OCR A A Level Physics | Young modulus solutions

Matching solutions and teaching prompts for the class review | 40 marks

Specification points covered

- 3.4.2(c) Define and use stress, strain and ultimate tensile strength. This worksheet focuses on stress and strain; ultimate tensile strength is reserved for a later worksheet.
- 3.4.2(d)(i) Define Young modulus, E = stress / strain.
- 3.4.2(d)(ii) Use techniques and procedures to determine the Young modulus for a metal.

Class review

Keep the language precise: force acts on a bolt, stress is force per cross-sectional area, strain is a fractional change in length, and Young modulus is stress divided by strain.

1. Building the Young modulus equation chain [4 marks]

Begin with the definitions that connect the load on a component to its deformation.

(a) Write expressions for tensile stress σ and tensile strain ε. Define every symbol that you use. **[2 marks]**

(b) Starting from E = σ / ε, derive ΔL = FL / (AE) for a uniform component in tension. **[2 marks]**

Answer / mark points: (a) σ = F / A, where F is tensile force and A is cross-sectional area. ε = ΔL / L, where ΔL is extension and L is original length. [2]
(b) E = σ / ε = (F / A) / (ΔL / L) [1]. Therefore E = FL / (AΔL), so ΔL = FL / (AE). [1]

Teacher prompt / common error: Stress is not force, and strain is not extension. Insist that the two ratios are named and kept separate before combining them.

2. Checking a trial bolt [4 marks]

A trial bolt has original gauge length 0.180 m and diameter 12.0 mm. A tensile force of 18.0 kN produces an extension of 0.140 mm.

(a) Calculate the tensile stress in the bolt. **[2 marks]**

(b) Calculate the tensile strain in the bolt. **[1 mark]**

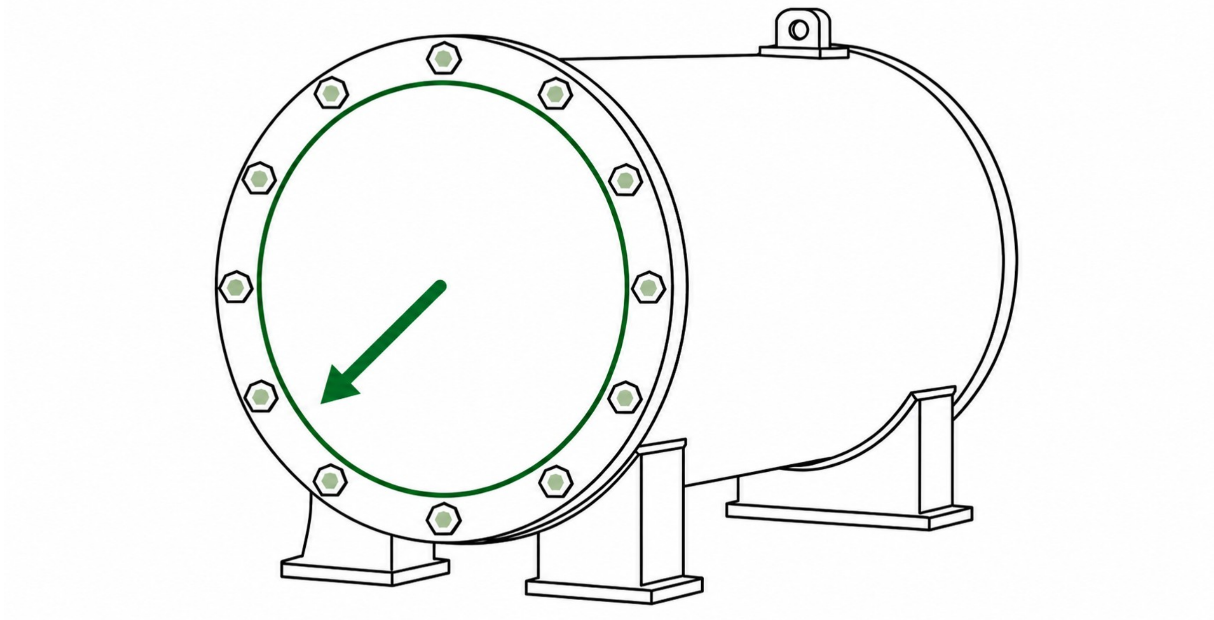
(c) Determine the Young modulus of the bolt material. **[1 mark]**

Answer / mark points: (a) $A = \pi d^2/4 = \pi(0.0120)^2/4 = 1.131 \times 10^{-4}$ m^2. $\sigma = F/A = 18\,000 / (1.131 \times 10^{-4}) = 1.592 \times 10^8$ Pa = 159 MPa. [2]
(b) $\varepsilon = \Delta L/L = 0.140 \times 10^{-3} / 0.180 = 7.778 \times 10^{-4}$. [1]
(c) $E = \sigma/\varepsilon = (1.592 \times 10^8) / (7.778 \times 10^{-4}) = 2.046 \times 10^{11}$ Pa ≈ 205 GPa. [1]

Teacher prompt / common error: The diameter must be halved before squaring if area is calculated as πr^2. Strain has no unit.

Scenario: the Helix composite-curing pressure chamber

The green arrow shows the resultant separating force exerted by the internal pressure; the twelve highlighted bolts share that load in the stated model.



Chamber quantity	Value
Internal pressure above external pressure	1.80 MPa
Circular plate diameter	0.620 m
Number of identical bolts	12
Bolt shank diameter	18.0 mm
Effective bolt length	95.0 mm
Young modulus of bolt material	205 GPa

3. From chamber pressure to bolt tension [3 marks]

Use the pressure difference across the circular access plate and assume that the separating force is shared equally by all twelve bolts.

(a) Calculate the area of the circular access plate. [1 mark]

(b) Calculate the resultant separating force exerted on the plate by the pressure difference. [1 mark]

(c) Calculate the tensile force carried by each bolt. [1 mark]

Answer / mark points: (a) $A = \pi d^2/4 = \pi(0.620)^2/4 = 0.302\ m^2$. [1]

(b) $F = pA = 1.80 \times 10^6 \times 0.302 = 5.434 \times 10^5$ N ≈ 543.4 kN. [1]

(c) Force per bolt = $5.434 \times 10^5 / 12 = 4.529 \times 10^4$ N ≈ 45.29 kN. [1]

Teacher prompt / common error: Use the pressure difference across the plate. The force is shared only after the total pressure force has been calculated.

4. Stress, strain and extension of one chamber bolt **[5 marks]**

The bolts are within the elastic region and each carries the force calculated in Question 3(c).

(a) Calculate the cross-sectional area of one bolt shank. **[1 mark]**

(b) Calculate the tensile stress in one bolt. Give the answer in MPa. **[1 mark]**

(c) Calculate the tensile strain in one bolt. **[1 mark]**

(d) Calculate the extension of one bolt in mm. **[2 marks]**

Answer / mark points: (a) $A = \pi(0.0180)^2/4 = 2.545 \times 10^{-4}$ m^2 = 254.5 mm^2. [1]
(b) $\sigma = F/A = (4.529 \times 10^4) / (2.545 \times 10^{-4}) = 1.780 \times 10^8$ Pa = 178 MPa. [1]
(c) $\varepsilon = \sigma/E = (1.780 \times 10^8) / (205 \times 10^9) = 8.681 \times 10^{-4}$. [1]
(d) $\Delta L = \varepsilon L = 8.681 \times 10^{-4} \times 0.0950$ [1] $= 8.247 \times 10^{-5}$ m = 0.0825 mm. [1]

Teacher prompt / common error: Young modulus links stress to strain. Do not divide force directly by Young modulus; area must be included through stress.

5. Verifying Young modulus using gap sensors **[4 marks]**

Four sensors measure the increase in the plate-to-chamber gap as 0.081 mm, 0.084 mm, 0.083 mm and 0.082 mm. Treat the chamber and plate as rigid, so the mean increase in the gap equals the bolt extension.

(a) Calculate the mean measured extension. **[1 mark]**

(b) Use the mean extension to calculate the experimental strain in a bolt. **[1 mark]**

(c) Use the stress from Question 4(b) to determine an experimental value for Young modulus. **[1 mark]**

(d) Calculate the percentage difference between the experimental value and 205 GPa. **[1 mark]**

Answer / mark points: (a) Mean extension = (0.081 + 0.084 + 0.083 + 0.082) / 4 = 0.0825 mm. [1]
(b) $\varepsilon = (0.0825 \times 10^{-3}) / 0.0950 = 8.684 \times 10^{-4}$. [1]
(c) $E = \sigma/\varepsilon = (1.780 \times 10^8) / (8.684 \times 10^{-4}) = 2.049 \times 10^{11}$ Pa = 204.9 GPa. [1]
(d) Percentage difference = $|204.9 - 205| / 205 \times 100 = 0.036\%$ (about 0.04%). [1]

Teacher prompt / common error: Use the unrounded calculated value when finding percentage difference. A small difference here does not prove every possible systematic effect is absent.

6. One bolt is not carrying load [4 marks]

An automated inspection finds that one bolt is incorrectly seated. Model the same separating force as being shared equally by the remaining eleven bolts.

(a) Calculate the new tensile force and tensile stress in each load-carrying bolt. **[2 marks]**

(b) Calculate the new extension of each load-carrying bolt. **[1 mark]**

(c) Explain why the stress and extension have both increased by the same factor compared with the twelve-bolt case. **[1 mark]**

Answer / mark points: (a) Force per bolt = $5.434 \times 10^5 / 11 = 4.940 \times 10^4$ N = 49.4 kN. [1] Stress = $(4.940 \times 10^4) / (2.545 \times 10^{-4}) = 1.941 \times 10^8$ Pa = 194 MPa. [1]

(b) $\Delta L = FL/(AE) = 4.940 \times 10^4 \times 0.0950 / (2.545 \times 10^{-4} \times 205 \times 10^9) = 0.0900$ mm. [1]

(c) Force per bolt is multiplied by 12/11 = 1.091. With A, L and E unchanged, both $\sigma = F/A$ and $\Delta L = FL/(AE)$ are multiplied by the same factor. [1]

Teacher prompt / common error: The total pressure force is unchanged; only the number of load-carrying bolts changes. Do not subtract one-twelfth of the pressure force.

7. Redesigning the bolt diameter [5 marks]

A redesigned access plate uses eight identical bolts. The same separating force acts on the plate. Each bolt has effective length 110 mm and Young modulus 205 GPa. The extension of each bolt must not exceed 0.100 mm.

(a) Calculate the minimum cross-sectional area required for each bolt. **[3 marks]**

(b) Calculate the corresponding minimum bolt diameter. **[1 mark]**

(c) Available standard diameters are 18.0 mm, 20.0 mm, 22.0 mm and 24.0 mm. Select the smallest suitable diameter. **[1 mark]**

Answer / mark points: (a) Force per bolt = $5.434 \times 10^5 / 8 = 6.793 \times 10^4$ N. [1] From $\Delta L = FL/(AE)$, $A = FL/(E\Delta L)$. [1] $A = 6.793 \times 10^4 \times 0.110 / (205 \times 10^9 \times 0.100 \times 10^{-3}) = 3.645 \times 10^{-4}$ m^2. [1]

(b) $d = \sqrt{(4A/\pi)} = \sqrt{(4 \times 3.645 \times 10^{-4} / \pi)} = 0.0215$ m = 21.5 mm. [1]

(c) Choose 22.0 mm, the smallest standard diameter greater than the 21.5 mm minimum. [1]

Teacher prompt / common error: The required area comes before the diameter. A value of 21.5 mm is a minimum, so a 20.0 mm standard bolt is not acceptable.

8. Quality-control tensile test [6 marks]

A sample from a new bolt batch has original gauge length 0.250 m. Five diameter readings are 6.01 mm, 5.99 mm, 6.00 mm, 6.02 mm and 5.98 mm. The force-extension data are shown below.

Force F / kN	Extension ΔL / mm
0	0
4.0	0.177
8.0	0.354
12.0	0.531
16.0	0.708
20.0	0.884

(a) Calculate the mean diameter of the sample. **[1 mark]**

(b) Calculate the cross-sectional area of the sample. **[1 mark]**

(c) Use the first and last data points to determine the gradient ΔF / ΔL in N m^{-1}. **[2 marks]**

(d) Use E = (L / A)(ΔF / ΔL) to determine the Young modulus of the sample. **[2 marks]**

Answer / mark points: (a) Mean diameter = (6.01 + 5.99 + 6.00 + 6.02 + 5.98) / 5 = 6.00 mm. [1]
(b) $A = \pi(6.00 \times 10^{-3})^2/4 = 2.827 \times 10^{-5}$ m^2. [1]
(c) Gradient = ΔF/ΔL = $20.0 \times 10^3 / (0.884 \times 10^{-3})$ [1] = 2.262×10^7 N m^{-1}. [1]
(d) E = (L/A)(ΔF/ΔL) = $(0.250 / (2.827 \times 10^{-5})) \times 2.262 \times 10^7$ [1] = 2.000×10^{11} Pa ≈ 200 GPa. [1]

Teacher prompt / common error: The force-extension gradient is not Young modulus by itself. Multiply by original length and divide by cross-sectional area.

9. Planning the production Young modulus check **[5 marks]**

A technician must determine Young modulus for a metal sample from each production batch using a tensile test machine.

(a) Describe the measurements, procedure and analysis required to obtain a reliable value for Young modulus. State how the load should be limited during the test. **[5 marks]**

Answer / mark points: (a) Measure the original gauge length L between two reference marks using a ruler or vernier calliper. [1]

Measure the diameter with a micrometer at several positions and orientations, calculate a mean diameter, then calculate $A = \pi d^2/4$. [1]

Apply a series of known tensile forces and measure the corresponding extension ΔL, allowing the reading to settle and repeating measurements where possible. [1]

Plot F against ΔL and determine the gradient of the best-fit straight line using a large triangle; then calculate $E = (L/A)(\Delta F/\Delta L)$. An equivalent stress-strain graph has gradient E. [1]

Keep the load below the limit of proportionality so the sample remains in the linear elastic region; secure the sample and shield the test area in case it slips. [1]

Teacher prompt / common error: A complete method must connect measured quantities to Young modulus. 'Plot a graph' is not enough unless both axes and the use of its gradient are stated.

Review emphasis

Full-credit calculations distinguish total pressure force from force per bolt, use the bolt shank area rather than the plate area, convert every prefix before substitution, and treat strain as dimensionless.