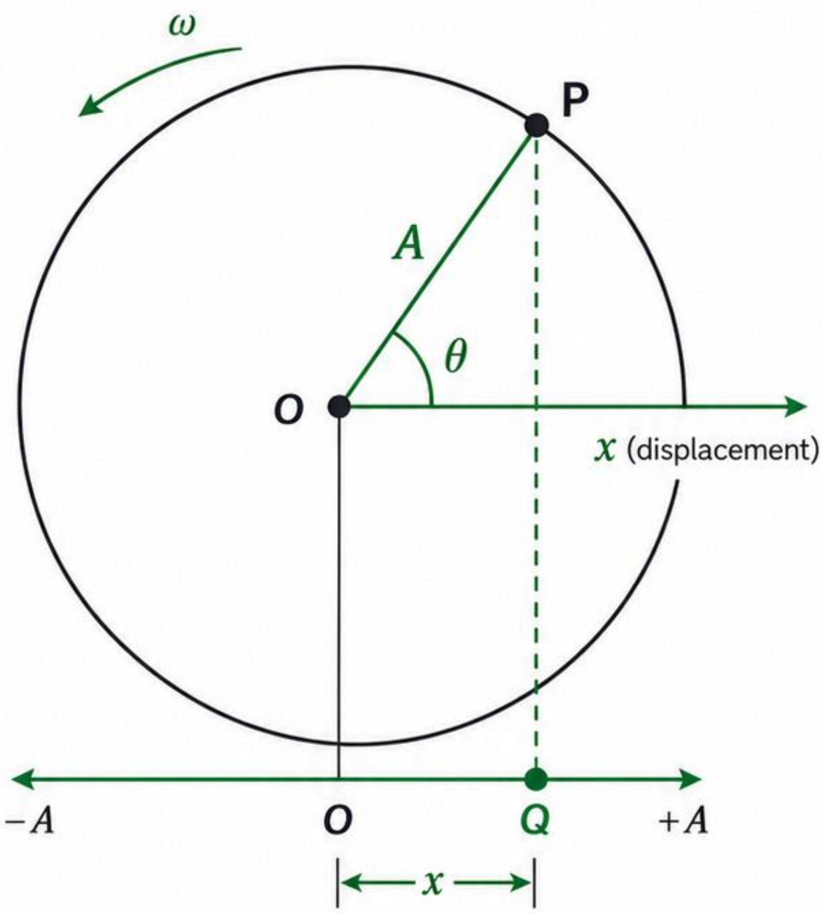
A mass–spring system oscillates with frequency 2.5 Hz.
Calculate the period and angular frequency.

From circular motion to SHM

Teacher demo: deriving displacement for simple harmonic motion

Simple harmonic motion (SHM) can be obtained from the horizontal projection of uniform circular motion.

1. Consider a point P moving with constant angular speed ω in a circle of radius A.
2. The horizontal projection of P performs simple harmonic motion.
3. From the geometry of the circle:
   $x = A\cos\theta$
4. For uniform circular motion:
   $\theta = \omega t$
5. Substitute to obtain:
   $x = A\cos(\omega t)$



Displacement equation for SHM: $\boxed{x = A\cos(\omega t)}$

Symbols

A = amplitude

ω = angular frequency

t = time

x = displacement from equilibrium

This is one valid form of the SHM displacement equation.
A sine form, $x = A\sin(\omega t)$ can also be used depending on the starting position.

Deriving velocity and acceleration in SHM

Teacher demo: differentiating the displacement equation

Starting from the displacement equation for SHM: $$x = A\cos(\omega t)$$

1 Differentiate once to find velocity

$$v = \frac{dx}{dt}$$

$$v = \frac{d}{dt}[A\cos(\omega t)]$$

$$v = -A\omega\sin(\omega t)$$

The speed is greatest when $|\sin(\omega t)| = 1$.

Maximum speed:

$$v_{\max} = A\omega$$

2 Differentiate again to find acceleration

$$a = \frac{dv}{dt}$$

$$a = \frac{d}{dt}[-A\omega\sin(\omega t)]$$

$$a = -A\omega^2\cos(\omega t)$$

Since $x = A\cos(\omega t)$, we can write

$$a = -\omega^2 x$$

The negative sign shows that the acceleration is always directed towards the equilibrium position.

Maximum acceleration:

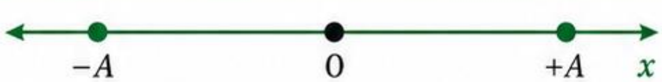
$$a_{\max} = \omega^2 A$$

Key SHM identity:

$$a = -\omega^2 x$$

Summary

Displacement	$x = A\cos(\omega t)$
Velocity	$v = -A\omega\sin(\omega t)$, maximum speed $v_{\max} = A\omega$
Acceleration	$a = -\omega^2 x$, maximum acceleration $a_{\max} = \omega^2 A$



At $x = -A$ or $+A$:
$|x|$ is maximum.
$|a|$ is maximum.
$v = 0$.

At $x = 0$ (equilibrium):
v is maximum.
$a = 0$.

At equilibrium ($x = 0$), speed is maximum.

At maximum displacement ($x = \pm A$), acceleration magnitude is maximum.

1. Which of the following best describes simple harmonic motion?
   A. Motion where the acceleration is inversely proportional to the velocity of an object.
   B. Motion where the velocity is directly proportional to the time period of the object.
   C. Motion where the acceleration is directly proportional to the displacement in the opposing direction.
   D. Motion where the displacement is inversely proportional to the acceleration of the object.

2. A pendulum is released from its maximum displaced position and left to oscillate freely. Which equation best describes the displacement of the oscillator after 2s?
   A. $x = Asin(2t)$
   B. $x = Acos(2t)$
   C. $x = Asin(2\omega)$
   D. $x = Acos(2\omega)$

3. Sketch a graph showing the relationship between acceleration and displacement for a simple harmonic oscillator.

SHM equations and acceleration

1 (ii) The displacement x, in mm, at a time t seconds after release is given by

$$x = 12\cos(7.85\,t)$$

Use this equation to show that the frequency of oscillation is 1.25 Hz.

USEFUL EQUATIONS

$$\omega = 2\pi f$$

$$v = -A\omega\sin(\omega t)$$

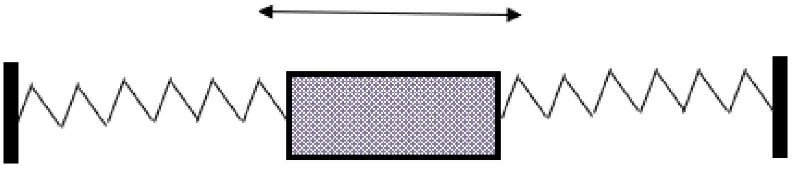
$$a = -\omega^2 x$$

$$a_{max} = \omega^2 A$$

2 **Grade B** A simple pendulum oscillates with SHM at a frequency of 0.60 Hz. Calculate the velocity of the pendulum bob 3.0 s after it is released from a maximum displacement of 6.0 cm.

3 **Grade A** State the relationship between acceleration and displacement during SHM, and calculate the maximum acceleration for the pendulum above.

A mass M oscillates in simple harmonic motion between two fixed supports. Frictional effects can be ignored. The maximum acceleration of the mass is a_1.



The mass is replaced with a mass of 4 M **and** the amplitude of the oscillation is doubled. The maximum acceleration of the mass in the revised system is a_2.

Which is the correct statement?

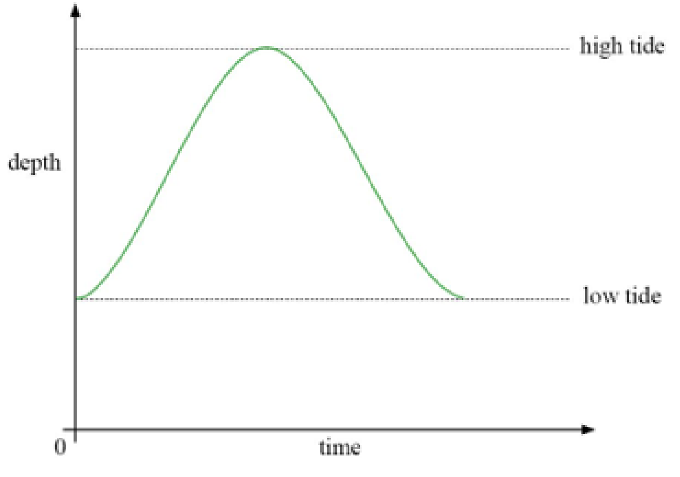
A $a_2 = 4a_1$

B $a_2 = 2a_1$

C $a_2 = a_1$

D $a_2 = ½\, a_1$

The rise and fall of water in a harbour is simple harmonic. The depth varies between 1.0 m at low tide and 3.0 m at high tide. The time between successive low tides is twelve hours.



A boat which requires a minimum water depth of 1.5 m approaches the harbour at low tide.

How long must it wait before being able to enter the harbour.

A 1.0 hour

B 1.5 hours

C 2.0 hours

D 3.0 hours

The motion of an oscillator is simple harmonic.

Which statement is correct about the period of the oscillator?

The period ...

A. is independent of the amplitude.

B. depends on the displacement of the oscillator.

C. is independent of the frequency of the oscillator.

D. depends on the force acting on the oscillator.

Your answer

Fig. 3.1 shows a metal plate attached to the end of a spiral spring. The end **A** of the spring is fixed to a rigid clamp. The plate is pulled down by a small amount and released. The plate performs simple harmonic motion in a vertical plane at a natural frequency of 8 Hz and the spring remains in tension at all times.

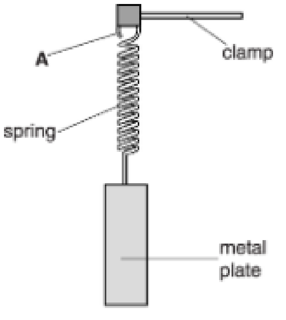


Fig. 3.1

i. On Fig. 3.2 sketch an acceleration a against displacement x graph for the motion of the metal plate. You are not required to give values on the axes.

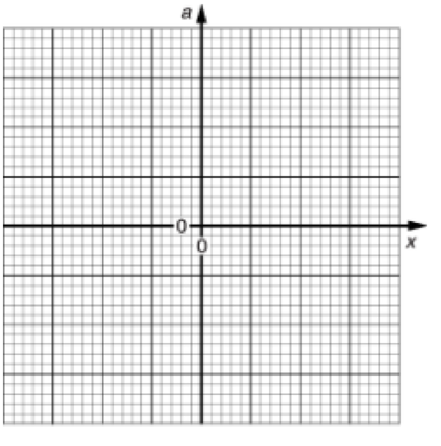


Fig. 3.2

[2]

ii. Explain how your graph could be used to determine the frequency of oscillation of the metal plate.

--- **[2]**

Fig. 3.1 shows a simple pendulum consisting of a steel sphere suspended by a light string from a rigid support. The sphere is displaced 50 mm from its vertical equilibrium position and released at time $t = 0$.

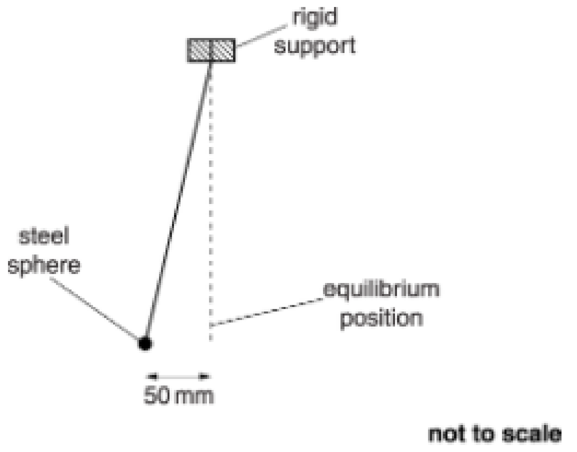


Fig. 3.1

Fig. 3.2 shows the graph of displacement x of the sphere against time t.

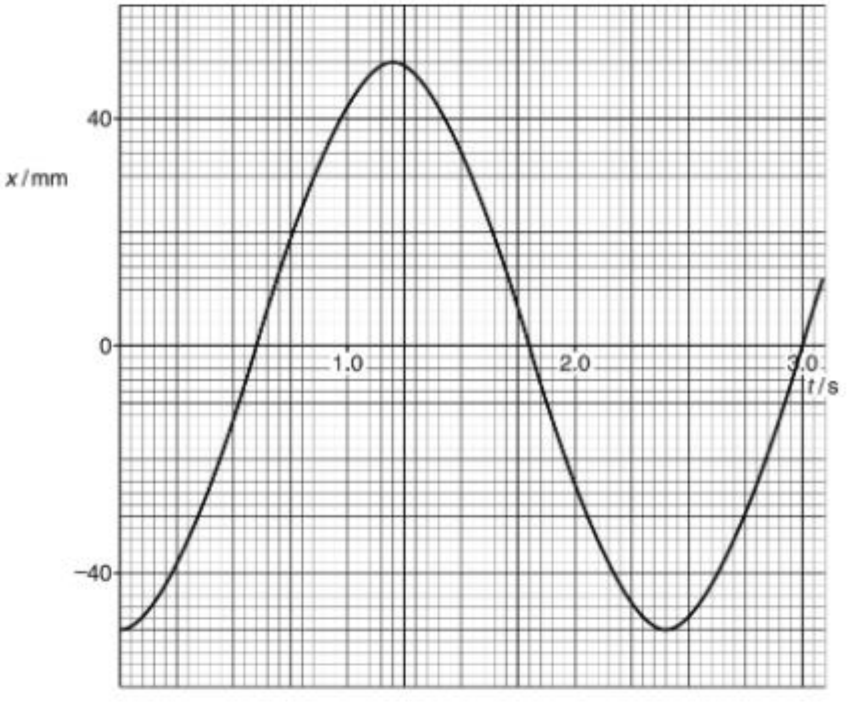


Fig. 3.2

i. Use Fig. 3.2 to determine the frequency of oscillation of the pendulum.

frequency = ________________________________ Hz **[1]**

ii. Use Fig. 3.2, or otherwise, to determine the maximum speed of the sphere.
Show your method clearly.

speed = ________________________________ m s^{-1} **[2]**

5.3.2 Energy of a simple harmonic oscillator

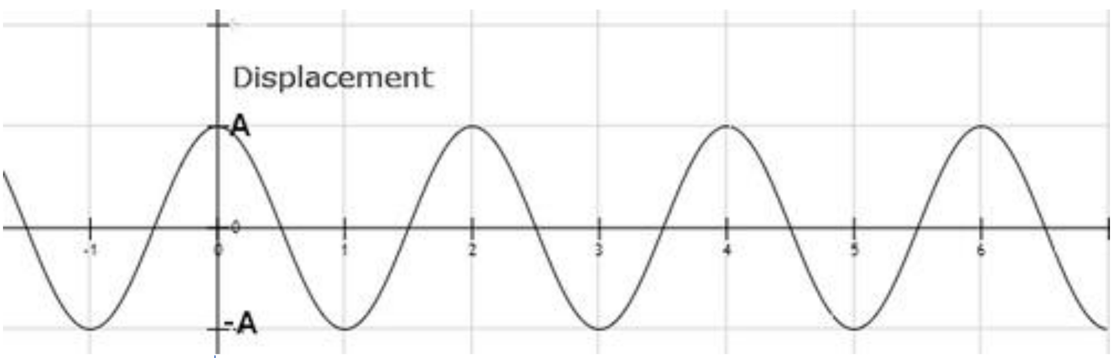
Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) interchange between kinetic and potential energy during simple harmonic motion

(b) energy-displacement graphs for a simple harmonic oscillator

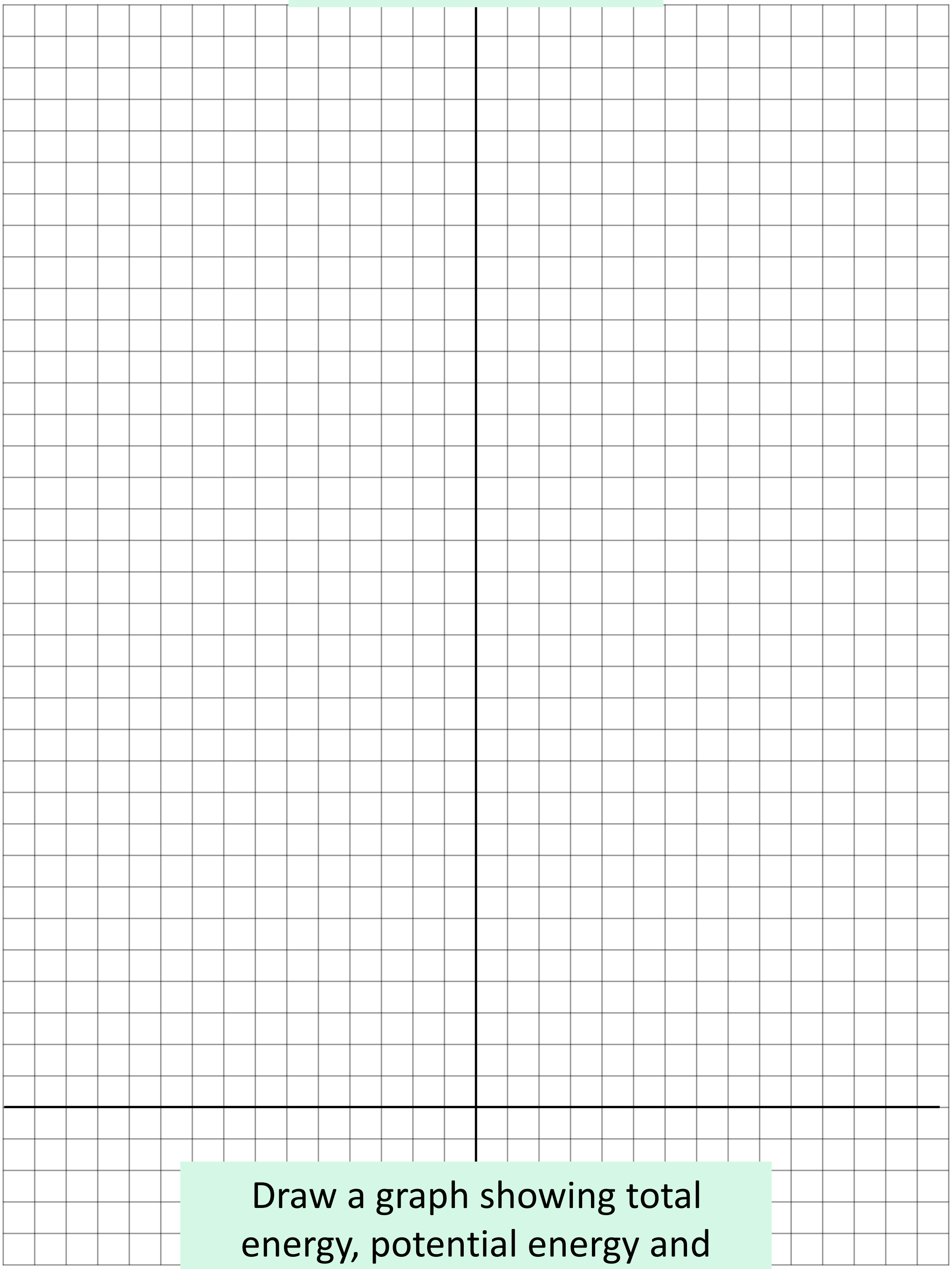
On the chart below, draw corresponding graphs for velocity and acceleration. Pay attention to +/- and the values on y axis.



https://phet.colorado.edu/sims/html/pendulum-lab/latest/pendulum-lab_all.html

The energy of any system is due to kinetic and potential energies.
The total energy of a system undergoing simple harmonic motion must remain constant(with no damping effects)

Energy of a simple harmonic oscillator



Draw a graph showing total energy, potential energy and kinetic energy

Find ‘T’ from the following; $x = Acos(5t)$

A) 2.6s
B) 1.3s
C) 0.80s
D) 1.6s

Knowing that ‘G’ is the acceleration of a bob on a pendulum, find an expression for the frequency ‘f’ given the equation

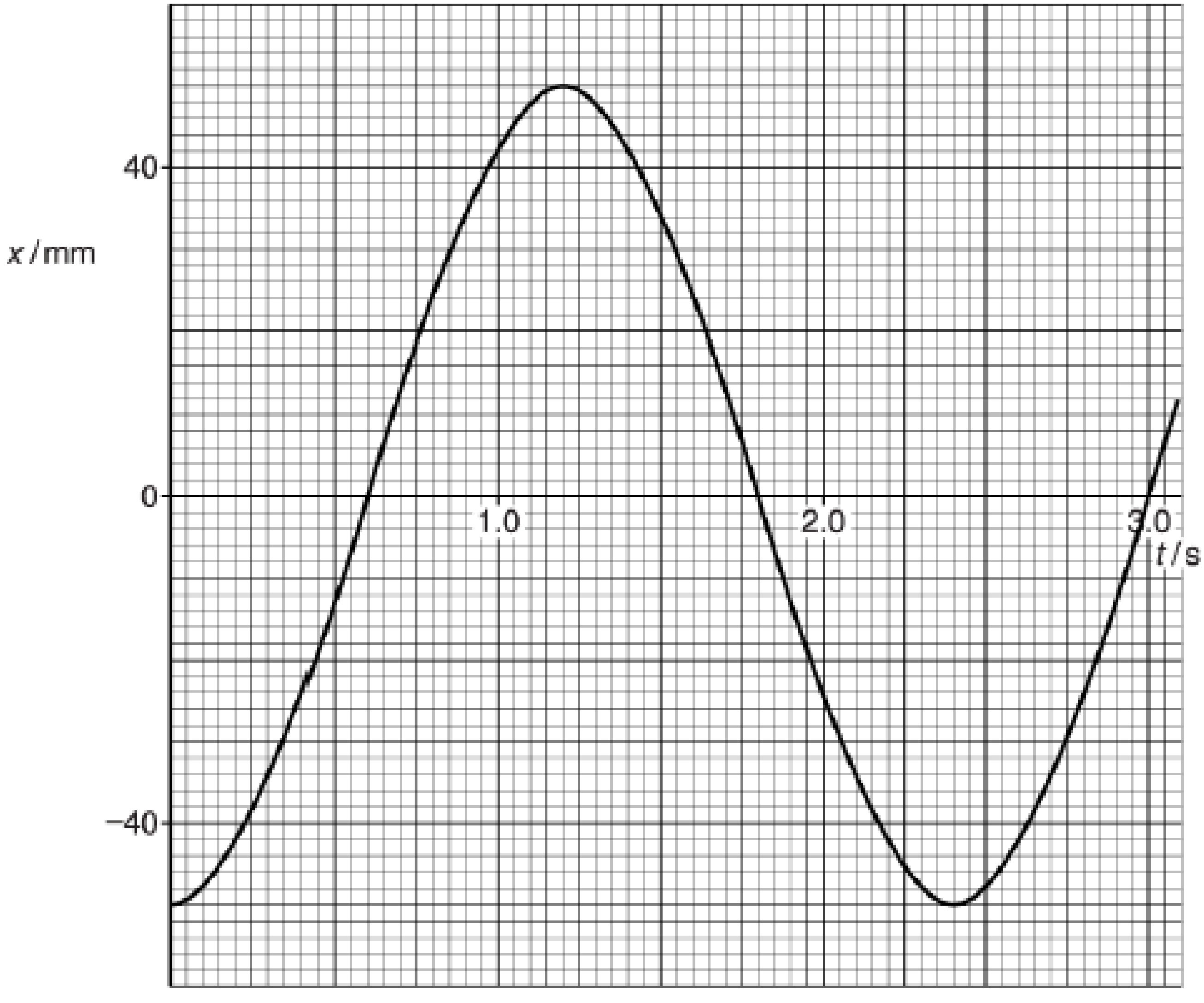
$$G = Q^2 A$$

A) $\frac{\pi}{2}$

B) $\sqrt{\frac{G}{4\pi^2 A}}$

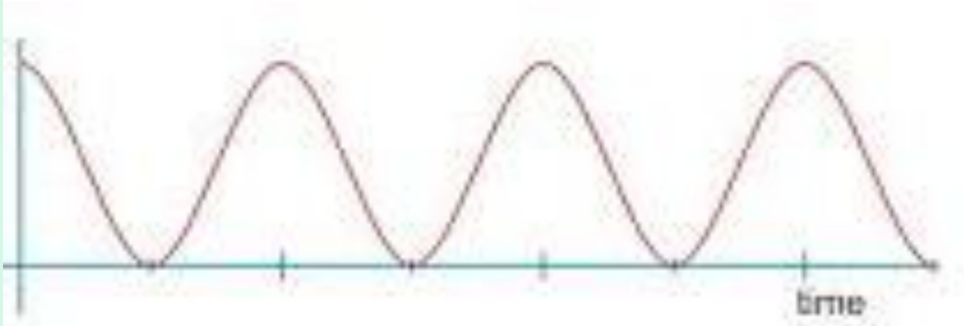
C) $\sqrt{\frac{G}{2\pi A}}$

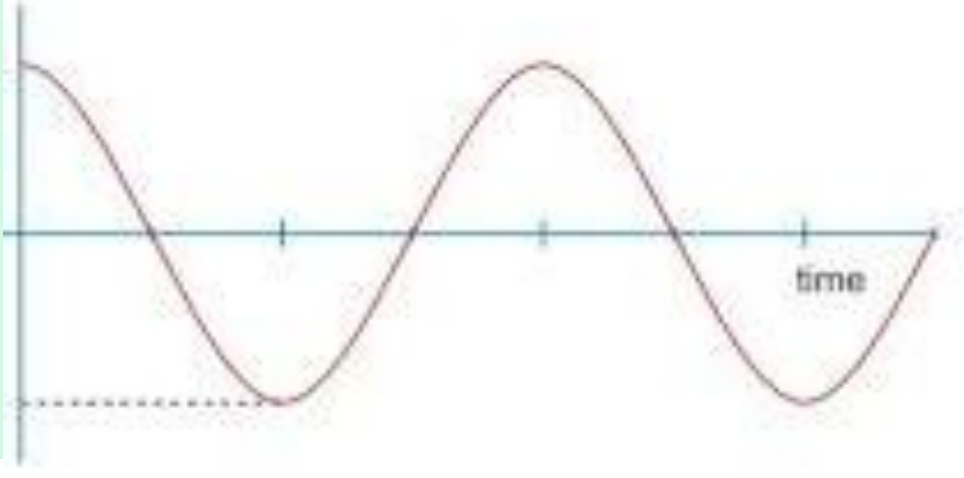
D) $G/2\pi$



Calculate the following
Angular velocity
Maximum velocity
Maximum acceleration

Which graph represents the KE of a harmonic oscillator?





Consolidation

Simple harmonic motion

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

- **a** displacement, amplitude, period, frequency, angular frequency and phase difference
- **b** angular frequency ω, where $\omega = \frac{2\pi}{T}$ or $\omega = 2\pi f$
- **c** (i) simple harmonic motion; defining equation $a = -\omega^2 x$
  (ii) techniques and procedures used to determine the period/frequency of simple harmonic oscillations
- **d** solutions to the equation $a = -\omega^2 x$
  e.g. $x = A\cos\omega t$
  or $x = A\sin\omega t$
- **e** velocity $v = \pm\omega\sqrt{A^2 - x^2}$
  hence $v_{max} = \omega A$
- **f** the period of a simple harmonic oscillator is independent of its amplitude (isochronous oscillator)
- **g** graphical methods to relate the changes in displacement, velocity and acceleration during simple harmonic motion.

Resources

PhysicsUK revision notes

physicsuk.co.uk/ocr-a-level-physics/module-5/simple-harmonic-motion

PhysicsUK SHM Explorer (simulation)

physicsuk.co.uk/tools/simple-harmonic-motion-simulator

1 A pendulum is displaced by 4 cm and left to swing freely. The pendulum completes 10 cycles in 8 s.

a) Calculate the period (T) and frequency (f) of the oscillation.

b) Calculate the angular frequency (ω) of the oscillation.

2 Draw three graphs on the same set of axes to show how the displacement x, velocity v and acceleration a of the oscillator vary with time.
Label the axes and indicate the amplitude.

3 Calculate the maximum velocity of the pendulum.

4 Calculate the maximum acceleration of the pendulum.

5 Calculate the displacement of the oscillator after 3.0 s.

6 Calculate the velocity of the oscillator at $t = 3.0$ s.

7 The bob of the pendulum has a mass of 40 g.

a) Calculate the maximum kinetic energy (KE) of the bob.

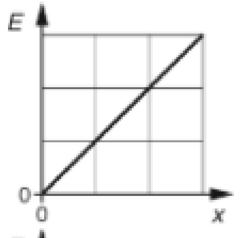
b) On separate axes, sketch a graph to show how the mechanical energy, kinetic energy and potential energy vary with time during the oscillation.

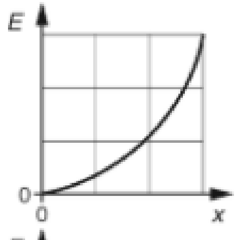
Extension

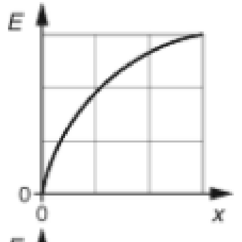
Explain where the greatest energy transfer takes place during each oscillation and what happens to the energy transfer over time.

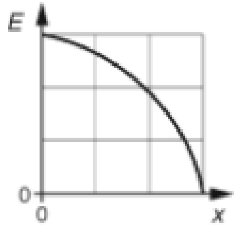
An object oscillates with simple harmonic motion.

Which graph **best** shows the variation of its potential energy *E* with distance *x* from the equilibrium position?

A


B


C


D


In which of the following lists are all three quantities constant when a particle moves in undamped simple harmonic motion?

A	acceleration	force	total energy
B	amplitude	force	kinetic energy
C	acceleration	period	kinetic energy
D	amplitude	period	total energy

A mass hanging from a vertical spring is pulled down.
It is then released from rest at time $t = 0$.
The mass oscillates vertically in a **vacuum** with simple harmonic motion about the equilibrium position. The spring is in tension at all times.

Fig. 18.1 shows the position of the mass at $t = 0$.

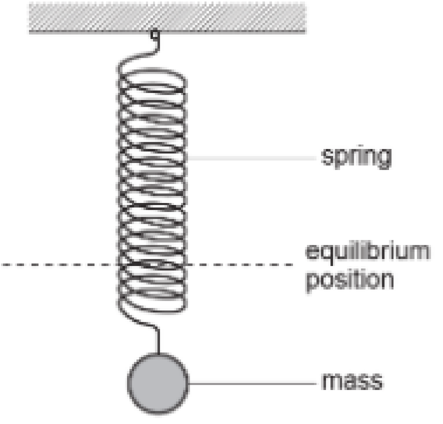


Fig. 18.1

At time $t = 6.5$ s the magnitude of the acceleration a of the mass is 3.6 m s^{-2} and its displacement x is 4.6×10^{-2} m.

The mass-spring system shown in Fig. 18.1 is now made to oscillate in **air**.

Different types of energy are involved in the oscillations of this mass-spring system.

Describe the energy changes that will take place as the mass moves from the lowest point in its motion through the equilibrium position to the highest point in its motion.

....

....

....

....

....

....

....

.... **[4]**

5.3.3 Damping

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) free and forced oscillations

(b) **(i)** the effects of damping on an oscillatory system

(ii) observe forced and damped oscillations for a range of systems

(c) resonance; natural frequency

(d) amplitude-driving frequency graphs for forced oscillators

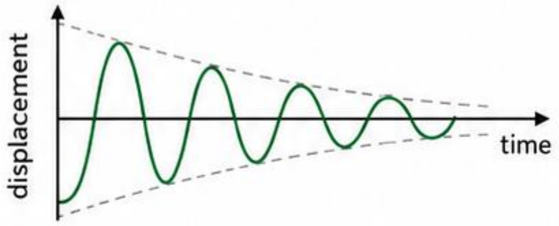
(e) practical examples of forced oscillations and resonance.

Damping

Damping is the reduction in amplitude of an oscillation caused by resistive forces transferring energy from the oscillator to the surroundings.

Types of damping

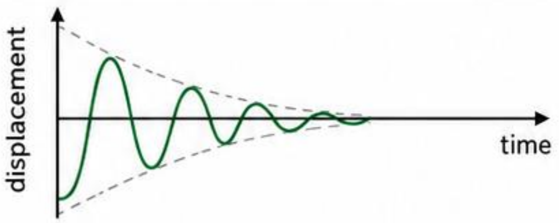
1



Light damping (underdamping)

Oscillations continue and the amplitude decreases gradually.

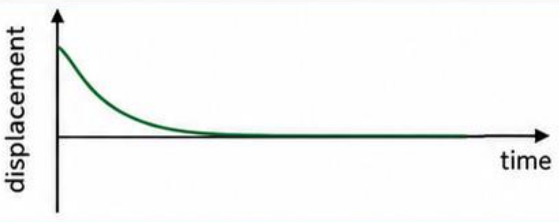
2



Heavy damping (still underdamped)

Oscillations continue but die away more rapidly.
The period increases slightly.

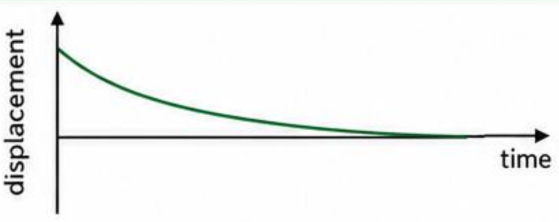
3



Critical damping

No oscillation occurs.
The system returns to equilibrium in the shortest possible time.

4

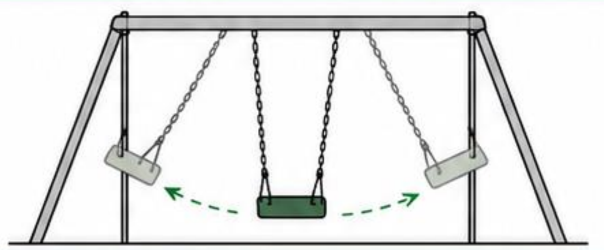


Overdamping

No oscillation occurs.
The system returns to equilibrium more slowly than for critical damping.

Example: playground swing

The swing oscillates with decreasing amplitude because air resistance and friction at the pivot transfer energy to the surroundings.



Key ideas

- Damping causes energy to be transferred from the oscillator to the surroundings.
- Greater damping reduces amplitude more quickly.
- Critical damping gives the fastest return to equilibrium without oscillation.
- Overdamping returns to equilibrium more slowly than critical damping.

Quick task: Sketch and label displacement-time graphs for light damping, critical damping and overdamping.

Damping

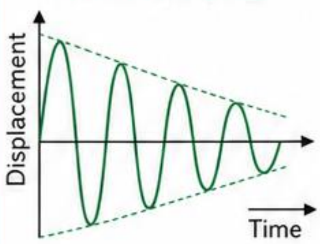
Teacher guide and pupil notes

1. TEACHER SHOW – DEFINITION

Damping is the reduction in amplitude of an oscillation because resistive forces transfer energy from the oscillator to the surroundings.

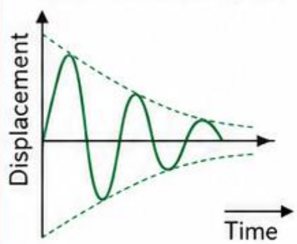
2. KEY IDEAS

Light damping (underdamping)



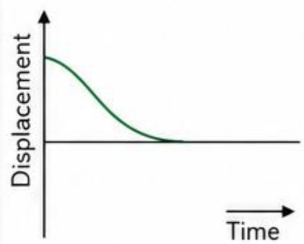
Oscillations continue. Amplitude decreases gradually with time.

Heavy damping (still underdamped)



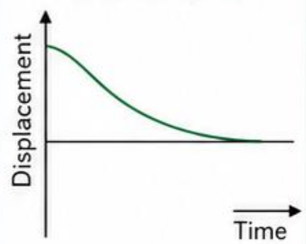
Oscillations continue. Amplitude decreases more quickly.

Critical damping



No oscillation. Returns to equilibrium in the shortest possible time.

Overdamping (overdamped)



No oscillation. Returns to equilibrium more slowly than critical damping.

3. OCR A EXAM NOTE

For light damping, the period is usually taken as almost unchanged. Only when damping is very large, for example in a highly viscous fluid such as thick oil, does the change in period become significant.

For critical damping and overdamping there is no oscillation, so there is no period.

4. MODEL WITH PUPILS

Teacher modelling prompt:

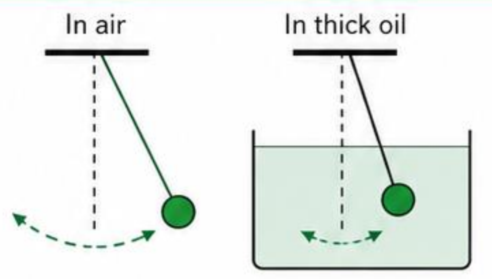
A **pendulum** swings in air and is then placed in thick oil. Describe how the amplitude and the motion change.

____

____

____

____



5. CHECK YOUR UNDERSTANDING

(a) Which type of damping gives the fastest return to equilibrium without oscillation? ____________________

(b) Which type of damping still allows oscillations but reduces amplitude more quickly? ____________________

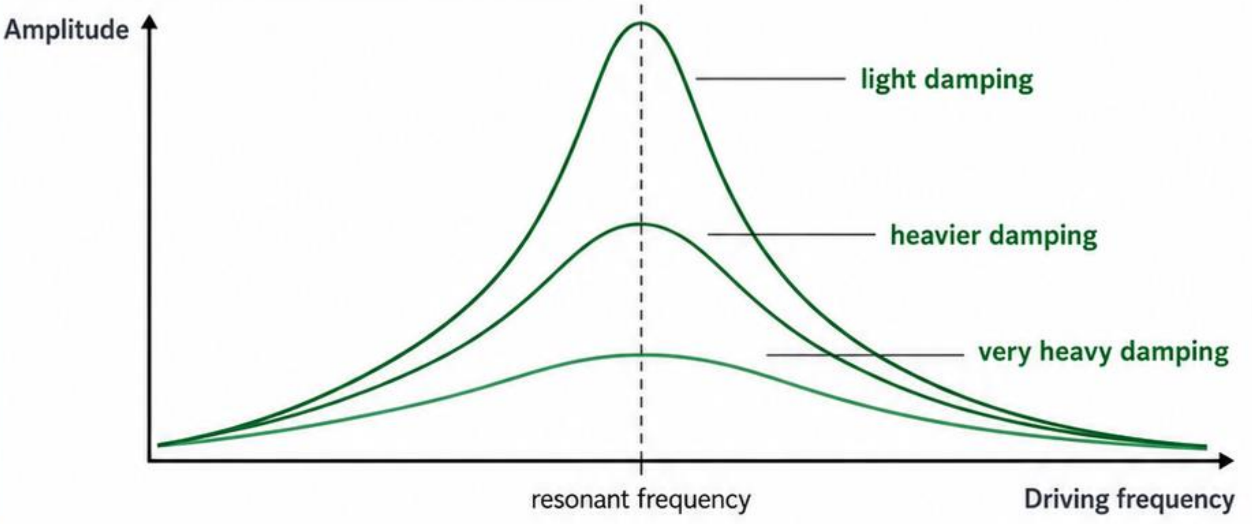
(c) Why does damping reduce the amplitude of an oscillation? ____________________

____

Effect of damping on resonance

Teacher guide and pupil notes

1. TEACHER SHOW – RESONANCE CURVES



2. KEY IDEAS

- As damping increases, the maximum amplitude decreases.
- As damping increases, the resonance peak becomes broader and less sharp.
- With greater damping, the resonant frequency is slightly lower.
- For light damping, the change in frequency or period is often treated as negligible in OCR A questions.
- Only when damping is very large, for example in a highly viscous fluid, does the change become more noticeable.

3. MODEL WITH PUPILS

Teacher modelling prompt:
Explain why car suspension should be damped, but not too heavily damped.
Refer to the size of the resonance peak and the motion of the car after a disturbance.

____

____

____

____

4. CHECK YOUR UNDERSTANDING

(a) Which damping condition gives the largest resonant amplitude? ______________________

(b) What happens to the shape of the resonance peak as damping increases? ______________________

(c) Why is resonance less dangerous in a heavily damped system? ______________________

(d) In OCR A, when is the change in period due to damping usually ignored? ______________________

Resonance

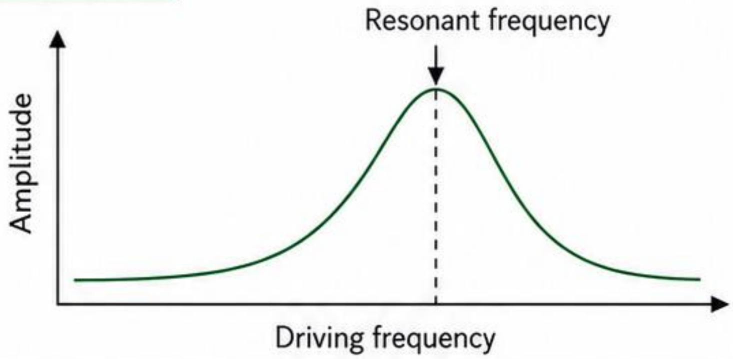
Teacher guide and pupil notes

1. TEACHER SHOW – WHAT IS RESONANCE?

Resonance occurs when the driving frequency equals the natural frequency of the system.

At resonance, the amplitude of oscillation is maximum and the transfer of energy is most efficient.

2. GRAPHICAL IDEA



The amplitude is greatest at resonance.

3. HOW TO GET MAXIMUM AMPLITUDE

- Drive the system at its natural frequency.
- Keep damping small so less energy is lost each cycle.

4. MODEL WITH PUPILS

Teacher modelling prompt:

A playground swing has a natural frequency of 0.50 Hz. Explain why pushing the swing once every 2.0 s produces the largest amplitude. Use the terms driving frequency, natural frequency and resonance.

____

____

____

____

5. CHECK YOUR UNDERSTANDING

(a) Complete the statement: Resonance happens when the driving frequency is equal to the ________ frequency. ____________________

(b) What happens to the amplitude at resonance? ____________________

(c) Give one useful example of resonance. ____________________

(d) Give one unwanted or dangerous example of resonance. ____________________

6. OCR A EXAM NOTE

In OCR A questions, resonance is usually described as the condition for maximum amplitude in a forced oscillation.

Applications and consolidation

Free oscillations, damping and resonance

1. TEACHER SHOW – USEFUL AND UNWANTED RESONANCE

Useful

musical instruments

tuning circuits / radio tuning

vibrations used for sensing or measurement

Unwanted

bridges or buildings

washing machines

car bodywork or suspension

Resonance can be useful when it is controlled, but dangerous when it is not controlled.

2. REDUCING UNWANTED OSCILLATIONS

- Increase damping.
- Avoid driving the system at its natural frequency.
- Change the natural frequency by changing mass or stiffness.

3. MODEL WITH PUPILS

Teacher modelling prompt:

A washing machine shakes violently during the spin cycle. Explain this using the ideas of forced oscillations and resonance, and suggest one way the amplitude could be reduced.

4. PUPIL QUESTIONS

(a) Define resonance.

(b) State one condition needed for maximum amplitude in a forced oscillation.

(c) Give one example of damping being useful.

(d) Explain one reason why critical damping is often useful in engineering systems.

(e) Suggest one method for reducing a dangerous resonance.

5. SUMMARY

- Free oscillations occur at the natural frequency.
- Forced oscillations need a periodic driving force.
- Resonance gives maximum amplitude when the driving frequency matches the natural frequency. Damping reduces amplitude and can reduce the risk of resonance.

Free and forced oscillations

Teacher guide and pupil notes

1. TEACHER SHOW – DEFINITIONS

Free oscillation: a system is displaced and released, then oscillates at its natural frequency.

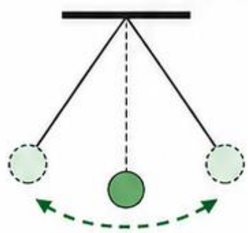
Forced oscillation: a periodic driving force acts on the system.

Natural frequency: the frequency at which a system oscillates when displaced and released.

In a real system, free oscillations usually die away because of damping.

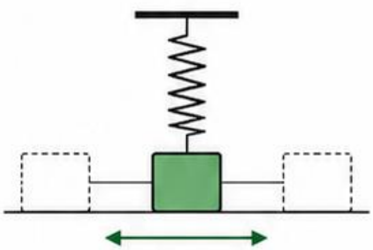
2. COMPARE THE TWO

Free oscillation



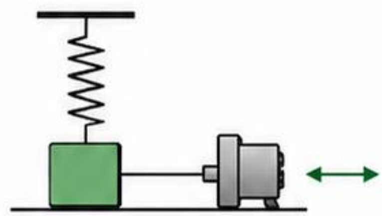
No periodic driving force

Oscillates at natural frequency



Forced oscillation

Periodic driving force



Energy supplied to maintain oscillation

3. KEY IDEA

A forced system can build up a large amplitude only when energy is transferred to it efficiently. This leads to resonance when the driving frequency matches the natural frequency.

4. MODEL WITH PUPILS

Teacher modelling prompt:

A child on a swing is pushed once every cycle.
Identify the driver, the driven system, and the natural frequency.
Explain why the oscillation can continue even though energy is lost by damping.

5. CHECK YOUR UNDERSTANDING

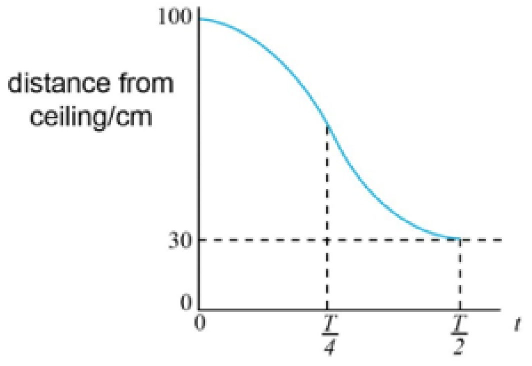
(a) State one difference between a free oscillation and a forced oscillation. ______

(b) What is meant by the natural frequency of a system? ______

(c) Why does a forced oscillator need a driving force? ______

A mass hanging from a spring suspended from the ceiling is pulled down and released.

The mass then oscillates vertically with simple harmonic motion, period T. The graph shows how the distance from the ceiling varies with time.



What can be deduced from this graph?

A The amplitude of the oscillation is 70 cm.

B The kinetic energy is maximum at $t = \frac{T}{2}$

C The restoring force on the mass increases between $t = 0$ and $t = \frac{T}{4}$

D The speed is maximum at $t = \frac{T}{4}$

1 A pendulum is oscillating in air and experiences damping.

Which of the following statements is/are correct for the damping force acting on the pendulum?

1 It is always opposite in direction to acceleration.
2 It is always opposite in direction to velocity.
3 It is maximum when the displacement is zero.

A Only 1 and 2
B Only 2 and 3
C Only 3
D 1, 2 and 3

Your answer [1]

2 Oscillations of an object can either be **free** or **forced**.

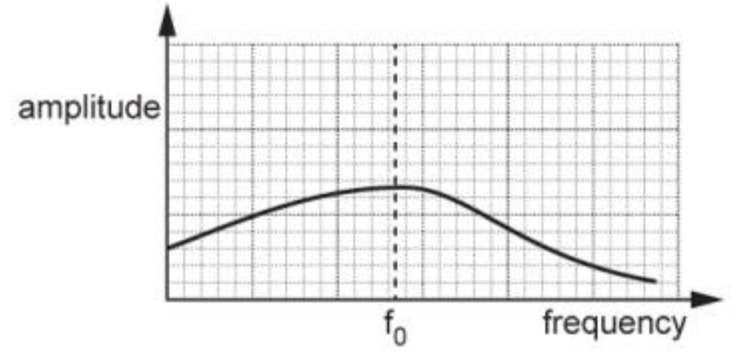
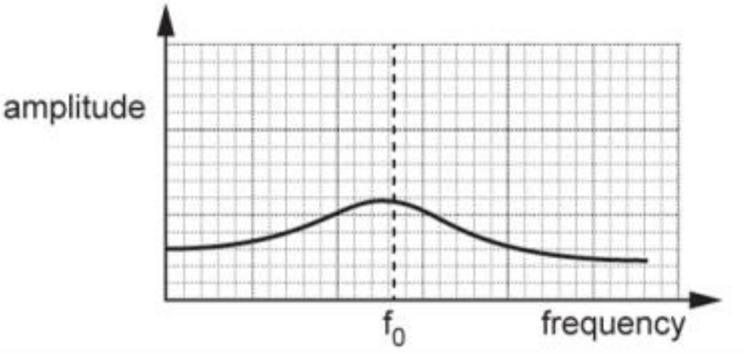
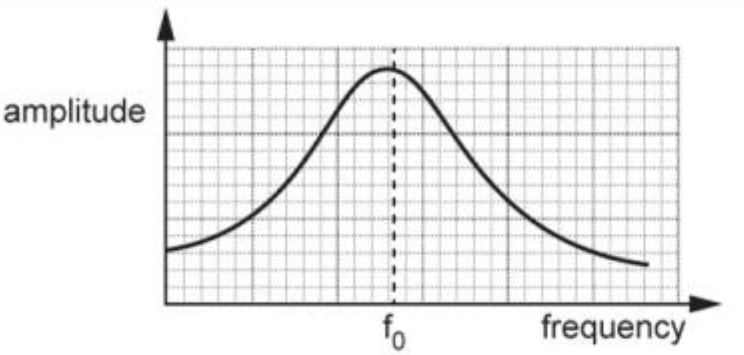
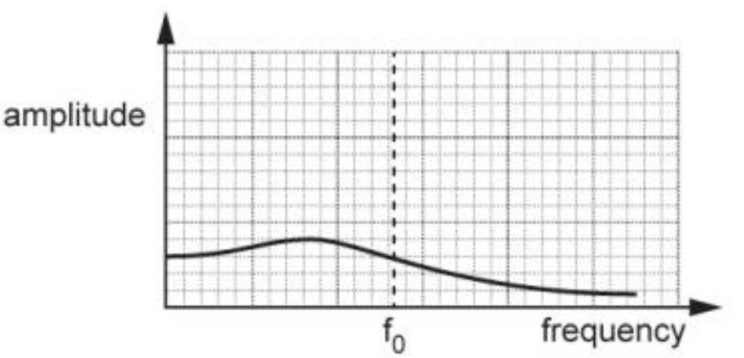
Which of the following is an example of a **forced** oscillation?

A A ball rolling to-and-fro on a curved track.
B A loudspeaker oscillating and producing a continuous note.
C A mass oscillating from the end of a suspended spring.
D A pendulum bob oscillating from the end of a fixed length of string.

Your answer [1]

3 Four different oscillator systems are forced to oscillate at various frequencies. The graphs show the amplitude of oscillation for each frequency. f_0 is the undamped resonant frequency for each oscillator. The vertical axes on the graphs are all to the same scale.

Which of the oscillators, **A** to **D**, is the most heavily damped?

A	
B	
C	
D	

4 Civil engineers are designing a floating platform to be used at sea. **Fig. 4.1** shows a model for one section of this platform, a sealed metal tube of uniform cross-sectional area, loaded with small pieces of lead, floating upright in equilibrium in water.

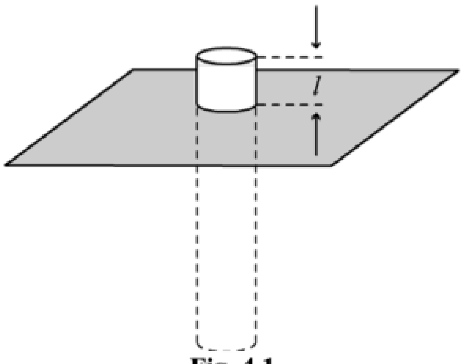


Fig. 4.1

When the tube is pushed down a small amount into the water and released it moves vertically up and down with simple harmonic motion. The period of these oscillations which quickly die away is about one second.

The oscillations of the tube can be maintained over a range of low frequencies by using a flexible link to a simple harmonic oscillator.

Fig. 4.2 shows a graph of amplitude of vertical oscillations of the tube against frequency obtained from this experiment.

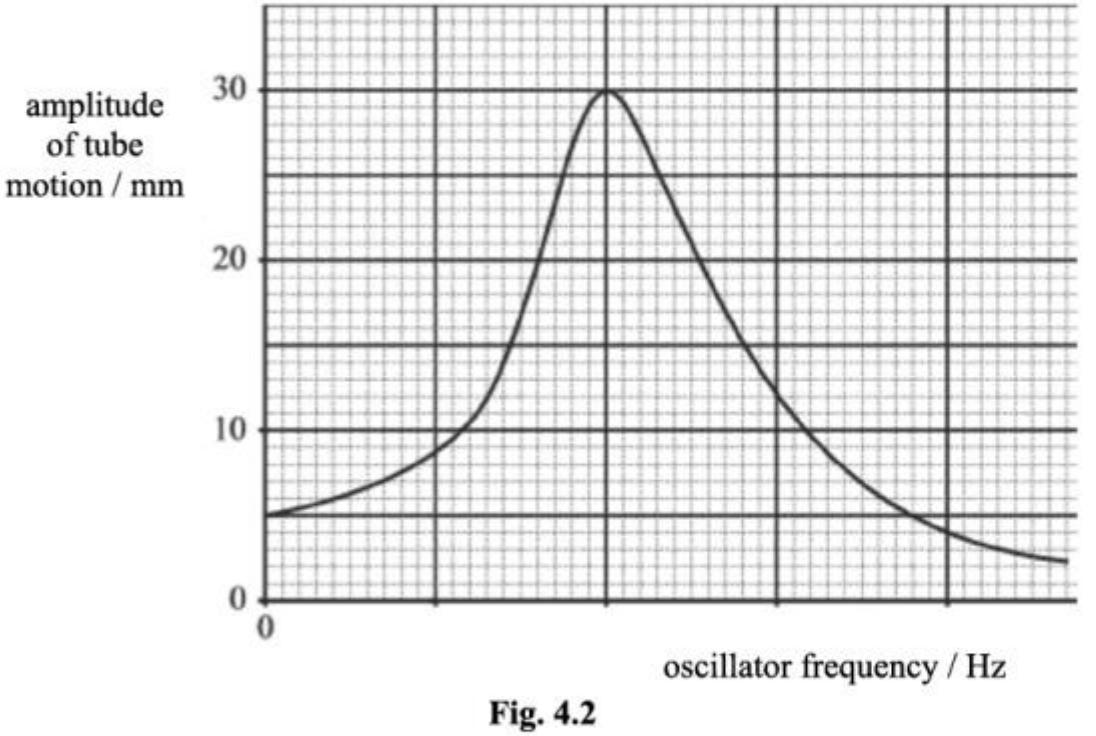


Fig. 4.2

(i) Use information from **Fig. 4.2** to state the amplitude of the motion of the oscillator.

amplitude = ____________________ mm **[1]**

(ii) Add a suitable scale to the frequency axis of **Fig. 4.2**.

[1]

(iii) The experiment is repeated in a much more viscous liquid such as motor oil.
On **Fig. 4.2** sketch the graph that you would predict from this experiment.

[2]

5.3.3 Damping

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) free and forced oscillations

(b) (i) the effects of damping on an oscillatory system

(ii) observe forced and damped oscillations for a range of systems

(c) resonance; natural frequency

(d) amplitude-driving frequency graphs for forced oscillators

(e) practical examples of forced oscillations and resonance.

1. Explain how time period, frequency and amplitude are all affected by light damping, heavy damping and critically damped systems.

2. Draw a graph of frequency against amplitude for systems with no damping and heavily damped systems.

3. Give an example where resonance can be useful and resonance can be a problem.

Additional practice

A stabilising mechanism for electrical equipment on board a high-speed train is modelled using a 5.0 g mass and two springs, as shown in **Fig. 21.1**. For testing purposes, the springs are horizontal and attached to two fixed supports in a laboratory.

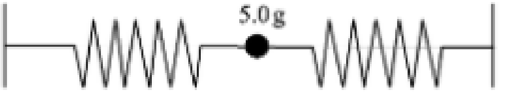


Fig. 21.1

Explain why the mass oscillates with simple harmonic motion when displaced horizontally.

----

[2]

A stabilising mechanism for electrical equipment on board a high-speed train is modelled using a 5.0 g mass and two springs, as shown in **Fig. 21.1**. For testing purposes, the springs are horizontal and attached to two fixed supports in a laboratory.

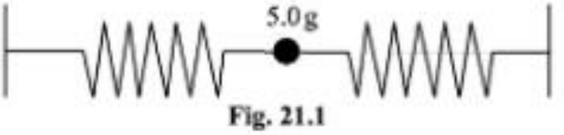


Fig. 21.1

Fig. 21.2 shows the graph of displacement against time for the oscillating mass.

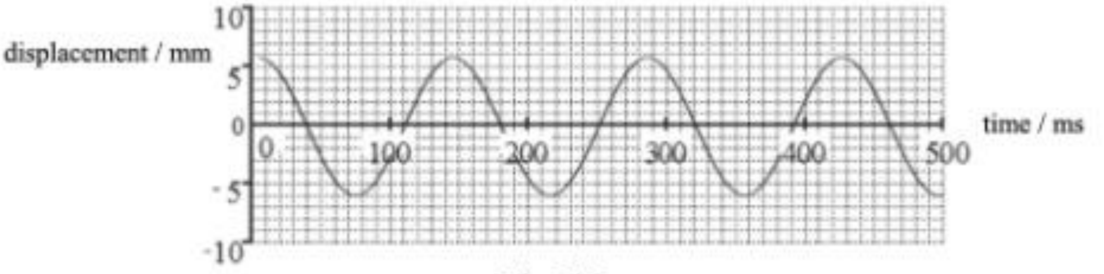


Fig. 21.2

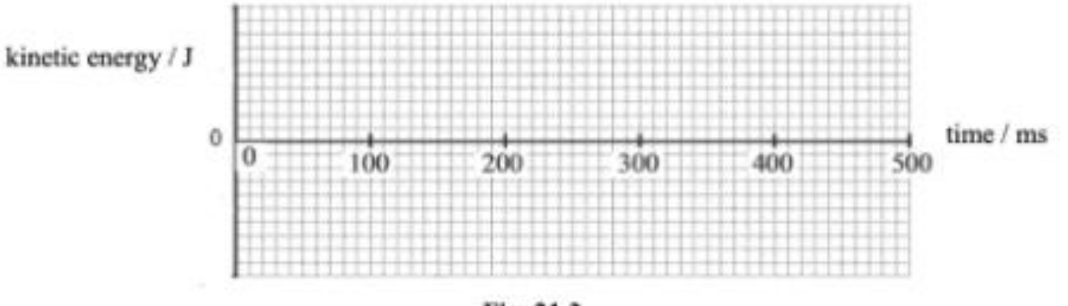


Fig. 21.3

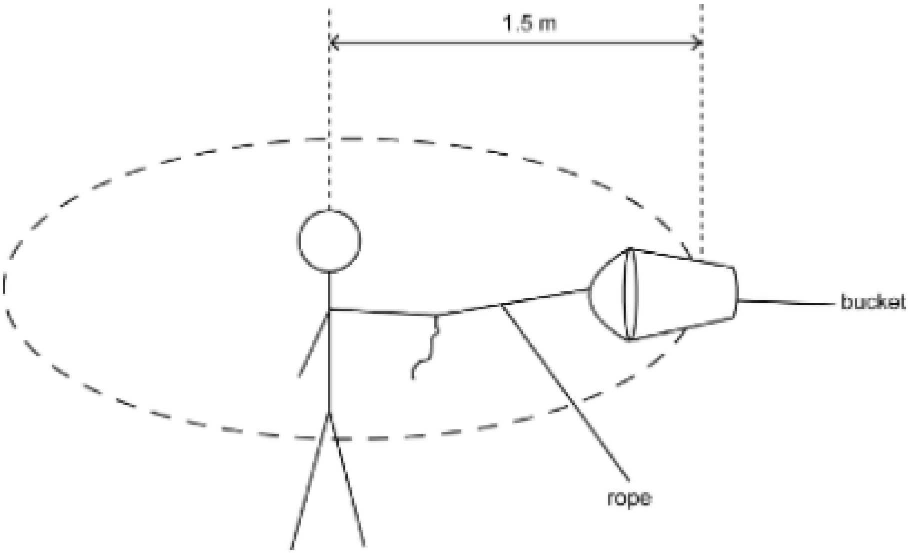
i. Determine the maximum acceleration of the mass during the oscillations.

maximum acceleration = _______________ ms^{-2} **[2]**

ii. Calculate the maximum kinetic energy of the mass during the oscillations.

maximum kinetic energy = _______________ J **[2]**

A rope is attached to a bucket. A man swings the bucket in a horizontal circle of radius 1.5 m. The bucket has a constant speed of 4.8 m s^{-1}. The mass of the bucket is 5.0 kg.



i. Calculate the tension *F* in the rope.

F = ______________________________ N **[2]**

ii. Calculate the angular velocity ω of the rotating bucket.

ω = ______________________________ rad s^{-1} **[2]**

* A student wishes to test the equation $F = \frac{mv^2}{r}$ for a constant force F using a whirling bung in the laboratory.

Describe with the aid of a labelled diagram how an experiment can be conducted, and how the data can be analysed to test the validity of this equation for a constant force.

....

....

....

....

....

....

....

....

....

.... **[6]**

Fig. 3.1 shows a metal plate attached to the end of a spiral spring. The end **A** of the spring is fixed to a rigid clamp. The plate is pulled down by a small amount and released. The plate performs simple harmonic motion in a vertical plane at a natural frequency of 8 Hz and the spring remains in tension at all times.

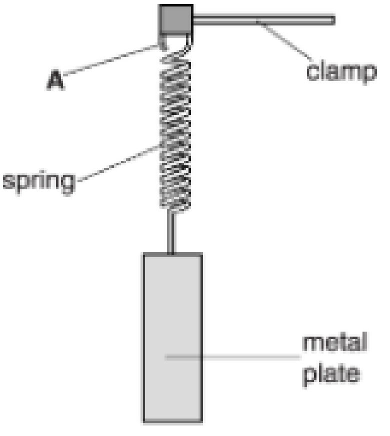


Fig. 3.1

i. the amplitude of the motion

amplitude = ... m **[2]**

ii. the maximum vertical acceleration of the plate.

acceleration = ... m s^{-2} **[2]**

Fig. 3.1 shows a simple pendulum consisting of a steel sphere suspended by a light string from a rigid support. The sphere is displaced 50 mm from its vertical equilibrium position and released at time $t = 0$.

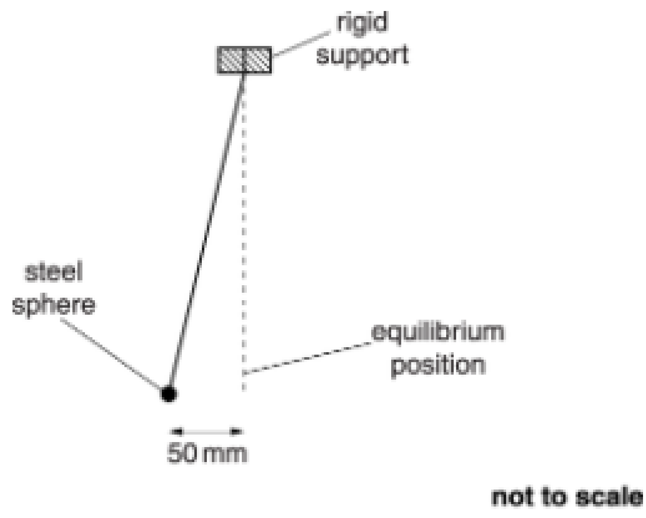


Fig. 3.1

Fig. 3.2 shows the graph of displacement x of the sphere against time t.

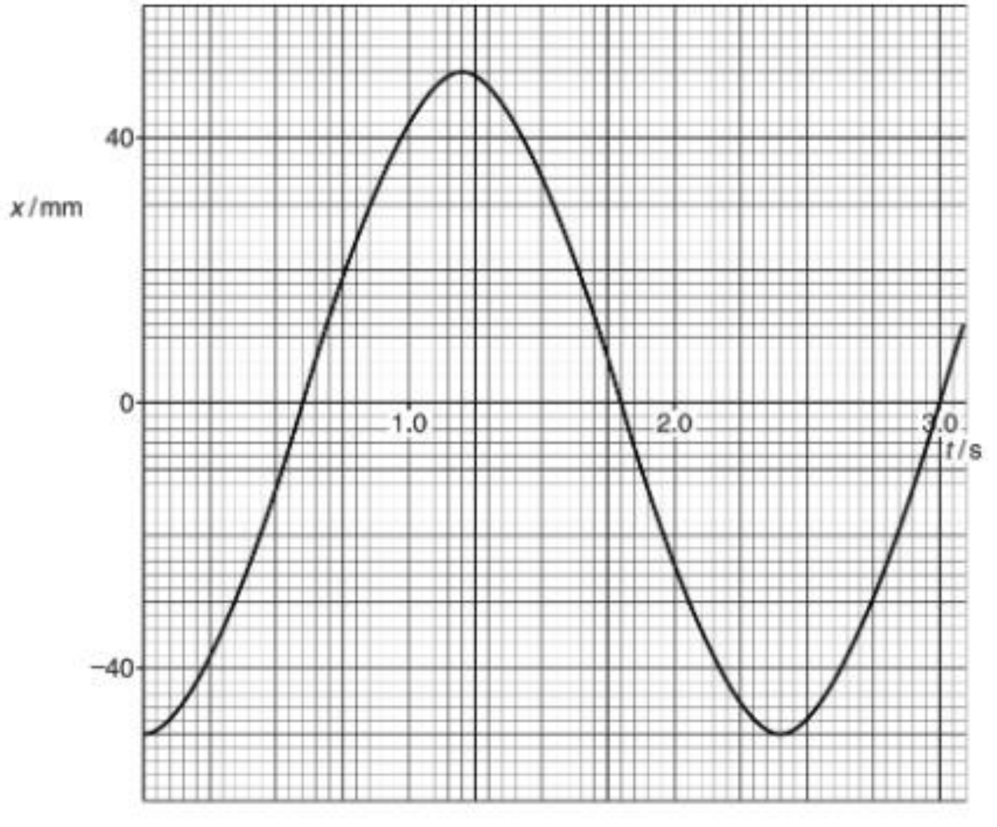


Fig. 3.2

The sphere is released from rest with a displacement x = 25 mm.
State with a reason, the change if any in

i. the frequency of the oscillations

--- **[1]**

ii. the maximum kinetic energy of the sphere.

--- **[2]**

An oscillator is in simple harmonic motion.

Which statement is **not** correct?

A The acceleration is directly proportional to the displacement.
B The acceleration is zero at maximum displacement.
C The maximum velocity is at zero displacement.
D The kinetic energy is zero at maximum displacement.

Your answer ☐

An object of mass m is attached to a string and then whirled in a horizontal circle. The speed of the object is slowly increased from zero. The string breaks when the object has a maximum speed of 1.00 m s^{-1}.
The experiment is repeated with an identical string but with an object of mass 1.5 m. The radius of the circle is kept constant.

What is the maximum speed of this object when the string breaks?

A 0.67 m s^{-1}
B 0.82 m s^{-1}
C 1.22 m s^{-1}
D 1.50 m s^{-1}

Your answer ☐

Consolidation 1: circular motion basics

Radian measure, period, frequency and angular velocity.

OCR A links: 5.2.1(a) radian measure • 5.2.1(b) period/frequency • 5.2.1(c) angular velocity

$\theta = s/r$	$f = 1/T$	$\omega = \Delta\theta/\Delta t$	$\omega = 2\pi/T = 2\pi f$	$v = \omega r$

A. WARM-UP: RADIAN AND FREQUENCY

1 Convert 135° into radians. Give your answer as a multiple of π.

2 A wheel completes 18 revolutions in 6.0 s. Calculate its frequency and period.

B. ANGULAR VELOCITY

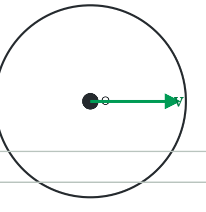
3 A centrifuge rotor completes one revolution every 4.0 ms. Calculate the angular velocity in rad s^{-1}.

4 A hard disk rotates at 7200 rpm. Calculate the angular velocity in rad s^{-1} and the number of revolutions in 0.50 s.

C. LINKING LINEAR AND ANGULAR SPEED

A point on the rim of a flywheel of radius 0.12 m has a linear speed of 18 m s^{-1}.

(a) Calculate the angular velocity of the flywheel. (b) Calculate its frequency. (c) State one assumption made in your calculation.



D. EXAM-STYLE PROBLEM

A model train travels around a circular track of radius 0.75 m. The train completes 12 laps in 36 s.

Calculate: (a) the period of the motion (b) the angular velocity (c) the speed of the train (d) the angular displacement after 9.0 s.

Checkpoint: explain why angular displacement must be measured in radians before using equations such as v = ωr.

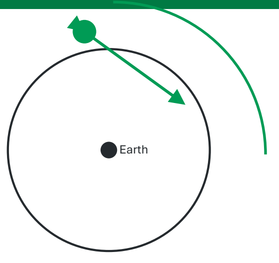
Consolidation 2: centripetal force

Centripetal acceleration, resultant force and circular-motion problems.

OCR A links: 5.2.2(a) net force perpendicular to velocity • 5.2.2(c) $a = v^2/r = \omega^2 r$ • 5.2.2(d) $F = mv^2/r = m\omega^2 r$

$a = v^2/r$	$a = \omega^2 r$	$F = ma$	$F = mv^2/r$	$F = m\omega^2 r$

A. DIAGRAM AND CONCEPTS



For the satellite, label the two arrows as instantaneous velocity and centripetal force. Then explain why the speed can remain constant even though the velocity changes.

B. EARTH ROTATION

Calculate the centripetal acceleration of a person standing on the equator as Earth rotates. Take Earth's radius as 6370 km. State the rotation period you are using.

C. HORIZONTAL CIRCULAR MOTION

A 0.080 kg rubber bung moves in a horizontal circle of radius 0.65 m. It completes 15 revolutions in 12 s. Calculate the tension in the string, assuming the tension provides the centripetal force.

D. CONICAL PENDULUM

A 0.20 kg mass moves as a conical pendulum. The string length is 0.90 m and it makes an angle of 35° to the vertical.

Calculate: (a) the radius of the circular path (b) the tension in the string (c) the centripetal force.

E. PRACTICAL METHOD

Describe how a whirling bung experiment can be used to test $F = mv^2/r$. Include the independent variable, dependent variable, controls, and how speed is determined.

Consolidation 3: simple harmonic motion

Definitions, equations, graphs and energy.

OCR A links: 5.3.1(a) SHM quantities • 5.3.1(c) $a = -\omega^2 x$ • 5.3.1(d/e) $x = A\cos\omega t$, $v_{max} = \omega A$ • 5.3.2 energy

$a = -\omega^2 x$	$x = A\cos(\omega t)$	$\omega = 2\pi f$	$v_{max} = \omega A$	$a_{max} = \omega^2 A$

A. CORE DEFINITION

Complete the definition: In simple harmonic motion, acceleration is _________ to displacement from equilibrium and is always directed _________ the equilibrium position.

B. GRAPH INTERPRETATION

Sketch one full cycle of x against t starting at +A.

For your sketch, mark: equilibrium positions, maximum displacement, where speed is maximum, and where acceleration magnitude is maximum.

C. EQUATION PROBLEM

The displacement of an oscillator is $x = 0.080\cos(12.0t)$, where x is in metres and t is in seconds.

Calculate: (a) amplitude (b) angular frequency (c) frequency (d) period (e) maximum speed (f) maximum acceleration.

D. DISPLACEMENT AND SPEED

A pendulum oscillates with amplitude 5.0 cm and frequency 0.80 Hz. It is released from maximum displacement at t = 0.

Calculate its displacement and speed at t = 0.50 s. Use a clear sign convention for displacement.

E. ENERGY IN SHM

For an undamped oscillator, explain how kinetic energy and potential energy change during one cycle. Then calculate the maximum kinetic energy of a 0.15 kg oscillator with amplitude 0.040 m and angular frequency 9.0 rad s^{-1}.

Consolidation 4: damping and resonance

Free oscillations, forced oscillations and amplitude–driving frequency graphs.

OCR A links: 5.3.3(a) free/forced oscillations • 5.3.3(b) damping • 5.3.3(c) resonance/natural frequency • 5.3.3(d/e) graphs and examples

A. DEFINITIONS

Define each term clearly: free oscillation • forced oscillation • natural frequency • resonance • damping.

B. DAMPING DESCRIPTIONS

A pendulum oscillates in air and is then placed into a highly viscous fluid such as thick oil. Describe how the amplitude and time period are affected in each case.

OCR A reminder: for light damping, period/frequency changes are usually treated as negligible; heavy damping can make the change noticeable.

C. RESONANCE CURVE

Sketch amplitude against driving frequency for light damping and heavy damping. Label the resonant frequency.

Explain how increasing damping changes: (a) maximum amplitude, (b) sharpness of the peak, and (c) resonant frequency.

D. APPLICATIONS

For each application, identify whether resonance is useful or unwanted, and explain why: (a) a musical instrument body (b) car suspension (c) a washing machine during spin cycle.

E. LONGER RESPONSE

A bridge is exposed to periodic forces from wind and traffic. Explain why resonance could be dangerous and describe two design changes that could reduce the risk.

Additional notes:

physicsUK

Continue your journey

Well done on completing this workbook!
Keep building your knowledge, skills and confidence with PhysicsUK.

Targeted physics practice
Focus on the topics that matter most.

Exam-style questions
Build exam technique and apply your learning.

Clear feedback
Understand your progress and learn from every step.

Build confidence step by step
Small steps today, big results tomorrow.

Ready for more?

Scan the QR code to continue your journey with PhysicsUK.

physicsUK.co.uk

Smarter practice, stronger understanding.