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SOLUTIONS

ATTEMPT CHECK CORRECT IMPROVE

OCR A A Level Simple Harmonic Motion 1 solutions

Matching solutions and teaching prompts for the class review

Specification points covered

- 5.3.1(a) Define displacement, amplitude, period, frequency, angular frequency and phase difference in oscillations.
- 5.3.1(b) Use angular frequency $\omega = 2\pi/T = 2\pi f$.
- 5.3.1(c)(i) Define simple harmonic motion using $a = -\omega^2 x$.
- 5.3.1(c)(ii) Use techniques and procedures to determine the period or frequency of simple harmonic oscillations.
- 5.3.1(d) Use $x = A\cos(\omega t)$ or $x = A\sin(\omega t)$, selected from the initial conditions.
- 5.3.1(g) Use graphical methods to relate displacement and acceleration during SHM.

Teaching focus

Tie sine and cosine to x(0) and the initial direction of motion. ‘Driven’ does not determine the phase convention.
Use the language chain: displacement has a sign; acceleration is towards equilibrium; velocity is zero at a turning point.

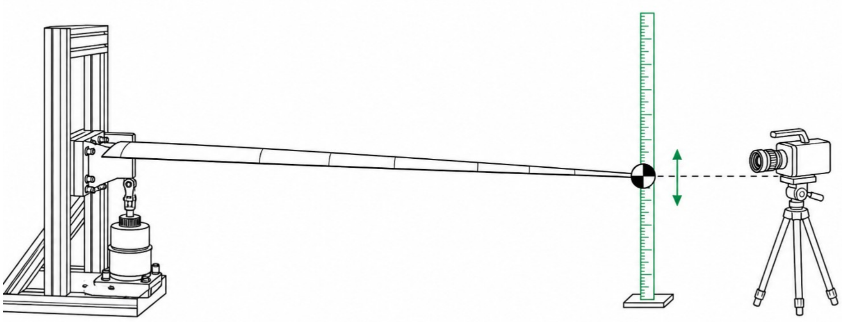

Figure 1: wing-tip optical measurement arrangement

1. The upper and lower turning positions of the wing-tip marker are 0.360 m apart. (a) Determine the amplitude. (b) Upward is positive. State the displacement when the marker is 0.120 m below equilibrium. (c) State one distinction between displacement and distance travelled. [3 marks]

Answer / mark points: (a) A = 0.360/2 = 0.180 m. (b) x = -0.120 m. (c) Displacement is the signed position measured from equilibrium in a specified direction; distance travelled is a scalar path length and is never negative.

Teacher prompt / common error: Amplitude is half the peak-to-peak separation. Keep the sign on displacement; do not attach a sign to distance travelled.

2. Slow-motion footage shows 18 complete oscillations in 12.0 s. Calculate the frequency, period and angular frequency of the wing tip. Explain why timing many oscillations gives a more reliable period than timing one oscillation. **[4 marks]**

Answer / mark points: $f = N/t = 18/12.0 = 1.50$ Hz. $T = 1/f = 0.667$ s. $\omega = 2\pi f = 2\pi \times 1.50 = 9.42$ rad s^{-1}. Timing many cycles makes the measured interval longer, so the same frame-selection or timing uncertainty is a smaller percentage of the result; divide the mean time by the number of cycles.
Teacher prompt / common error: Angular frequency is 2π times frequency and has unit rad s^{-1}. Reward an explicit percentage-uncertainty argument for timing many cycles.

1. Angular frequency and the SHM condition

3. State the condition for motion to be simple harmonic. Include the defining equation and explain the meaning of its negative sign. [3 marks]

Answer / mark points: For SHM, acceleration is directly proportional to displacement from equilibrium and is opposite in direction: $a = -\omega^2 x$. The negative sign shows that the acceleration is always directed towards equilibrium, so it is a restoring acceleration.

Teacher prompt / common error: Require both proportionality and the opposite direction. 'Moves backwards and forwards' is not a definition of SHM.

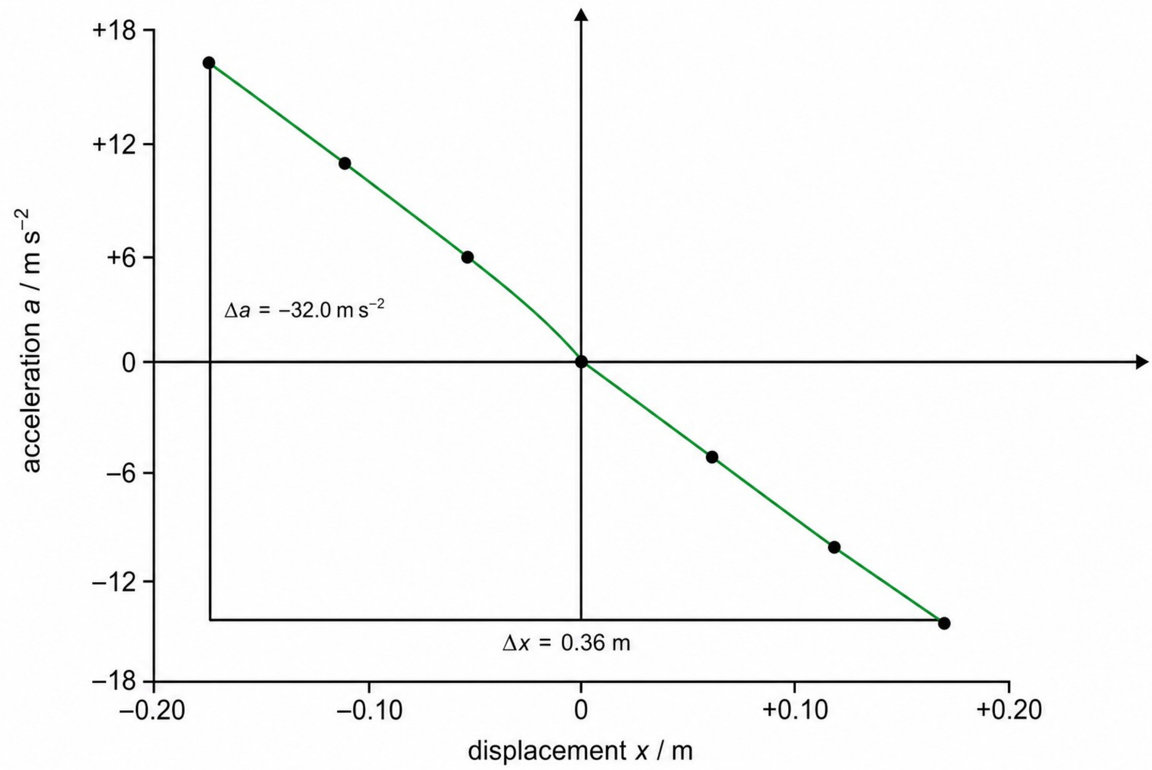

Figure 2 solution: a large gradient triangle uses widely separated points

4. Figure 2 shows acceleration a plotted against displacement x for the free wing-tip oscillation. (a) Use widely separated points on the line to determine its gradient, including the unit. (b) Hence calculate the angular frequency and frequency. (c) Explain how the graph supports the claim that the motion is simple harmonic. [6 marks]

Answer / mark points: Using the endpoints shown, gradient = $\Delta a/\Delta x$ = [-16.0 - (+16.0)]/[0.180 - (-0.180)] = -32.0/0.360 = -88.9 s^{-2}. Since gradient = $-\omega^2$, $\omega = \sqrt{88.9}$ = 9.43 rad s^{-1}. $f = \omega/(2\pi)$ = 1.50 Hz. The points lie on a straight line through the origin with a negative gradient, so a is directly proportional to -x, which is the defining condition for SHM.

Teacher prompt / common error: The gradient must be negative and have unit s^{-2}. Use its magnitude only when taking the square root to obtain ω.

2. Free oscillation from a turning point

5. Run A begins when the wing tip is released from rest at its upper turning point. The amplitude is 0.180 m and the frequency is 1.50 Hz. (a) Explain why a cosine model is suitable and write an equation for displacement x at time t. (b) Calculate x at t = 0.250 s. **[5 marks]**

Answer / mark points: At $t = 0$ the marker is at its positive maximum displacement, so $\cos(0) = 1$ gives the required initial condition. $\omega = 2\pi f = 9.42$ rad s^{-1}, hence $x = 0.180 \cos(9.42t)$, with x in metres and t in seconds. At $t = 0.250$ s, $x = 0.180 \cos(9.42 \times 0.250) = -0.127$ m.
Teacher prompt / common error: Cosine follows from $x(0) = +A$, not from the fact that the motion is free. Calculator angles must be in radians.

6. For Run A, calculate the first time after release at which the wing tip passes through equilibrium. Then calculate the acceleration at t = 0.250 s and state its direction. **[3 marks]**

Answer / mark points: The first equilibrium crossing occurs after one quarter-period: $t = T/4 = 1/(4f) = 1/(4 \times 1.50) = 0.167$ s. At 0.250 s, $a = -\omega^2 x = -88.9 \times (-0.127) = +11.3$ m s^{-2}. Positive is upward, so the acceleration is upward, towards equilibrium.
Teacher prompt / common error: At negative displacement, acceleration is positive. Train the direction statement ‘towards equilibrium’ rather than ‘velocity is accelerating’.

3. Trigger position, phase and driven motion

7. In Run B, a shaker drives the wing at 1.20 Hz with amplitude 0.125 m. The timer is triggered as the marker passes through equilibrium moving upward. (a) Write the displacement equation. (b) Calculate the displacement at t = 0.150 s. (c) Calculate the first time after the trigger at which the marker reaches its upper turning point. [5 marks]

Answer / mark points: The trigger has $x = 0$ and positive velocity, so use sine with a positive sign. $\omega = 2\pi \times 1.20 = 7.54$ rad s^{-1}. (a) $x = 0.125 \sin(7.54t)$. (b) $x = 0.125 \sin(7.54 \times 0.150) = +0.113$ m. (c) The first upper maximum occurs after $T/4$: $t = 1/(4 \times 1.20) = 0.208$ s.

Teacher prompt / common error: Sine follows from $x(0) = 0$ with positive velocity. A downward equilibrium crossing would need the negative sign.

8. A repeat of Run B is timed from the instant the marker passes through equilibrium moving downward. Write the new displacement equation and explain why describing an oscillator as 'driven' does not, by itself, decide whether sine or cosine should be used. [2 marks]

Answer / mark points: The marker begins at $x = 0$ with negative velocity, so $x = -0.125 \sin(7.54t)$. Sine or cosine is selected by the displacement and direction at the chosen $t = 0$; being driven does not fix that phase convention.

Teacher prompt / common error: The time origin fixes phase. A driven steady-state oscillation may be written with sine, cosine or a phase constant depending on the reference chosen.

9. Run C is a free oscillation of the same wing, but its amplitude is 0.090 m. Compare its period with Run A and calculate its maximum acceleration. State the physical principle used in the comparison. [3 marks]

Answer / mark points: For the same oscillator in SHM, the period is independent of amplitude (isochronous behaviour), so T remains 0.667 s. The angular frequency is unchanged and maximum acceleration $= \omega^2 A = 88.9 \times 0.090 = 8.00$ m s^{-2}, half the Run A value.

Teacher prompt / common error: Isochronous means period is independent of amplitude for ideal SHM. Maximum acceleration remains proportional to amplitude at fixed ω.

10. A stroboscope flashing at 1.50 Hz makes the marker appear stationary. Explain what this observation suggests and why the observation alone does not prove that the wing frequency is exactly 1.50 Hz. [2 marks]

Answer / mark points: The marker is sampled at the same phase on successive flashes, which is consistent with a 1.50 Hz oscillation. However, stroboscopic aliasing can also give a stationary image when the oscillator completes an integer number of cycles between flashes, so the true frequency may be an integer multiple of the flash frequency. Vary the flash rate or confirm with timed video.

Teacher prompt / common error: A stationary stroboscopic image is not a unique frequency measurement because of aliasing. A small change in flash rate helps reveal the apparent drift.

4. Practical: measuring and testing SHM

11. Scaffolded six-mark practical. Describe how slow-motion video can be used to determine the amplitude and frequency of the wing-tip motion and test whether it is consistent with SHM. Organise your answer around: the fiducial marker and scale; camera position and time base; release or drive conditions; measurements, repeats and uncertainty; calculation of A, T, f and ω; and a graph or model that tests $a = -\omega^2 x$. [6 marks]

Answer / mark points: Fix a bright fiducial marker to the wing tip and place a vertical scale in the same plane as the marker. Mount the camera securely, perpendicular to the plane of motion, far enough away to reduce perspective error; use a known slow-motion frame rate or visible time stamps. Mark equilibrium, displace the wing by a small measured amount and release it from rest without a push, or start a repeatable controlled drive. Record many complete cycles. Calibrate pixels against the scale and read the upper and lower extremes; A is half the peak-to-peak displacement. Use corresponding same-phase frames separated by N cycles to find $T = \Delta t/N$; repeat clips or intervals, calculate a mean, then $f = 1/T$ and $\omega = 2\pi f$. More cycles reduce percentage timing uncertainty; quote scale and frame-time resolution. Use tracking software to obtain x against t and fit $x = A\cos(\omega t + \varphi)$, checking residuals. Alternatively obtain acceleration values from a smooth fitted curve and plot a against x. A straight line through the origin with negative gradient $-\omega^2$, consistent with the independently measured ω within uncertainty, supports SHM.

Teacher prompt / common error: Full credit needs geometry and calibration, a known time base, many cycles and repeats, quantitative A/T/f/ω analysis, and a genuine proportionality/model test with uncertainty.

5. Language precision

12. A student writes, 'At the upper turning point, the velocity is accelerating downwards and is greatest.' Rewrite this statement using precise language for velocity and acceleration. [3 marks]

Answer / mark points: At the upper turning point the velocity is zero. The acceleration has its maximum magnitude and is directed downward, towards the equilibrium position. A quantity does not 'accelerate': velocity changes, while acceleration is the rate of change of velocity.

Teacher prompt / common error: Use the nouns correctly: velocity can increase, decrease or change direction; acceleration has magnitude and direction. At a turning point, velocity is zero.

Language and method check

Say 'the acceleration has maximum magnitude and is towards equilibrium'; never 'the velocity is accelerating'.
A convincing SHM test uses a range of x values and the predicted negative straight-line relationship, not one matching time or one stroboscopic image.