OCR A AS Mean Drift Velocity solutions

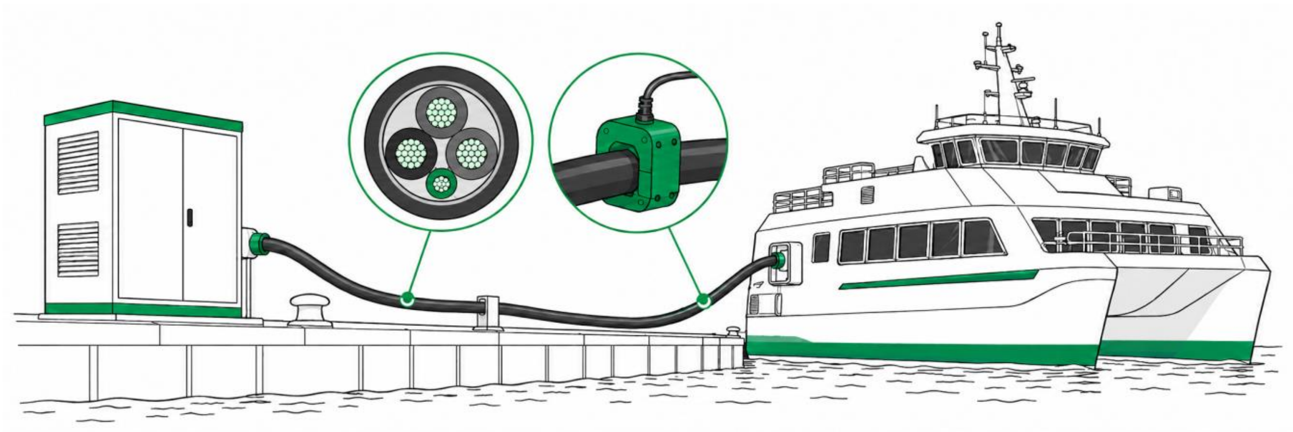
Matching solutions and teaching prompts for the class review

Specification points covered

• 4.1.2(a) Define and understand mean drift velocity of charge carriers.
• 4.1.2(b) Use and apply I = Anev, where n is the number density of charge carriers.
• 4.1.2(c) Distinguish conductors, semiconductors and insulators in terms of the number density of charge carriers.

Teaching focus

Require pupils to distinguish rapid random electron motion, mean drift velocity and rapid electrical response.
Award calculation marks for the correct perpendicular area, SI conversion, proportional reasoning and final unit—not simply the calculator display.



Context diagram: the rapid-charging berth, four-core cable and current sensor.

1. Meaning and model

1. Define mean drift velocity of charge carriers. [1 mark]

Answer / mark points: Mean drift velocity is the average velocity of the charge carriers in a particular direction through a material.

Teacher prompt / common error: The definition needs both 'average velocity' and a direction; 'the speed of the electrons' is not precise enough.

2. Free electrons in a metal have rapid random motion even when there is no current. Explain how applying a potential difference produces a small mean drift velocity with a definite direction. [2 marks]

Answer / mark points: Without a potential difference, the random electron velocities have no resultant direction, so their mean velocity is zero. An applied potential difference establishes an electric field; the field exerts a force on the electrons and produces a small average velocity opposite to the conventional-current direction.

Teacher prompt / common error: Separate random microscopic motion from the small resultant drift caused by an electric field.

2. Using I = Anev

3. The equation I = Anev models current in a conductor. State what A and n represent and give the SI unit of each quantity. [2 marks]

Answer / mark points: A is the cross-sectional area perpendicular to the drift, measured in m^2. n is the number of mobile charge carriers per unit volume, measured in m^{-3}.

Teacher prompt / common error: A is an area, not amplitude, and n counts mobile carriers per cubic metre, not the total number in the cable.

4. The shore-side copper busbar carries 360 A. Its cross-sectional area is 4.8×10^{-4} m^2 and its carrier number density is 8.5×10^{28} m^{-3}. Calculate the mean drift velocity. Use $e = 1.60 \times 10^{-19}$ C. [2 marks]

Answer / mark points: $v = I/(Ane) = 360 / [(4.8 \times 10^{-4})(8.5 \times 10^{28})(1.60 \times 10^{-19})] = 5.51 \times 10^{-5}$ m s^{-1}, so 5.5×10^{-5} m s^{-1}.

Teacher prompt / common error: Use square metres for area and keep A, n and e together in the denominator.

5. Each circular copper core in the flexible charging cable has diameter 11.0 mm and carries 92 A. The carrier number density is 8.5×10^{28} m^{-3}. Calculate the cross-sectional area of one core and its mean drift velocity. [3 marks]

Answer / mark points: $A = \pi(d/2)^2 = \pi(5.50 \times 10^{-3})^2 = 9.50 \times 10^{-5}$ m^2. $v = 92 / [(9.50 \times 10^{-5})(8.5 \times 10^{28})(1.60 \times 10^{-19})] = 7.12 \times 10^{-5}$ m s^{-1}.

Teacher prompt / common error: The stated diameter must be halved before using πr^2, and millimetres must be converted before squaring.

3. Geometry and shared current

6. A replacement core is made from the same material and carries the same current, but its diameter is 8.0 mm rather than 11.0 mm. Determine the factor by which its mean drift velocity changes and calculate the new value using your answer to Question 5. **[2 marks]**

Answer / mark points: At fixed I and n, v is inversely proportional to A and therefore to d^2. $v_{new}/v_{old} = (11.0/8.0)^2 = 1.89$. $v_{new} = 1.89(7.12 \times 10^{-5}) = 1.35 \times 10^{-4}$ m s^{-1}.

Teacher prompt / common error: Drift velocity varies with the inverse square of diameter, not simply the inverse of diameter.

7. Four identical cores share a total current equally. One core is taken out of service while the total current is maintained. Determine the factor by which the mean drift velocity in each remaining core changes. Explain your reasoning. **[2 marks]**

Answer / mark points: Initially each core carries one quarter of the total current. With three cores, each carries one third. Since A and n are unchanged, v changes by (1/3)/(1/4) = 4/3. The mean drift velocity increases by a factor of 1.33.

Teacher prompt / common error: Maintaining the total current forces the remaining cores to carry a greater share; do not assume the current in each core stays fixed.

8. Two sections of the charging link are in series and carry the same current. Section P has $A = 4.2 \times 10^{-4}$ m^2 and $n = 6.0 \times 10^{28}$ m^{-3}. Section Q has $A = 2.4 \times 10^{-4}$ m^2 and $n = 8.5 \times 10^{28}$ m^{-3}. Calculate v_Q/v_P and state which section has the greater mean drift velocity. **[2 marks]**

Answer / mark points: For the same series current, v is inversely proportional to An. Therefore $v_Q/v_P = (A_P n_P)/(A_Q n_Q) = [(4.2 \times 10^{-4})(6.0 \times 10^{28})] / [(2.4 \times 10^{-4})(8.5 \times 10^{28})] = 1.24$. Section Q has the greater mean drift velocity.

Teacher prompt / common error: Because the current is common to both series sections, compare the products An rather than area alone.

4. Comparing sections

9. The table gives data for three parts of the charging system. Calculate the mean drift velocity in each section and rank the sections from smallest to largest mean drift velocity. [3 marks]

Section	Current I / A	Area A / m^2	Number density n / m^{-3}
P	200	4.0×10^{-4}	8.5×10^{28}
Q	150	2.0×10^{-4}	8.5×10^{28}
R	5.0×10^{-3}	1.0×10^{-6}	2.0×10^{22}

Answer / mark points: P: $v = 200/[(4.0 \times 10^{-4})(8.5 \times 10^{28})e] = 3.68 \times 10^{-5}$ m s^{-1}. Q: $v = 5.51 \times 10^{-5}$ m s^{-1}. R: $v = 1.56$ m s^{-1}. Ranking: P, Q, R. The much smaller n in R dominates despite its small current.
Teacher prompt / common error: A small current does not guarantee a small drift velocity; carrier number density and cross-sectional area also matter.

10. A copper link has cross-sectional area 6.0×10^{-5} m^2, carrier number density 8.5×10^{28} m^{-3} and mean drift velocity 1.1×10^{-4} m s^{-1}. Calculate the current in the link. [2 marks]

Answer / mark points: $I = Anev = (6.0 \times 10^{-5})(8.5 \times 10^{28})(1.60 \times 10^{-19})(1.1 \times 10^{-4}) = 89.8$ A, so 90 A.
Teacher prompt / common error: The equation produces amperes when all quantities are in SI units.

11. A semiconductor strip in the current sensor is 5.0 mm wide and 0.20 mm thick. It carries 12 mA and its charge carriers have mean drift velocity 0.035 m s^{-1}. Calculate the carrier number density and explain why the result is consistent with a semiconductor rather than a metal. [3 marks]

Answer / mark points: $A = (5.0 \times 10^{-3})(0.20 \times 10^{-3}) = 1.0 \times 10^{-6}$ m^2. $n = I/(Aev) = 0.012/[(1.0 \times 10^{-6})(1.60 \times 10^{-19})(0.035)] = 2.14 \times 10^{24}$ m^{-3}. This is many orders of magnitude below the number density typical of a metal, so it is consistent with a semiconductor.
Teacher prompt / common error: Convert both dimensions to metres before multiplying. The classification must refer to mobile carrier number density.

5. Finding n from measurements

12. A 1.20 m length of braided conductor has cross-sectional area 1.85×10^{-4} m^2 and contains 1.88×10^{25} free electrons. It carries 280 A. Determine its carrier number density and then calculate its mean drift velocity. [3 marks]

Answer / mark points: Volume = AL = $(1.85 \times 10^{-4})(1.20) = 2.22 \times 10^{-4}$ m^3. n = N/(AL) = 1.88×10^{25} / 2.22×10^{-4} = 8.47×10^{28} m^{-3}. v = $280/[(1.85 \times 10^{-4})(8.47 \times 10^{28})(1.60 \times 10^{-19})] = 1.12 \times 10^{-4}$ m s^{-1}.	
Teacher prompt / common error: Find the conductor volume AL before dividing the total number of free electrons by it.	

13. Use your mean drift velocity from Question 12 to calculate the time taken for an electron's net drift displacement to be 2.0 m. The ferry begins charging almost immediately after connection. Explain why these facts are consistent. [3 marks]

Answer / mark points: t = displacement/v = $2.0/(1.12 \times 10^{-4}) = 1.8 \times 10^{4}$ s, about 5.0 h. Charge carriers are already present throughout the complete circuit. The electric field is established rapidly, so carriers all along the cable begin to drift; one electron does not need to travel from shore to the ferry before charging begins.	
Teacher prompt / common error: Do not claim that individual electrons travel along the full cable at the speed of the electrical signal.	

6. Temperature, materials and graphs

14. Heating the semiconductor current sensor increases its carrier number density by a factor of 6 while its current and dimensions remain constant. Determine the new mean drift velocity as a fraction of its original value. Evaluate the statement: 'the current stays constant because the charge carriers move faster'. [2 marks]

Answer / mark points: With I and A fixed, v is inversely proportional to n. The new mean drift velocity is v/6. The statement is incorrect: the carriers drift more slowly, while the six-times-greater number density compensates so that Anev and the current remain constant.

Teacher prompt / common error: At fixed current, an increase in n requires a decrease in v. Current is not a speed.

15. Three candidate materials have the carrier number densities shown. Match A, B and C to the metal busbar, semiconductor sensor and insulating cable sheath. Justify each match using number density. [3 marks]

Candidate	Carrier number density n / m^{-3}
A	8.5×10^{28}
B	2.0×10^{22}
C	less than 1×10^{8}

Answer / mark points: A is the metal busbar because its very large carrier number density supports a large current. B is the semiconductor sensor because its carrier density is much lower than a metal but appreciably above zero. C is the insulating sheath because it has a negligible density of mobile charge carriers.

Teacher prompt / common error: Use number density rather than naming a familiar material property such as hardness or transparency.

16. For one cable core, temperature, cross-sectional area and carrier number density are constant. State the shape of a graph of mean drift velocity v against current I, and write an expression for its gradient. [2 marks]

Answer / mark points: v = I/(Ane), so v is directly proportional to I. The graph is a straight line through the origin with gradient 1/(Ane).

Teacher prompt / common error: The gradient is not Ane; rearrange to make v the subject before comparing with y = mx.

7. Engineering limits

17. A revised charger must supply 400 A through four identical copper cores. The maximum permitted mean drift velocity in a core is 8.0×10^{-5} m s^{-1} and n = 8.5×10^{28} m^{-3}. Calculate the minimum cross-sectional area and minimum diameter of each core. Standard core diameters are 9.0 mm, 10.0 mm and 11.0 mm; select the smallest suitable size. **[4 marks]**

Answer / mark points: Each core carries I = 400/4 = 100 A. $A_{min} = I/(nev_{max}) = 100/[(8.5 \times 10^{28})(1.60 \times 10^{-19})(8.0 \times 10^{-5})] = 9.19 \times 10^{-5}$ m^2. $d_{min} = \sqrt{(4A/\pi)} = 1.08 \times 10^{-2}$ m = 10.8 mm. The smallest suitable standard core is 11.0 mm.

Teacher prompt / common error: The calculated diameter is a minimum. A standard size below it is not acceptable even if it is close.

18. During one charging pulse, the mean drift velocity is 5.5×10^{-5} m s^{-1} for 180 s and then 2.75×10^{-5} m s^{-1} for 300 s, in the same direction. Calculate the net drift displacement of an electron. Explain why this displacement is not the distance travelled in its rapid random motion. **[3 marks]**

Answer / mark points: Drift displacement = $(5.5 \times 10^{-5} \times 180) + (2.75 \times 10^{-5} \times 300)$ = 0.01815 m, so 1.8×10^{-2} m or 1.8 cm. Mean drift describes the small resultant displacement in one direction. The electron's actual path is much longer and irregular because of its rapid random motion and repeated collisions.

Teacher prompt / common error: Mean drift displacement is the resultant of random motion, not the total microscopic path length.

Language and method check

Use 'mean drift velocity increases/decreases/stays constant'; current itself does not 'move faster'.
Use 'number density of mobile charge carriers per unit volume'; do not use the total number of electrons as n.
When diameter changes, the cross-sectional area changes with diameter squared.