Parallel plate capacitors solutions

Electric fields 3 A robotic gripper's capacitive sensor

Matching answers and examiner-style commentary

OCR A H556 | A-level physics | Module 6 | 44 marks

Specification points covered

6.2.3(a–b) Uniform fields, parallel plates, permittivity and relative permittivity.

6.1.1(a–b), 6.1.2(b) Capacitance, plate charge and stored energy.

Use $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}. 1 pF = 10^{-12} F; 1 nC = 10^{-9} C.

1 Read the sensor

A robotic gripper senses compression using two electrodes separated by an insulating pad. The overlap area is 40.0 cm^2, the initial thickness is 1.20 mm and the pad's relative permittivity is 3.20.

Initially a 3.00 V supply is connected. Treat the pad as completely filling the gap. Assume fixed overlap, constant ε_r, parallel plates and negligible edge effects unless a question asks you to evaluate these assumptions.

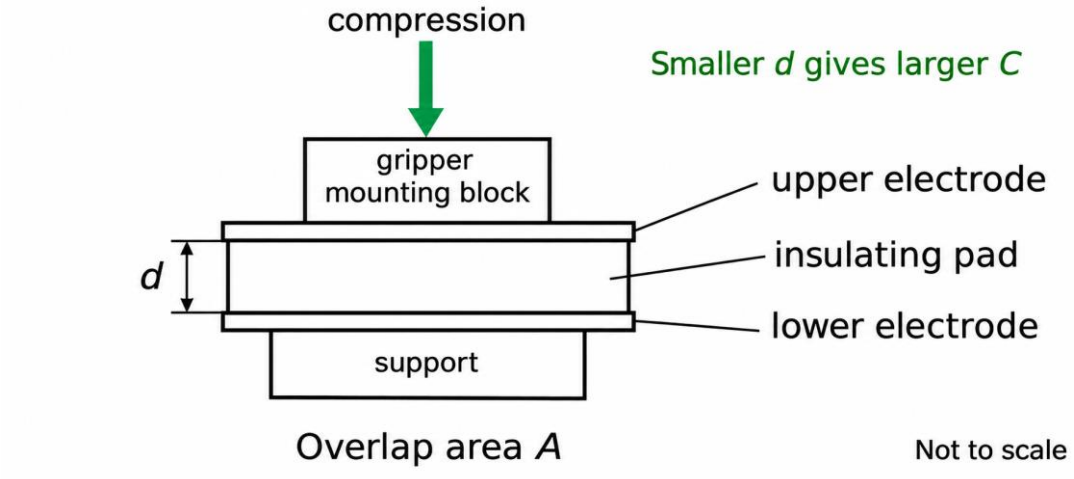


Figure 1 Capacitive compression sensor

1(a) In $C = \varepsilon_0\varepsilon_r A/d$, explain what A and d represent for the sensor in Figure 1. **[1 mark]**

A is the area of overlap of the facing electrode surfaces, not the sum of both areas. d is their inner-face separation, equal to the pad thickness in this fully filled model.

Marking guidance 1: both geometric quantities correct.

1(b) Convert the overlap area of 40.0 cm^2 and the initial gap of 1.20 mm into SI units. **[2 marks]**

1 $cm^2 = (10^{-2}\ m)^2 = 10^{-4}\ m^2$, so $A = 40.0 \times 10^{-4} = 4.00 \times 10^{-3}\ m^2$.

$d = 1.20 \times 10^{-3}$ m.

Marking guidance 1: correct area conversion. 1: correct gap conversion.

2 Find the baseline reading

2(a) Calculate the capacitance if the same geometry had air between its plates. Take the relative permittivity of air as 1.00. **[1 mark]**

$C_{air} = \varepsilon_0 A/d = (8.85 \times 10^{-12})(4.00 \times 10^{-3})/(1.20 \times 10^{-3}) = 2.95 \times 10^{-11}$ F = 29.5 pF.

Marking guidance 1: 29.5 pF.

2(b) Calculate the capacitance with the insulating pad present and explain the numerical relationship to your air-gap result. **[2 marks]**

$C = \varepsilon_0\varepsilon_r A/d = (8.85 \times 10^{-12})(3.20)(4.00 \times 10^{-3})/(1.20 \times 10^{-3}) = 9.44 \times 10^{-11}$ F = 94.4 pF.

It is 3.20 times the air capacitance because C is proportional to ε_r for unchanged geometry.

Marking guidance 1: 94.4 pF. 1: factor 3.20 linked to relative permittivity.

2(c) Find the magnitude of charge on either electrode at 3.00 V. Explain how this can be non-zero even if the total net charge on the two-electrode capacitor is zero. **[2 marks]**

$Q = CV = (9.44 \times 10^{-11})(3.00) = 2.832 \times 10^{-10}$ C = 0.283 nC.

The electrodes carry +Q and −Q. These add to zero net charge, but Q in Q = CV is the magnitude on one electrode, not their algebraic sum.

Marking guidance 1: charge magnitude. 1: correct distinction between one-plate charge and net charge.

2(d) Calculate the initial electric field magnitude and the energy stored in the sensor. **[2 marks]**

$E = V/d = 3.00/(1.20 \times 10^{-3}) = 2.50 \times 10^3\ V\ m^{-1}$.

$U = \frac{1}{2}CV^2 = \frac{1}{2}(9.44 \times 10^{-11})(3.00)^2 = 4.248 \times 10^{-10}$ J = 0.425 nJ.

Marking guidance 1: correct field. 1: correct stored energy.

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3 Choose the electrode geometry

3(a) A redesigned pad has d = 1.00 mm and $\varepsilon_r = 3.20$. Find the minimum overlap area needed for C = 120 pF. Give the answer in cm^2. **[2 marks]**

Step 1 Rearrange the geometry equation.

$C = \varepsilon_0\varepsilon_r A/d$ gives $A = Cd/(\varepsilon_0\varepsilon_r)$.

Step 2 Convert the data before substituting.

$A = (120 \times 10^{-12})(1.00 \times 10^{-3})/[(8.85 \times 10^{-12})(3.20)] = 4.237 \times 10^{-3}\ m^2$.

Since $1\ m^2 = 10^4\ cm^2$, $A = 42.4\ cm^2$.

Marking guidance 1: valid rearrangement/substitution. 1: about 42.4 cm^2.

3(b) Two fully overlapping electrode pairs are available: rectangles 80.0 mm by 50.0 mm, or circles of diameter 75.0 mm. At the redesigned thickness, which reaches at least 120 pF? Justify using the areas or capacitances. **[2 marks]**

The rectangular overlap is $0.0800 \times 0.0500 = 0.00400\ m^2 = 40.0\ cm^2$. Its capacitance is 113 pF, below the target.

The circular radius is half the diameter: 0.0375 m.

$A = \pi(0.0375)^2 = 0.004418\ m^2 = 44.2\ cm^2$; $C = \varepsilon_0\varepsilon_r A/d = 125$ pF.

Choose the circular electrodes: only their area exceeds 42.4 cm^2, so only they meet the 120 pF target.

Marking guidance 1: both areas or capacitances correct. 1: correct justified choice.

3(c) For an ideal sensor with fixed geometry and dielectric, does increasing its supply p.d. increase its capacitance? Explain. **[1 mark]**

No. $C = \varepsilon_0\varepsilon_r A/d$ depends on the geometry and dielectric. A larger p.d. produces a proportionally larger charge, leaving Q/V unchanged.

Marking guidance 1: unchanged capacitance with a valid reason.

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4 Compress the connected sensor

4(a) Return to the original 40.0 cm^2 sensor. Its gap decreases from 1.20 mm to 0.900 mm while the 3.00 V supply stays connected. Calculate its new capacitance. **[1 mark]**

$C = \varepsilon_0\varepsilon_r A/d = (8.85 \times 10^{-12})(3.20)(4.00 \times 10^{-3})/(0.900 \times 10^{-3}) = 1.2587 \times 10^{-10}$ F ≈ 126 pF.

Marking guidance 1: about 126 pF.

4(b) Calculate the new plate-charge magnitude and the increase in that magnitude. Explain where the additional charge transfer comes from. **[3 marks]**

Step 1 The connected supply holds V at 3.00 V.

$Q_{new} = C_{new}V = (1.2587 \times 10^{-10})(3.00) = 3.776 \times 10^{-10}$ C = 0.3776 nC.

Step 2 Compare with the original one-plate magnitude.

$\Delta Q = 0.3776 - 0.2832 = 0.0944$ nC.

The supply transfers extra charge between electrodes through the external circuit to maintain its p.d. Charge does not flow through the ideal insulating pad.

Marking guidance 1: new charge. 1: increase of 0.0944 nC. 1: supply/circuit explanation.

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4 Compress the connected sensor continued

4(c) Calculate the new field magnitude and state its ratio to the original field. **[1 mark]**

$E = V/d = 3.00/(0.900 \times 10^{-3}) = 3.33 \times 10^{3}$ V m^{-1}.

$E_{new}/E_{old} = 1.20/0.900 = 4/3$.

Marking guidance 1: field and ratio correct.

4(d) Find the new stored energy and explain whether it rises or falls. Do not equate this stored-energy change to the gripper's mechanical work. **[2 marks]**

$U = \frac{1}{2}C_{new}V^2 = \frac{1}{2}(1.2587 \times 10^{-10})(3.00)^2 = 5.664 \times 10^{-10}$ J.

It rises from 0.425 nJ to 0.566 nJ, a factor 4/3, because C rises while V stays constant. The connected supply also exchanges energy, so the stored-energy change alone is not the mechanical work.

Marking guidance 1: new energy. 1: correct trend linked to fixed V and increased C.

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5 Compare a disconnected sensor

5(a) Reset the original sensor to 1.20 mm and charge it to 3.00 V. Disconnect both supply and meter, so its plate-charge magnitude remains fixed. Then compress it to 0.900 mm. Find its new p.d. **[2 marks]**

Step 1 Use the original charge, not the connected-case charge.

$Q = C_{old}V_{old} = (94.4 \times 10^{-12})(3.00) = 2.832 \times 10^{-10}$ C.

Step 2 The geometry still gives $C_{new} = 1.2587 \times 10^{-10}$ F, so use V = Q/C.

$V_{new} = (2.832 \times 10^{-10})/(1.2587 \times 10^{-10}) = 2.25$ V.

The p.d. falls because the same charge is stored on a larger capacitance.

Marking guidance 1: uses fixed original charge. 1: 2.25 V.

5(b) Calculate the stored energy after this isolated compression and compare it with the initial stored energy. **[2 marks]**

Choose $U = Q^2/(2C)$ because Q is fixed, or use $\frac{1}{2}CV^2$ with the new V.

$U_{new} = (2.832 \times 10^{-10})^2/[2(1.2587 \times 10^{-10})] = 3.186 \times 10^{-10}$ J.

It falls to 0.319 nJ, which is 3/4 of the original 0.425 nJ. At fixed Q, energy is inversely proportional to C.

Marking guidance 1: correct energy. 1: correct comparison/reason.

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5 Compare a disconnected sensor continued

5(c) Show that the field stays unchanged in this ideal isolated compression. Then state how V and E would change if, instead, ε_r doubled at a fixed gap and fixed charge. **[3 marks]**

After compression, $E = V_{new}/d_{new} = 2.25/(0.900 \times 10^{-3}) = 2.50 \times 10^{3}$ V m^{-1}, unchanged.

More generally, $Q = CV$ and $C = \varepsilon A/d$ give $E = V/d = Q/(\varepsilon A)$, independent of d at fixed Q, ε and A.

If ε_r doubles at fixed gap, C doubles. With Q fixed, $V = Q/C$ halves, and $E = V/d$ also halves.

Marking guidance 1: unchanged-field numerical check. 1: fixed-charge/dielectric reasoning gives half V. 1: half E at fixed gap.

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6 Test the geometry model

Use the separate rigid-air-plate apparatus in Figure 2 for this investigation.

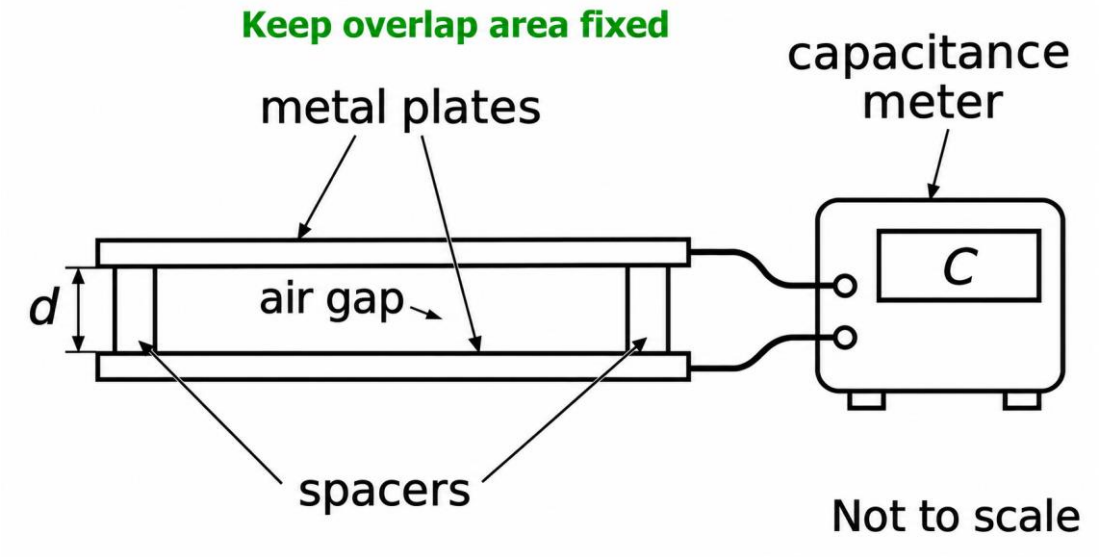


Figure 2 Rigid air-gap test

6(a) Figure 2 shows a separate rigid-air-plate investigation. Describe how to vary d and measure C while controlling two variables that matter to the model. Include a useful way to measure a small gap. **[3 marks]**

Use measured insulating spacers to set several gaps and read C from the capacitance meter. Measure spacer thickness using a suitable micrometer, checking zero and repeating readings where the thickness may vary.

Keep the overlap area and the medium (air) unchanged; keep the plates parallel and the leads in fixed positions so their stray contribution is approximately constant. Repeated readings help judge random variation.

Marking guidance 1: workable varying-gap/capacitance method. 1: suitable small-gap measurement. 1: two relevant controls identified.

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6 Process the air-gap readings

For this separate air-gap test, the leads are kept fixed and the stray capacitance is approximately constant.

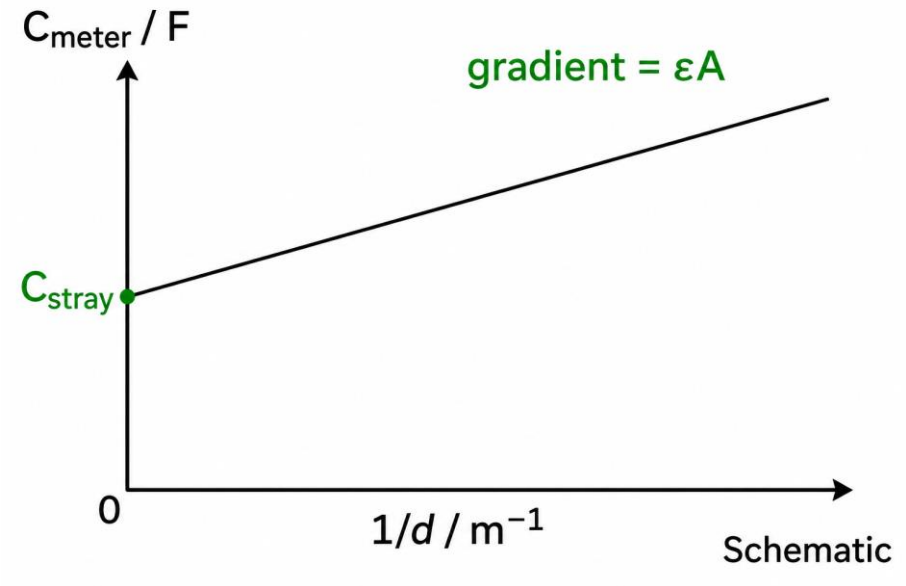
Table 1

d / mm	C_{meter} / pF
1.00	47.4
1.50	35.6
2.00	29.7
2.50	26.2
3.00	23.8

6(b) Table 1 contains results for A = 4.00 × 10^{-3} m^2. The meter includes a constant stray capacitance: $C_{meter} = \varepsilon A/d + C_{stray}$. Sketch C_{meter} against 1/d and label the meaning of its intercept. **[2 marks]**

Write the model as $C_{meter} = (\varepsilon A)(1/d) + C_{stray}$. It has the form y = mx + c.

The expected plot is a straight line with a positive vertical intercept, C_{stray}; the gradient is εA. It should not be forced through the origin.



Model sketch Numerical calculations use the stated values and data tables

Marking guidance 1: straight-line plot with correct axes. 1: positive intercept identified as stray capacitance.

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6 Process the air-gap readings continued

6(c) Use the readings at d = 1.00 mm and 2.00 mm to determine the gradient in F m, then infer ε for the air. **[2 marks]**

Step 1 Calculate the horizontal coordinates in SI units.

$1/d = 1000\ m^{-1}$ at 1.00 mm and $500\ m^{-1}$ at 2.00 mm.

Step 2 Use the change in readings; a constant stray contribution cancels.

gradient = $[(47.4 - 29.7) \times 10^{-12}]/(1000 - 500) = 3.54 \times 10^{-14}$ F m.

Step 3 Divide by overlap area because gradient = εA.

$\varepsilon = (3.54 \times 10^{-14})/(4.00 \times 10^{-3}) = 8.85 \times 10^{-12}$ F m^{-1}, consistent with air.

Marking guidance 1: correct SI gradient. 1: correct ε.

6(d) Explain why repeating each meter reading and taking an average does not remove a constant stray capacitance. **[1 mark]**

Averaging reduces the effect of random scatter, but the same constant offset is present in every reading. It must be measured/corrected or separated using the graph's intercept.

Marking guidance 1: distinguishes random scatter from the fixed offset.

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7 Interpret the gripper reading

7(a) The gripper's original $\varepsilon_r = 3.20$ sensor now gives a meter reading of 145 pF. Its separately calibrated parallel stray capacitance is 25 pF. Determine the pad thickness. **[3 marks]**

Step 1 Separate the sensor capacitance from the meter reading.

$C_{sensor} = 145 - 25 = 120$ pF $= 1.20 \times 10^{-10}$ F.

Step 2 Rearrange the geometry model for the thickness.

$d = \varepsilon_0 \varepsilon_r A / C_{sensor}$.

$d = [(8.85 \times 10^{-12})(3.20)(4.00 \times 10^{-3})]/(1.20 \times 10^{-10}) = 9.44 \times 10^{-4}$ m $= 0.944$ mm.

Marking guidance 1: correct stray correction. 1: correct rearrangement/substitution. 1: 0.944 mm.

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7 Interpret the gripper reading continued

7(b) Compression is sufficient when d ≤ 0.950 mm. The meter's uncertainty is ±1 pF; take all other values as exact for this check. Is sufficient compression guaranteed? Use the limiting reading. **[2 marks]**

The nominal result 0.944 mm passes. The largest inferred thickness occurs at the smallest sensor capacitance.

$C_{sensor,min} = (145 - 1) - 25 = 119$ pF.

$d_{max} = (8.85 \times 10^{-12})(3.20)(4.00 \times 10^{-3})/(119 \times 10^{-12}) = 0.9519$ mm.

Because 0.9519 mm exceeds 0.950 mm, the nominal reading passes but the uncertainty does not guarantee sufficient compression.

Marking guidance 1: correct limiting reading/thickness. 1: justified no-guarantee conclusion.

7(c) Explain two reasons why a real compressed pad might give a different thickness from the ideal calculation, even with a perfect capacitance meter. **[2 marks]**

The pad may bulge, changing overlap area A, or its relative permittivity may vary with compression. Either changes C independently of d, so assuming the original A and ε_r can give a wrong thickness. Non-parallel plates or a non-uniform gap also break the single-gap model.

Marking guidance 1 each for two distinct explained model limitations.

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