OCR A AS EMF and Internal Resistance solutions

Matching solutions and teaching prompts for the class review

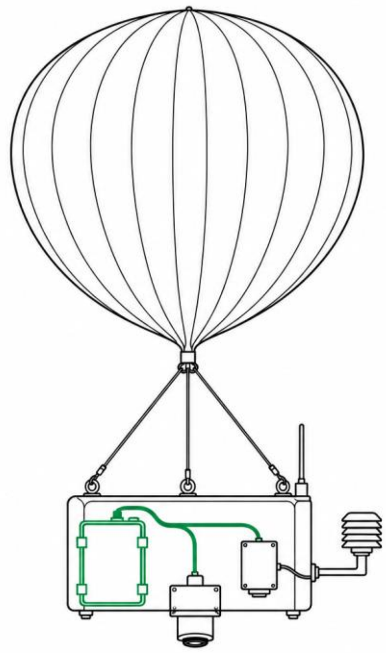
Specification points covered

- 4.3.1(f) Analyse circuits with more than one source of e.m.f.
- 4.3.2(a) Define a source of e.m.f. and internal resistance.
- 4.3.2(b) Define terminal potential difference and lost volts.
- 4.3.2(c)(i) Use and apply $E = I(R + r)$ and $E = V + Ir$.
- 4.3.2(c)(ii) Use techniques and procedures to determine the internal resistance of a cell or power supply.

Teaching focus

Keep definitions anchored to energy transferred per unit charge and distinguish energy transferred inside and outside the source.

When a source delivers current, use $V = E - Ir$. On a V against I graph, the intercept is E and the gradient is negative with magnitude r.



Context diagram: the Asteria balloon and its battery-powered instrument gondola.

1 Energy and source language

1. The Asteria balloon battery transfers 25.2 J from chemical energy to electrical energy when 2.10 C of charge passes through it. Calculate the e.m.f. of the battery. [1 mark]

Answer / mark points: $E = W/Q = 25.2/2.10 = 12.0$ V.	
Teacher prompt / common error: E.m.f. is energy transferred to electrical energy per unit charge, so use $E = W/Q$.	

2. State what is meant by the internal resistance of a source. Define terminal potential difference and explain the term lost volts. [3 marks]

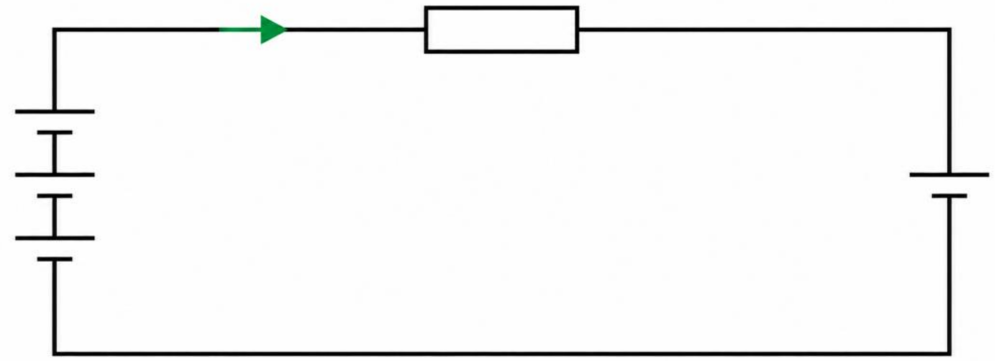
Answer / mark points: Internal resistance is the resistance to current within the source. Terminal p.d. is the energy transferred from electrical energy per unit charge in the external circuit, measured across the source terminals. Lost volts is the p.d. across the internal resistance, Ir, caused by energy dissipated within the source.	
Teacher prompt / common error: Keep the energy language. Voltage does not flow and lost volts does not mean that charge disappears.	

3. A high-resistance voltmeter connected across the battery reads 12.6 V when the external circuit is open. Explain why this reading equals the e.m.f. of the battery. [1 mark]

Answer / mark points: With the external circuit open, $I = 0$ and therefore $Ir = 0$. From $E = V + Ir$, $V = E = 12.6$ V.	
Teacher prompt / common error: A high-resistance voltmeter draws negligible current, so the internal voltage drop is negligible.	

2 Opposing and aiding sources

4. The diagram shows the main pack of e.m.f. 12.0 V and a backup source of e.m.f. 3.0 V connected in opposition. Their internal resistances are negligible. Determine the resultant e.m.f. and the direction of conventional current. [2 marks]



Solution circuit: the larger left-hand source drives clockwise current through the opposing source.

Answer / mark points: The sources oppose, so the resultant e.m.f. is 12.0 − 3.0 = 9.0 V. Conventional current is clockwise: it leaves the positive terminal of the 12.0 V main pack and enters the positive terminal of the 3.0 V backup source.	
Teacher prompt / common error: Subtract opposing e.m.f.s and use the larger source to set the current direction.	

5. The external resistor in the circuit in Question 4 has resistance 18.0 Ω. Calculate the current in the resistor. [1 mark]

Answer / mark points: $I = E/R = 9.0/18.0 = 0.500$ A.	
Teacher prompt / common error: Use the resultant e.m.f., not 12.0 V or 15.0 V.	

6. For each coulomb that passes around the circuit, state the energy supplied by the main pack, the energy transferred to the backup source and the energy dissipated in the resistor. Show that energy is conserved. [2 marks]

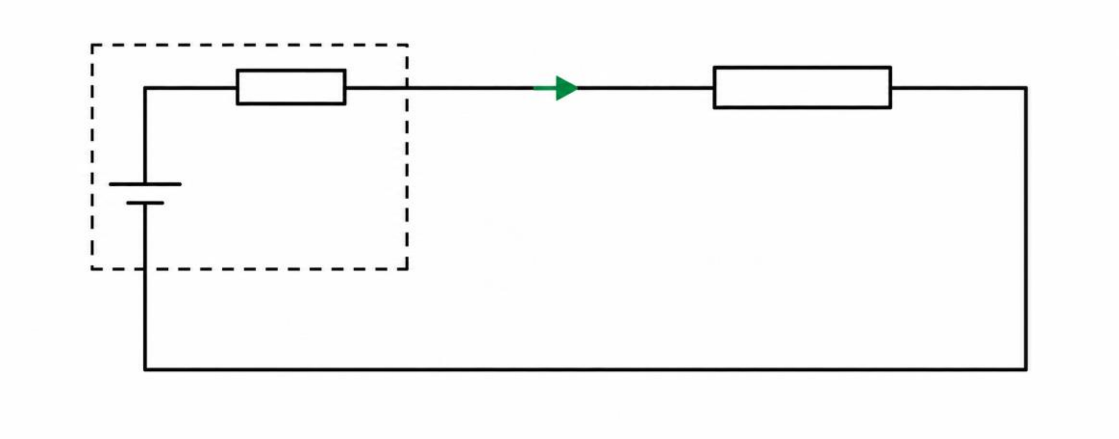
Answer / mark points: The main pack supplies 12.0 J C^{-1}. The backup source receives 3.0 J C^{-1} and the resistor receives 9.0 J C^{-1}. The outputs total 3.0 + 9.0 = 12.0 J C^{-1}, equal to the energy supplied.	
Teacher prompt / common error: The smaller opposing source absorbs energy; current enters its positive terminal, so it is being charged.	

7. The backup source is reversed so that the two e.m.f.s act in the same direction. Calculate the new current. Determine the factor by which the power dissipated in the 18.0 Ω resistor increases. [3 marks]

Answer / mark points: The combined e.m.f. is 12.0 + 3.0 = 15.0 V, so $I = 15.0/18.0 = 0.833$ A. Initially $P = 0.500^2 \times 18.0 = 4.50$ W; after reversal $P = 0.833^2 \times 18.0 = 12.5$ W. The factor is 12.5/4.50 = 2.78.	
Teacher prompt / common error: Power changes with I^2 for the unchanged resistor, so compare powers rather than currents alone.	

3 A real source under load

8. The diagram models the Asteria battery as an ideal source of e.m.f. 12.6 V in series with internal resistance 0.600 Ω. The external instrument load has resistance 5.70 Ω. Calculate the current using $E = I(R + r)$. **[1 mark]**



Solution model: current passes through the internal resistance before reaching the external load.

Answer / mark points: $I = E/(R + r) = 12.6/(5.70 + 0.600) = 2.00$ A.
Teacher prompt / common error: The internal and external resistances are in series in the real-source model.

9. Calculate the lost volts and the terminal p.d. of the battery. Verify the terminal p.d. using the external load resistance. **[2 marks]**

Answer / mark points: Lost volts $= Ir = 2.00 \times 0.600 = 1.20$ V. Terminal p.d. $V = E - Ir = 12.6 - 1.20 = 11.4$ V. Also $V = IR = 2.00 \times 5.70 = 11.4$ V.
Teacher prompt / common error: Terminal p.d. is across the external load. Lost volts is across the internal resistance.

10. Account for the 12.6 J transferred by the source for each coulomb of charge by stating the energy per coulomb transferred inside the battery and in the external load. **[2 marks]**

Answer / mark points: The source transfers 12.6 J C^{-1} from chemical to electrical energy. The internal resistance dissipates 1.20 J C^{-1} and the external load receives 11.4 J C^{-1}. These add to 12.6 J C^{-1}.
Teacher prompt / common error: Account for energy per coulomb; do not say that voltage or current is used up.

4 High current operation

11. When the camera and heater operate together, the external load resistance falls to 1.50 Ω. The battery e.m.f. and internal resistance remain 12.6 V and 0.600 Ω. Calculate the current and terminal p.d. [2 marks]

Answer / mark points: $I = 12.6/(1.50 + 0.600) = 6.00$ A. $V = E - Ir = 12.6 - (6.00 \times 0.600) = 9.00$ V. Equivalently, $V = IR = 6.00 \times 1.50 = 9.00$ V.
Teacher prompt / common error: A lower external resistance increases current, increasing Ir and reducing terminal p.d.

12. For the operating condition in Question 11, calculate the power dissipated by the internal resistance and the percentage of the source power delivered to the external load. [2 marks]

Answer / mark points: Internal power = $I^2r = 6.00^2 \times 0.600 = 21.6$ W. The fraction delivered externally is $V/E = 9.00/12.6 = 0.714$, so the percentage is 71.4%.
Teacher prompt / common error: The source power is EI and the load power is VI, so the efficiency can be written V/E.

13. At high altitude the internal resistance increases to 1.50 Ω while the external load remains 1.50 Ω. Calculate the new terminal p.d. and decide whether an instrument that resets below 8.0 V will continue to operate. [3 marks]

Answer / mark points: $I = 12.6/(1.50 + 1.50) = 4.20$ A. $V = IR = 4.20 \times 1.50 = 6.30$ V. This is below 8.0 V, so the instrument resets and does not continue operating normally.
Teacher prompt / common error: A lower current does not guarantee a higher terminal p.d.; the increased internal resistance changes the voltage division.

5 Load test data

14. Rearrange $E = V + Ir$ into the straight-line form for a graph of terminal p.d. V against current I. State what the y-intercept and gradient represent. **[3 marks]**

Answer / mark points: $V = E - Ir$, or $V = -rI + E$. On a graph of V on the y-axis against I on the x-axis, the y-intercept is E and the gradient is $-r$.
Teacher prompt / common error: For V against I, the gradient is negative. Internal resistance is the magnitude of the gradient.

15. The table shows readings from a load test on the balloon battery. Use two widely separated points to calculate the gradient of the V against I graph and hence determine the internal resistance. **[2 marks]**

Current I / A	Terminal p.d. V / V
0.50	12.25
1.00	11.90
1.50	11.55
2.00	11.20
2.50	10.85

Answer / mark points: Gradient = $(10.85 - 12.25)/(2.50 - 0.50) = -1.40/2.00 = -0.700$ V A^{-1}. Since gradient = $-r$, the internal resistance is 0.700 Ω.
Teacher prompt / common error: Use widely separated points and include the sign of the gradient before taking its magnitude for r.

16. Use one row of the table and your value of internal resistance to determine the e.m.f. of the battery. **[1 mark]**

Answer / mark points: Using $I = 0.50$ A and $V = 12.25$ V, $E = V + Ir = 12.25 + (0.50 \times 0.700) = 12.60$ V. Any row gives 12.60 V.
Teacher prompt / common error: The e.m.f. is the intercept at $I = 0$, not one of the loaded terminal p.d. values.

6 Measuring internal resistance

17. A pupil has a cell, an ammeter, a voltmeter, a variable resistor, a switch and connecting wires. Describe an experiment to determine the e.m.f. and internal resistance of the cell. Include a circuit diagram, the measurements, how readings are varied, the graph and how both quantities are obtained. **[6 marks]**

Answer / mark points: Connect the cell, switch, ammeter and variable resistor in series. Connect the voltmeter across the cell terminals, which is also across the variable resistor when meter resistance is very large. Close the switch briefly and record I and V. Change the variable resistance to obtain a suitable range of paired readings, opening the switch between readings to limit heating. Repeat readings where useful. Plot V on the y-axis against I on the x-axis. The y-intercept gives E and the magnitude of the gradient gives r.

Teacher prompt / common error: Essential points are the correct meter positions, paired readings over a range, heating control, V against I, intercept E and gradient magnitude r.

18. The pupil leaves the circuit connected while changing the variable resistor and the cell becomes warm. Explain why this can reduce the validity of the straight-line analysis. State two improvements to the procedure. **[3 marks]**

Answer / mark points: The model $V = E - Ir$ assumes that E and r remain constant. Heating can change one or both, so later readings no longer belong to one straight-line relationship. Improvements include opening the switch between readings, taking readings promptly, limiting the maximum current, allowing the cell to cool and repeating measurements.

Teacher prompt / common error: Do not assume warming only changes the current. The problem is that source properties may vary during the data set.

Language and method check

Current passes through the internal resistance. Potential difference is measured across it.
Terminal p.d. decreases as the current increases because the lost volts Ir increases.
The graph gradient is −r; the internal resistance is the positive magnitude of that gradient.