OCR A A Level Simple Harmonic Motion 3 worksheet

35-40 minutes independent practice + 15-20 minutes review | 47 marks

Specification points covered

- 5.3.1(c)(i) Define simple harmonic motion using $a = -\omega^2 x$.
- 5.3.1(e) Derive velocity for SHM as $v = \pm\omega\sqrt{(A^2 - x^2)}$ and identify $v_max = \omega A$.
- 5.3.2(a) Describe interchange between kinetic and potential energy during SHM.
- 5.3.2(b) Interpret energy-displacement graphs for a simple harmonic oscillator.
- 5.3.3(a) Distinguish between free and forced oscillations.
- 5.3.3(b)(i) Describe the effects of damping on an oscillating system.
- 5.3.3(b)(ii) Observe and describe forced and damped oscillations for a range of systems.
- 5.3.3(c) Define resonance and natural frequency and describe resonance conditions.
- 5.3.3(d) Sketch and interpret amplitude-frequency response graphs for forced oscillations.
- 5.3.3(e) Give practical examples of forced oscillations and resonance in real systems.

Offshore wind-turbine stabiliser

A sliding carriage inside a wind-turbine nacelle is attached to two identical pre-tensioned springs. An adjustable hydraulic damper transfers oscillation energy thermally to its fluid and the surroundings.
The rotor and flexible tower can exert a continuing periodic driving force. Take displacement to the right as positive. Ignore friction except when damping is stated.

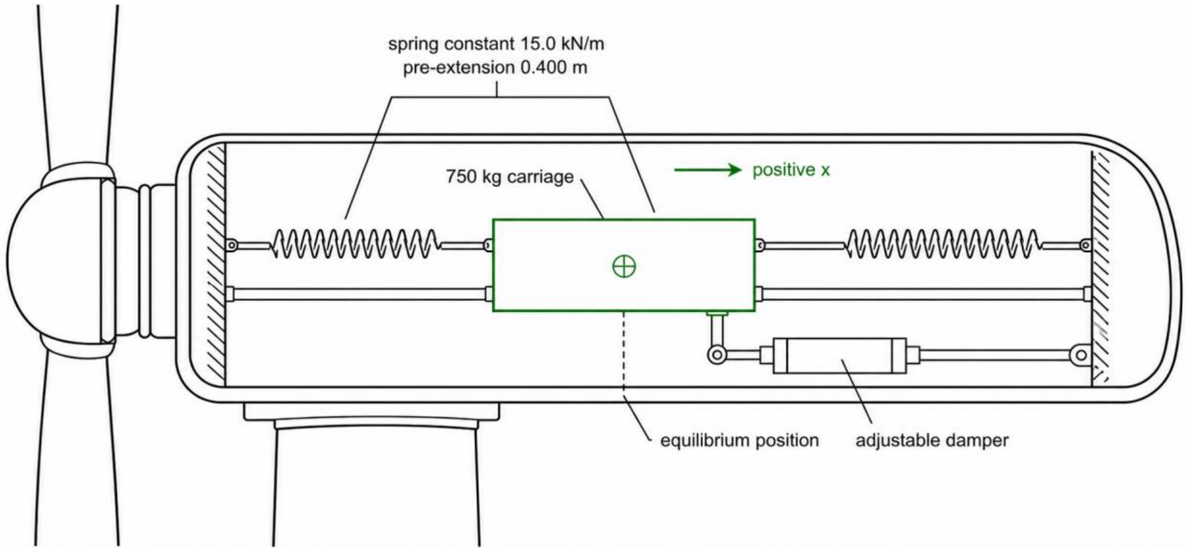

Figure 1: pre-tensioned two-spring carriage and adjustable damper

1. Natural motion of the carriage

1. The carriage is displaced a distance x to the right. The left spring extension becomes $e_0 + x$ and the right spring extension becomes $e_0 - x$. Use the two spring forces to show that the resultant horizontal force is $F = -2k_s x$. Explain how this establishes SHM and state the effective force constant of the spring combination. **[4 marks]**

2. The carriage mass is 750 kg and each spring has force constant $k_s = 15.0$ kN m^{-1}. Calculate the natural angular frequency ω_0, natural frequency f_0 and natural period T_0 of the carriage. **[3 marks]**

3. In a low-damping release test the carriage amplitude is 0.120 m. Calculate the maximum speed and maximum acceleration magnitude of the carriage. **[2 marks]**

2. Springs that are always in tension

4. At equilibrium, each spring is already extended by $e_0 = 0.400$ m. (a) Starting with $U = 1/2k_s(e_0 + x)^2 + 1/2k_s(e_0 - x)^2$, show that the total elastic potential energy is $U = k_se_0^2 + k_sx^2$. (b) Calculate the minimum value of U and its value at x = +0.120 m. (c) Calculate both spring extensions at x = +0.120 m and use them to explain why neither spring becomes slack and why the elastic potential energy is never zero. (d) Explain why an SHM energy graph may nevertheless show potential energy equal to zero at equilibrium. **[7 marks]**

3. Damped free oscillation

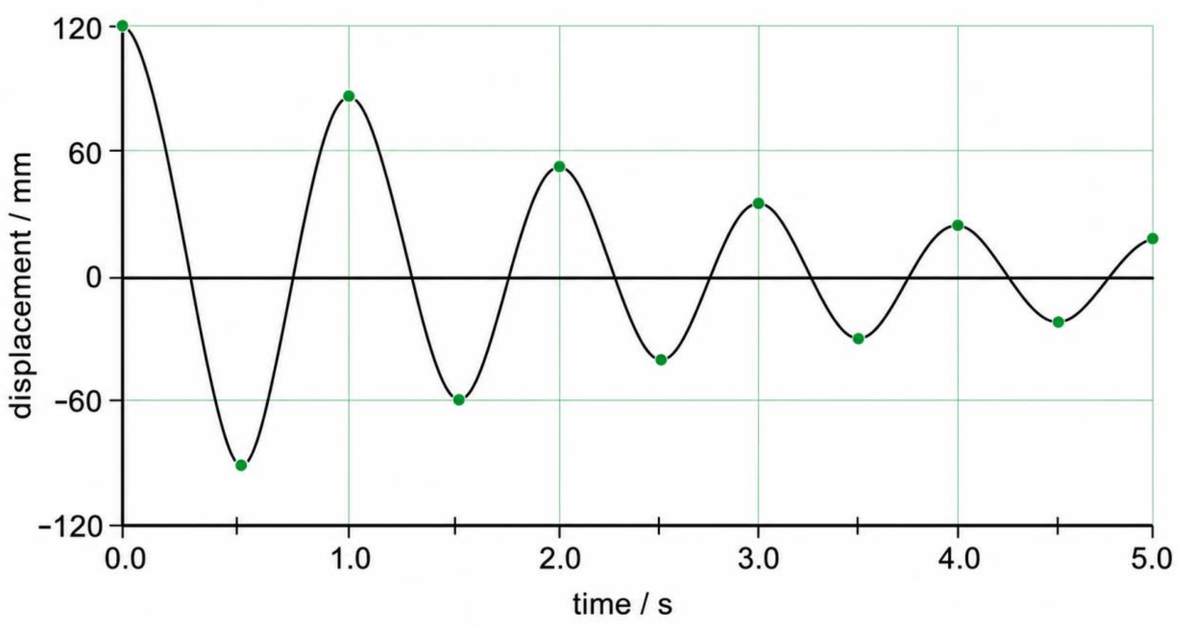

Figure 2: displacement-time record after release

5. Figure 2 is a displacement-time record after the carriage is released from rest. (a) Determine the period. (b) Read the amplitude at t = 2.0 s and calculate the fraction of the initial oscillation energy that remains, using E proportional to A^2. (c) At the first equilibrium crossing, state the directions of the carriage velocity and the damping force. [4 marks]

6. Describe the energy transfers from the initial positive turning position, through the first equilibrium crossing, to the first negative turning position. Your answer must distinguish kinetic energy, the displacement-dependent elastic potential energy and energy dissipated by the damper. [4 marks]

4. Free, forced and resonant motion

7. Engineers use two tests. In Test 1 the carriage is displaced and released while the nacelle is stationary. In Test 2 a rotating imbalance produces a continuing periodic force on the nacelle. State which test is a free oscillation and which is a forced oscillation, and give the reason for each choice. [2 marks]

8. Define natural frequency and resonance. Explain why a forced oscillator has a large steady-state amplitude when the driving frequency is close to its natural frequency. [3 marks]

5. Reading the resonance curves

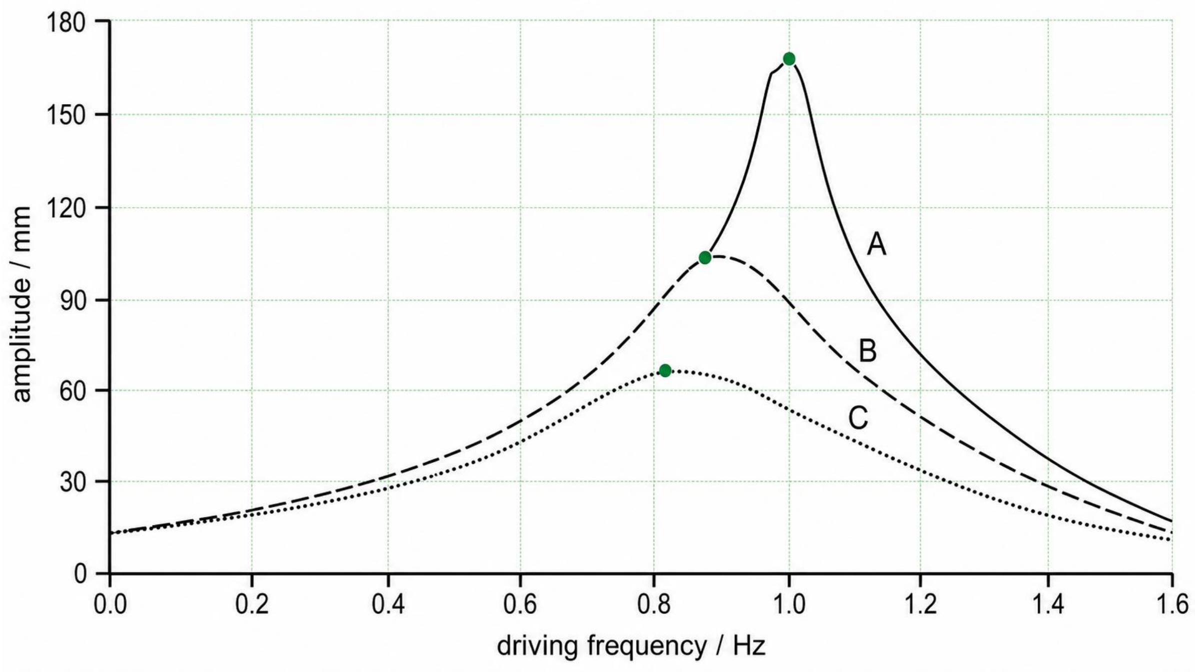

Figure 3: response of the same carriage at three damper settings

9. Figure 3 shows the amplitude-frequency responses for damper settings A, B and C. (a) Determine the resonant frequency for setting A and identify the most heavily damped setting. (b) Describe how increasing the damping changes the height, width and position of the resonance peak. [5 marks]

6. Choosing a safe damper setting

10. At one operating condition the rotor produces one force pulse per revolution and rotates at 58.0 rev min^{-1}. Calculate the driving frequency and explain why this operating condition requires careful damping design. [2 marks]

11. Table 1 gives measured performance at 58.0 rev min^{-1}. The design requires carriage amplitude below 120 mm and nacelle acceleration below 0.025 m s^{-2}. Select the only acceptable damper setting and justify your decision using both requirements. [3 marks]

Table 1: measured performance at 58.0 rev min^{-1}

Damper setting	Carriage amplitude / mm	Nacelle acceleration / m s^{-2}
A	160	0.014
B	105	0.019
C	62	0.030

12. With the selected setting, the damper dissipates 210 J during each steady oscillation cycle at 58.0 rev min^{-1}. Calculate the mean power dissipated. Explain why the amplitude can remain constant even though energy is continuously dissipated. [3 marks]

7. Measuring a response curve

13. Describe a method the engineers could use to obtain an amplitude-frequency response curve and compare two damper settings. Include how the oscillator is driven and measured, the variables that must be controlled, how the frequency range should be sampled, and how the results should be processed. [5 marks]

Review and improve

- ☐ I distinguish a free oscillation from a continuing periodic drive.
- ☐ I describe damping as an energy transfer and use precise quantity language.
- ☐ I remember that choosing zero potential energy at equilibrium does not mean the pre-tensioned springs store no energy.