OCR A A Level Simple Harmonic Motion 2 worksheet

35-40 minutes independent practice + 15-20 minutes review | 45 marks

Specification points covered

- 5.3.1(e) Derive velocity for SHM as $v = \pm\omega\sqrt{(A^2 - x^2)}$ and identify $v_{max} = \omega A$.
- 5.3.1(f) Recognise that the period of a simple harmonic oscillator is independent of amplitude (isochronous behaviour).
- 5.3.1(g) Use graphical methods to relate displacement, velocity and acceleration during SHM.
- 5.3.2(a) Describe the interchange between kinetic and potential energy during SHM.
- 5.3.2(b) Interpret energy-displacement graphs for a simple harmonic oscillator.

AquaSol piston-pump calibration

A solar motor turns a flywheel and Scotch-yoke to drive an irrigation pump. The crank pin has radius 40.0 mm, so the working piston displacement is sinusoidal when the flywheel rotates uniformly.
For energy measurements, engineers disconnect the motor and water chamber. The low-friction piston-spring assembly is then a free simple harmonic oscillator of effective mass 1.60 kg. Take displacement to the right as positive.

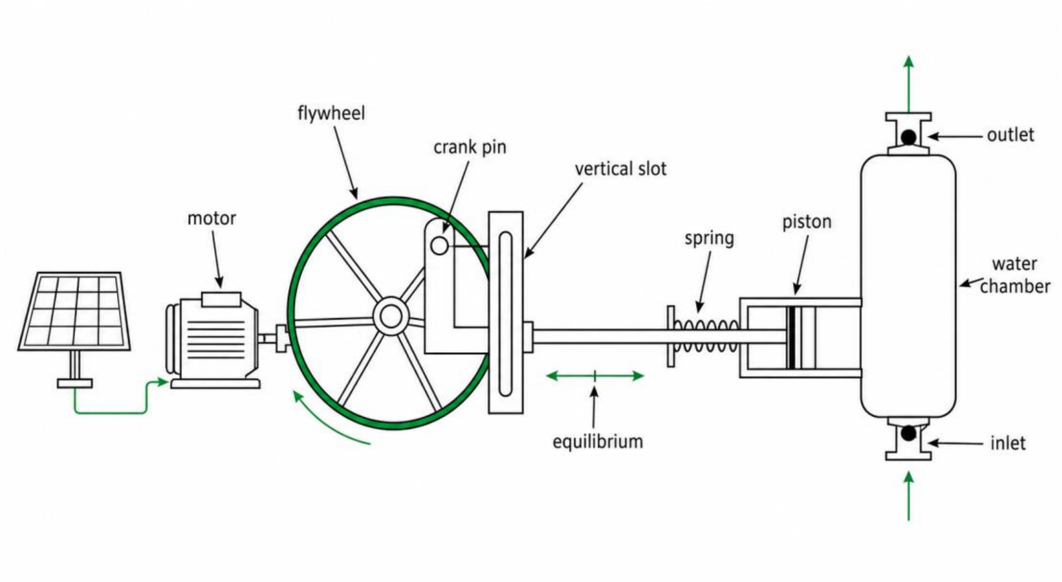

Figure 1: solar-powered Scotch-yoke water pump and calibration spring

1. During normal operation the pump flywheel rotates uniformly at 75.0 rev min^{-1}. One revolution produces one complete piston oscillation. Calculate the oscillation frequency f, period T and angular frequency ω. [3 marks]

1. Reading and modelling the oscillation

2. For a low-friction calibration run, the motor and water chamber are disconnected. The piston-spring assembly is released from rest at x = +40.0 mm when t = 0. Using your angular frequency from Question 1, write an equation for x in terms of t and calculate x at t = 0.100 s. [3 marks]

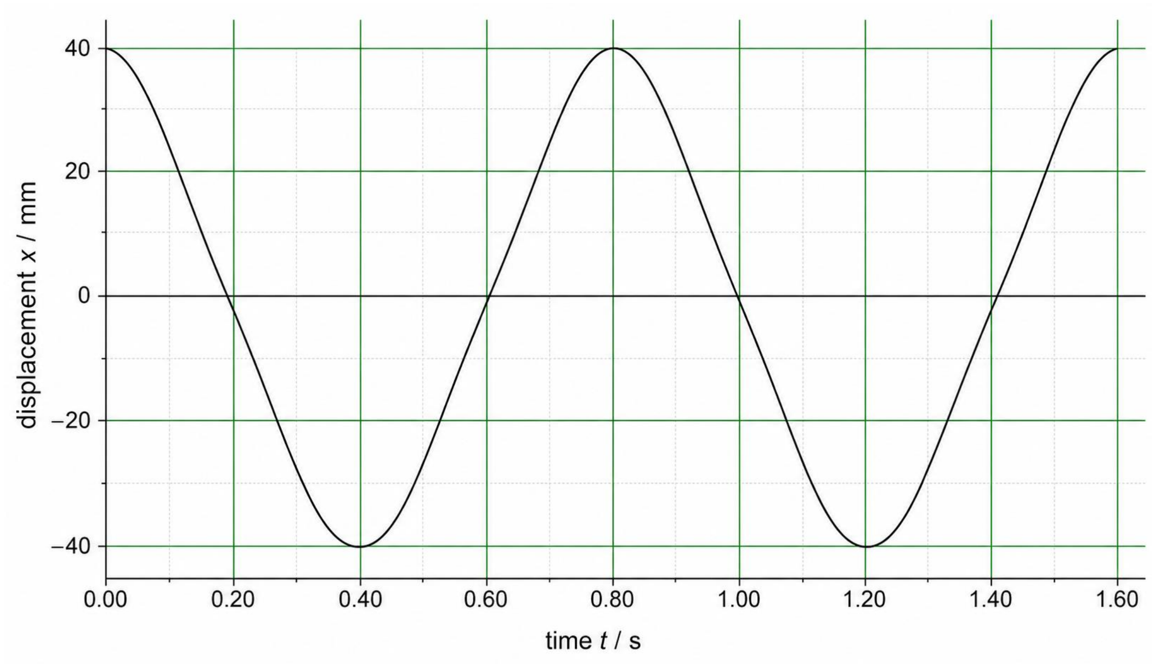

Figure 2: displacement-time record for the free calibration run

3. Figure 2 is the displacement-time record from the calibration run. (a) Determine the amplitude and period. (b) The positive direction is to the right. Use the gradient of the graph to state the direction of the piston velocity at t = 0.300 s. [3 marks]

2. Maximum values and signed velocity

4. Calculate the maximum speed and the maximum magnitude of the acceleration of the piston. State the position or positions at which each maximum occurs. [4 marks]

5. At one instant the piston has displacement $x = -24.0$ mm and is moving in the negative direction. Use $v = \pm\omega\sqrt{(A^2 - x^2)}$ to calculate its signed velocity. Calculate its acceleration at the same instant and state how the acceleration direction compares with the velocity direction. [4 marks]

6. Starting with $x = A\cos(\omega t)$, derive $v = \pm\omega\sqrt{(A^2 - x^2)}$. Your derivation must show why there are two possible signs and how $v_m$ax $= \omega A$ follows. [3 marks]

3. Amplitude and energy calculations

7. The calibration is repeated with amplitude 60.0 mm but with the same piston-spring assembly. State the new period, calculate the new maximum speed, and determine the factor by which the maximum kinetic energy changes. [3 marks]

8. The effective oscillating mass is 1.60 kg. For the 40.0 mm amplitude calibration, calculate the total mechanical energy. At x = 24.0 mm, calculate the elastic potential energy and kinetic energy, then use the kinetic energy to calculate the piston speed. [5 marks]

4. Energy-displacement graphs

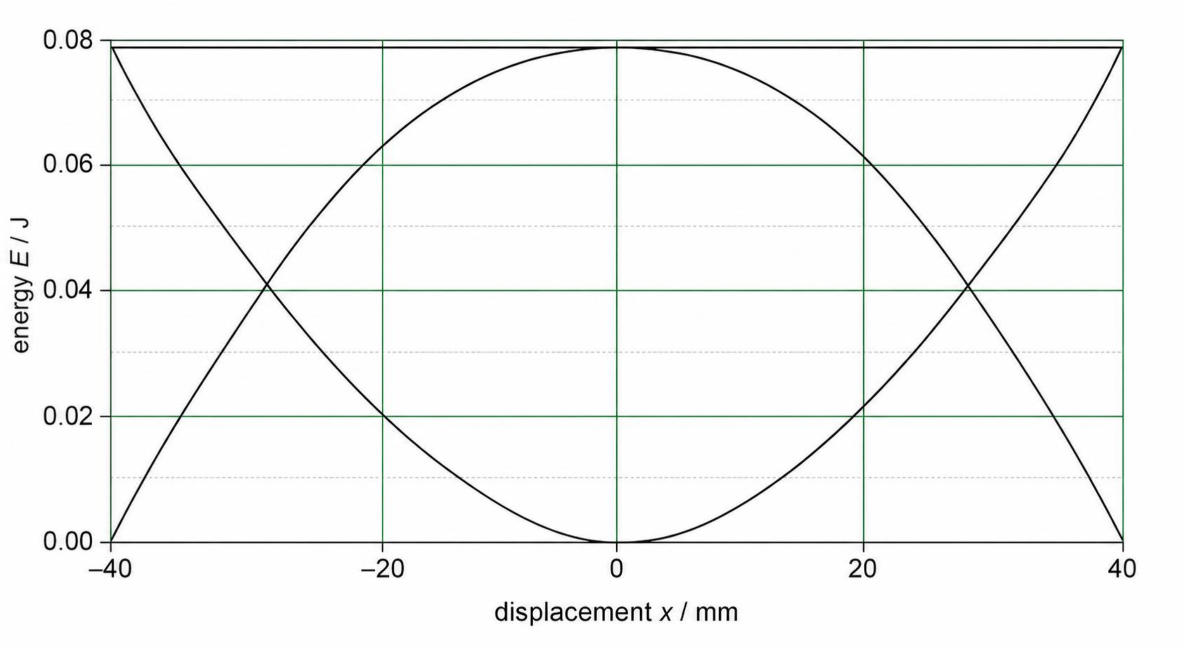

Figure 3: energy against displacement for the free calibration run

9. Figure 3 shows three energy-displacement curves for the free calibration run. (a) Label the total mechanical energy, elastic potential energy and kinetic energy curves. (b) Estimate the magnitude of the displacement where the kinetic and elastic potential energies are equal. (c) Describe the energy transfers as the piston moves from the negative extreme, through equilibrium, to the positive extreme. [5 marks]

5. Displacement, velocity and acceleration

10. On Figure 4, draw displacement x, velocity v and acceleration a against time for one complete cycle. The piston begins at x = +A at t = 0. Show the correct zero crossings and label the positive and negative extrema of all three quantities. **[6 marks]**

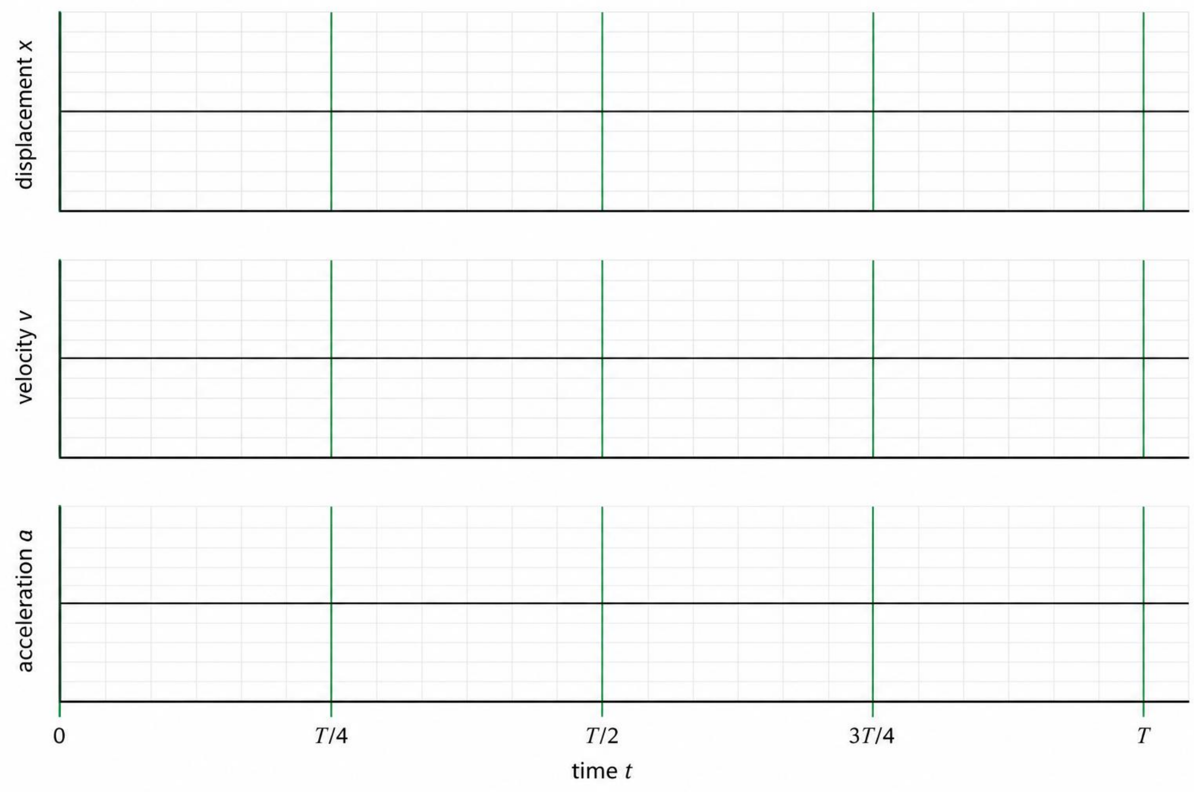

Figure 4: draw the three quantities over the same complete cycle

6. Reading the phase relationships

11. Use your graphs in Figure 4. (a) State x, v and a at $t = T/4$. (b) State the time offset between corresponding extrema of displacement and velocity. (c) State the phase relationship between displacement and acceleration. **[3 marks]**

12. During water pumping the crank mechanism enforces an approximately sinusoidal piston displacement. A student concludes that the piston's total mechanical energy must therefore be constant. Explain why this conclusion does not follow and identify the important energy transfers during pumping. **[3 marks]**

Review and improve

- ☐ I use the sign of the graph gradient to state the velocity direction.
- ☐ I place maximum speed at equilibrium and maximum acceleration magnitude at the turning positions.
- ☐ I keep the ideal free-oscillator energy model separate from the driven, dissipative pump.