Radioactive decay calculations solutions

Activity, half-life and a time-critical delivery

Matching answers and examiner-style commentary

OCR A H556 | A-level physics | Module 6 | 36 marks

Specification points covered

6.4.3(d): activity and the becquerel; $A = \lambda N$.

6.4.3(e)(i): decay constant and half-life, $\lambda t_{1/2} = \ln 2$.

6.4.3(f)(i): exponential changes in N and A, including time calculations.

For the illustrative tracer X, $t_{1/2}$ = 90.0 min. Use ln 2 = 0.693, one minute = 60 s and one hour = 3600 s. The detector signal is due only to X; detection fraction is constant unless stated.

1 Rates and probabilities

An isotope laboratory sends tracer X to an island measurement centre. The instrument requires a specified minimum activity when its scheduled measurement begins.

Dispatch is t = 0. The laboratory initially considers a vial with activity 480 MBq.

Every interval after dispatch counts towards the decay time. The route times in Figure 1 are also stated in Table 1.

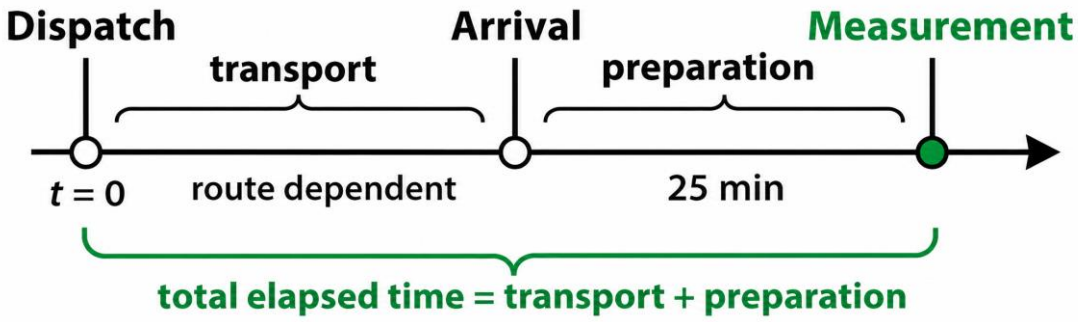


Figure 1 The full elapsed time starts at dispatch and ends at measurement.

1(a) A vial has activity 480 MBq when it leaves the laboratory. State what this activity means, including the number and type of events per second. **[1 mark]**

480 MBq means an average of 480 × 10^6 nuclear decays each second. It does not mean that the detector must record 480 million counts per second.

Marking guidance 1: 4.80 × 10^8 nuclear decays per second.

1(b) Calculate the decay constant of X in min^{-1} and in s^{-1}. Retain an unrounded value for later calculations. **[2 marks]**

Half-life and decay constant are related by $\lambda = \ln 2 / t_{1/2}$.

$\lambda = 0.693 / 90.0 = 0.007700\ min^{-1}$.

To use seconds, $t_{1/2} = 90.0 \times 60 = 5400$ s, so $\lambda = 0.693 / 5400 = 1.2833 \times 10^{-4}\ s^{-1}$.

Marking guidance 1: 7.70 × 10^{-3} min^{-1}. 1: 1.28 × 10^{-4} s^{-1} with consistent conversion.

2 Choose a route using the activity on arrival

Table 1 Scheduled intervals after dispatch

Route	Transport / min	Preparation / min
Ferry	95	25
Aircraft	55	25

2(a) Use Table 1 to determine the elapsed time from dispatch until measurement for each route. **[1 mark]**

Ferry: t = 95 + 25 = 120 min. Aircraft: t = 55 + 25 = 80 min. Both include preparation after transport.

Marking guidance 1: both elapsed times, 120 min and 80 min.

2(b) Calculate the activity at the measurement time for the ferry route, starting from 480 MBq. **[2 marks]**

Step 1 Choose the exponential activity equation and consistent minute units.

$A = A_0\, e^{-\lambda t} = 480\, e^{-0.007700 \times 120}$ MBq.

Step 2 Evaluate the surviving activity: A = 190.5 MBq ≈ 191 MBq.

The exponent is dimensionless because λ is in min^{-1} and t is in min.

Marking guidance 1: correct exponential with full 120 min. 1: 191 MBq (allow 190–191 depending on ln 2 precision).

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2(c) The instrument test requires at least 240 MBq when measurement starts. Calculate the aircraft-route activity and decide which route meets the requirement for this 480 MBq vial. **[2 marks]**

For the aircraft route, $A = 480\ e^{-0.007700 \times 80} = 259.2$ MBq ≈ 259 MBq.

The aircraft route meets 240 MBq; the ferry route does not. This is a decision about the activity at measurement, rather than at arrival or dispatch.

Marking guidance 1: aircraft activity 259 MBq. 1: aircraft passes and ferry fails, using the threshold.

2(d) Calculate the fraction of the original nuclei that has decayed by the ferry measurement time. Explain why it is not the same as A/A_0. **[2 marks]**

$A/A_0 = N/N_0 = e^{-0.924} = 0.39693$ is the fraction remaining.

The fraction that has decayed is 1 − 0.39693 = 0.60307, or 60.3%. A/A_0 counts the undecayed parent fraction, so it is the complement of the requested fraction.

Marking guidance 1: surviving fraction about 0.397. 1: decayed fraction about 0.603, with correct distinction.

3 How many nuclei are in the vial?

3(a) Calculate the number of undecayed X nuclei in the vial at dispatch. Use the initial activity 480 MBq. **[2 marks]**

Step 1 Convert activity to decays per second: $A_0 = 4.80 \times 10^8$ Bq.

Step 2 Rearrange $A = \lambda N$. Because activity is in Bq, use λ in s^{-1}.

$N_0 = A_0 / \lambda = (4.80 \times 10^8) / (1.2833 \times 10^{-4}) = 3.740 \times 10^{12}$ nuclei ≈ 3.74×10^{12} nuclei.

Marking guidance 1: A in Bq with λ in s^{-1} and correct rearrangement. 1: 3.74×10^{12} nuclei.

3(b) Each X nucleus has an approximate mass of 1.99×10^{-25} kg. Calculate the mass of undecayed X in the vial at dispatch. The vial also contains a carrier liquid; do not include that liquid. **[1 mark]**

Multiply the number of X nuclei by the mass of one X nucleus.

$m = (3.740 \times 10^{12})(1.99 \times 10^{-25}) = 7.44 \times 10^{-13}$ kg.

This is the mass of the active isotope, rather than the total vial contents.

Marking guidance 1: 7.44×10^{-13} kg.

3(c) Calculate the number of X nuclei that decay during the full ferry interval. **[2 marks]**

Step 1 Use the fraction decayed, not the fraction remaining.

$\Delta N = N_0[1 - e^{-\lambda t}] = (3.740 \times 10^{12})(1 - e^{-0.924})$.

Step 2 Evaluate: $\Delta N = 2.256 \times 10^{12} \approx 2.26 \times 10^{12}$ nuclei. Each parent nucleus is counted once in this loss.

Marking guidance 1: N_0(1 − exponential) or equivalent difference. 1: 2.26×10^{12}.

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3(d) A second isotope Y has half-life 30.0 min. Compare its activity with X when the samples contain equal numbers of undecayed nuclei. Then compare their numbers of nuclei if instead their activities are equal. **[2 marks]**

Y has one third of X's half-life, so its decay constant is three times as large. With equal N, $A = \lambda N$ means $A_Y = 3A_X$.

If the activities are equal, $N = A/\lambda$ means $N_Y = N_X/3$. Equal activity does not imply equal numbers of nuclei.

Marking guidance 1: activity ratio 3 for equal N. 1: nucleus-number ratio 1/3 for equal A, with relationship.

4 Plan the dispatch, rather than hope it arrives

4(a) Calculate the minimum dispatch activity needed for the ferry route to provide 240 MBq at measurement, assuming the scheduled 120 min elapsed time. **[2 marks]**

Step 1 Make the dispatch activity the unknown in $A = A_0 e^{-\lambda t}$.

$A_0 = A e^{\lambda t} = 240\ e^{0.007700 \times 120}$ MBq.

Step 2 Evaluate: $A_0 = 604.6$ MBq, so approximately 605 MBq is needed. The dispatch activity must be greater than the required later activity.

Marking guidance 1: correct rearrangement with 120 min. 1: about 605 MBq.

4(b) A 600 MBq vial is now available. Find the latest elapsed time after dispatch at which it can meet the 240 MBq requirement. Show how the logarithm is used. **[3 marks]**

Step 1 Form a dimensionless ratio: $A/A_0 = 240/600 = 0.400$.

Step 2 Take natural logarithms of $A/A_0 = e^{-\lambda t}$: $\ln(0.400) = -\lambda t$.

Step 3 Isolate the positive time: $t = -\ln(0.400)/0.007700 = 118.999$ min ≈ 119 min.

A negative logarithm divided by $-\lambda$ gives a positive elapsed time. Do not take the logarithm of an activity with units on its own.

Marking guidance 1: ratio 0.400. 1: logarithmic rearrangement with correct sign. 1: 119 min.

4(c) For the 600 MBq vial, the ferry route takes 120 min before measurement and the aircraft route takes 80 min. Calculate how much additional delay the aircraft route can tolerate. Explain why the ferry route already fails. **[2 marks]**

The latest acceptable elapsed time is 119 min, measured from dispatch.

Aircraft delay allowance = 119 − 80 = 39.0 min. The ferry route already takes 120 min, which is beyond the 119 min limit; even its scheduled journey gives slightly less than 240 MBq.

Marking guidance 1: 39.0 min additional aircraft delay. 1: ferry time compared with full maximum elapsed time.

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4(d) For the original 480 MBq vial, calculate the activity after 4.50 h. Give a quick half-life check on the answer. **[1 mark]**

4.50 h = 270 min = three half-lives. Therefore $A = 480 / 2^3 = 60.0$ MBq.

The exponential form gives the same result to rounding: $480\ e^{-0.007700 \times 270} \approx 60.0$ MBq.

Marking guidance 1: 60.0 MBq.

5 Check the measurement system

5(a) At one measurement time the gross rate is 3200 s^{-1} and the background is 200 s^{-1}. The apparatus records a fraction 1.25×10^{-5} of the source's decays. Calculate the activity. **[2 marks]**

Step 1 Remove background: $R = 3200 - 200 = 3000\ s^{-1}$.

Step 2 Use $R = \varepsilon A$, where ε is the detected fraction.

$A = R/\varepsilon = 3000 / (1.25 \times 10^{-5}) = 2.40 \times 10^8$ Bq = 240 MBq. Dividing by a small detected fraction reconstructs all decays from the few recorded ones.

Marking guidance 1: correct net rate. 1: 240 MBq from division by detected fraction.

5(b) Between two readings 45.0 min apart, the net rate decreases from 3000 s^{-1} to 2120 s^{-1}. Use the ratio to estimate the half-life and decide whether the reading is consistent with X. **[3 marks]**

Step 1 The same detector fraction cancels in the ratio: $R_2/R_1 = 2120/3000 = 0.70667$.

Step 2 Find the decay constant: $\lambda = -\ln(0.70667)/45.0 = 0.0077157\ min^{-1}$.

Step 3 Convert to a half-life: $t_{1/2} = \ln 2 / \lambda = 89.8$ min. This is close to 90.0 min and is consistent with X within the rounding of the readings; it is not proof of isotope identity by itself.

Marking guidance 1: logarithmic use of the net-rate ratio. 1: λ about 0.00772 min^{-1} and $t_{1/2}$ about 89.8 min. 1: justified consistency judgement.

5(c) The source is moved closer to the detector between readings. Explain why its count rate may no longer follow the same measured exponential, even though the nuclear decay constant is unchanged. **[2 marks]**

Moving the source changes the fraction of emissions that reach and are recorded by the detector. The relationship $R = \varepsilon A$ now has a changing ε.

The activity of X still decreases exponentially with the same λ, but the detected count rate combines that decrease with a geometry change. A higher detected rate does not imply new nuclei were created.

Marking guidance 1: detected fraction changes. 1: distinguishes unchanged activity law from altered count-rate record.

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5(d) A technician proposes warming an old vial to restore its original activity. Explain why this is not a workable way to meet the dispatch requirement. **[1 mark]**

Warming does not replace the parent nuclei that have decayed and does not normally change this nuclear decay constant. The remaining activity is set by λN; a new appropriately prepared sample would be needed.

Marking guidance 1: cannot restore parent N/activity by ordinary heating.

6 Explain the shape of the decay

6(a) The same fraction of X nuclei survives each 90 min interval, but the number decaying in successive 90 min intervals becomes smaller. Explain both statements using the microscopic meaning of the decay constant and A = λN. **[3 marks]**

For this isotope each remaining nucleus has the same constant probability per unit time of decaying. In a large sample, half of the remaining parent nuclei survive each half-life interval, so the surviving fraction per equal interval is constant.

After each interval there are fewer parent nuclei left. Since A = λN and λ stays constant, the activity decreases as N decreases. For example, a sample falls from N_0 to $N_0/2$ to $N_0/4$: it loses $N_0/2$ in the first half-life but only $N_0/4$ in the next.

A constant fractional decrease is an exponential, non-linear decrease in N or A, not a constant number lost per interval.

Marking guidance 1: constant decay probability per nucleus linked to a constant surviving fraction. 1: fewer remaining nuclei with A = λN. 1: smaller absolute loss/non-linear exponential distinction.

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