The nuclear atom solutions

Scattering, nuclear size and a target-film analyser

Matching answers and examiner-style commentary

OCR A H556 | A-level physics | Module 6 | 36 marks

Specification points covered

6.4.1(a–d) Alpha scattering, nuclear structure, relative sizes, nuclide notation and isotopes.

6.4.1(e–g) Strong nuclear force, $R = r_0A^{1/3}$, and mean atomic/nuclear density.

Prior electric-potential, energy and graph skills are applied to an explicitly stated head-on model.

Use $e = 1.60 \times 10^{-19}$ C, $k = 1/(4\pi\varepsilon_0) = 8.99 \times 10^9$ N m^2 C^{-2}, $r_0 = 1.20$ fm and nucleon mass $= 1.67 \times 10^{-27}$ kg. 1 fm $= 10^{-15}$ m; 1 MeV $= 1.60 \times 10^{-13}$ J.

1 Prepare the target record

A materials-analysis centre is commissioning an alpha-beam instrument for ultrathin target films. It uses the beam counts to test an atomic model, then compares estimates of nuclear size.

Figure 1 is a plan view. The detector can be moved around the foil while keeping its distance and counting time fixed. An alpha particle has charge +2e. All diagrams are schematic; numerical work uses the supplied data.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

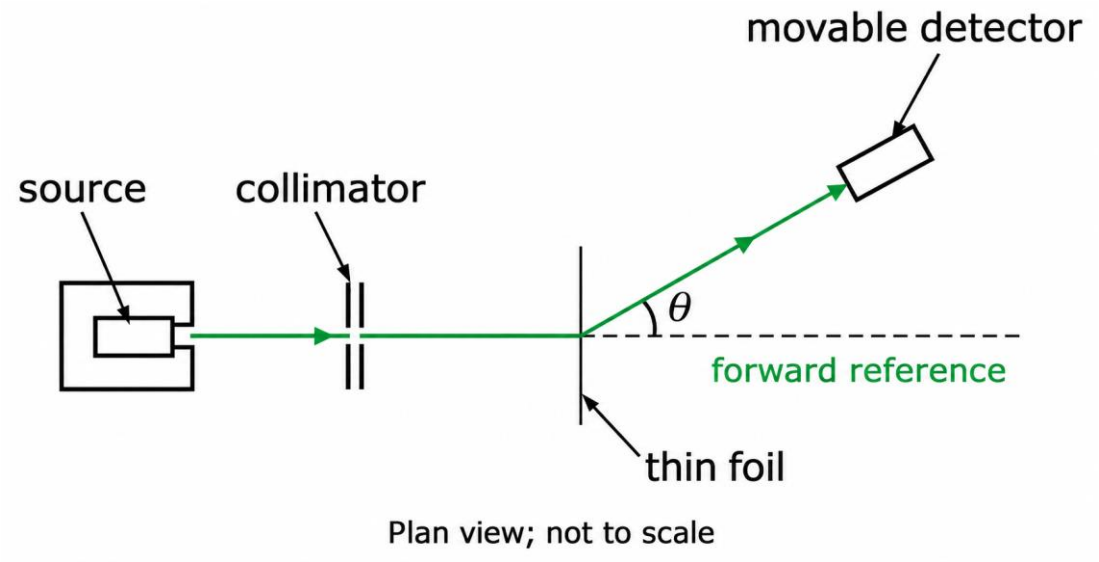


Figure 1 Alpha-beam target analyser

1(a) The analyser uses neutral tin-120 atoms, with proton number Z = 50. State the numbers of protons, neutrons and electrons in one atom, then represent its nucleus using A/Z nuclear notation. Use Sn for tin. **[2 marks]**

There are 50 protons, 120 − 50 = 70 neutrons, and 50 electrons. Neutrality requires the electron and proton counts to be equal.

The nucleus is $^{120}_{50}\mathrm{Sn}$. The upper number is A and the lower number is Z.

Marking guidance 1: all three counts correct. 1: correct nuclear notation.

1(b) A second film contains tin-118. Explain why it is an isotope of tin-120 and state how its neutron count differs. **[1 mark]**

Both have the same proton number, 50, but different nucleon numbers. Tin-118 has 68 neutrons, two fewer than tin-120.

Marking guidance Same proton number/different neutron number, with two fewer neutrons.

2 Connect observations to the model

2(a) For equal counting times, the mean detector counts at 30° and 150° are 2400 and 12. Calculate their ratio, N_{30}/N_{150}. **[1 mark]**

N_{30}/N_{150} = 2400/12 = 200. Large-angle counts are much less common in this comparison.

Marking guidance 200.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

2(b) A calibrated forward counter records about 96% of the incident particles within a narrow cone around the original beam direction. Separate measurements show a very small fraction scattered backwards. Explain what these observations support about atomic structure. **[3 marks]**

Most particles pass through with little deflection, supporting the conclusion that most of the atom is empty space.

The rare large deflections require a small region containing concentrated positive charge, which strongly repels a nearby positive alpha particle.

The large changes of alpha-particle momentum are consistent with a dense, massive nucleus containing most of the atomic mass. Link each inference to the relevant observation rather than listing features without evidence.

Marking guidance Mostly empty linked to near-straight transmission; small concentrated positive nucleus linked to rare large deflections; concentrated mass/dense nucleus linked to large momentum changes.

2(c) Can the two counts in 2(a) alone establish the percentage of all particles scattered through angles greater than 90°? Explain. **[1 mark]**

No. They sample only two detector positions, not all directions or the total number in the backward hemisphere. A ratio of selected counts is not the required overall fraction.

Marking guidance Insufficient angular/total-count coverage explained.

2(d) Explain why the target should be very thin and why the detector distance and counting time should be kept fixed when angular counts are compared. **[2 marks]**

A thin target reduces repeated scattering and energy loss before a particle leaves the film, so a single-scattering interpretation is more reasonable.

Fixed distance and time keep the collection geometry and observation duration comparable; otherwise count changes could have causes other than angle.

Marking guidance Thin target linked to reduced multiple scattering/energy loss; fixed geometry/time linked to a fair comparison.

3 Test a head-on approach model

This is a separate idealised head-on calculation. It does not describe every particle in the angular-count measurements.

3(a) A 5.00 MeV alpha particle approaches a stationary tin nucleus directly. Convert its initial kinetic energy to joules. **[1 mark]**

$E_k = 5.00 \times 1.60 \times 10^{-13} = 8.00 \times 10^{-13}$ J.

Marking guidance 8.00×10^{-13} J.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk



3(b) Neglect recoil of the heavy target and all non-electrostatic effects. Take electric potential energy as zero at infinite separation. Calculate the distance of closest approach for a tin nucleus of charge +50e. **[2 marks]**

Step 1 At closest approach the incident alpha particle is momentarily at rest

Its initial kinetic energy becomes electric potential energy: $E_k = k(2e)(50e)/r$.

Step 2 Rearrange for the separation

$r = k(2e)(50e)/E_k = (8.99 \times 10^9)(100)(1.60 \times 10^{-19})^2/(8.00 \times 10^{-13}) = 2.8768 \times 10^{-14}$ m = 28.8 fm.

Marking guidance Correct energy relationship with both charges; about 2.88×10^{-14} m.

3(c) If the incident kinetic energy is doubled, what happens to this closest-approach distance? Explain using the model. **[1 mark]**

At fixed charges, $r = kQq/E_k$, so r halves to 14.4 fm. The repulsive potential energy must reach twice the previous value.

Marking guidance Halves, with inverse-energy relationship.

3(d) A technician reports the 28.8 fm result as an exact measurement of nuclear radius. Explain why this is not justified. **[2 marks]**

The alpha particle can be turned back by electrostatic repulsion without reaching the nuclear surface. The separation is therefore an upper bound on a radius inferred by this method, not an observed point of contact.

The calculation also depends on its stated head-on, stationary-target and electrostatic approximations. It does not locate a sharp nuclear boundary.

Marking guidance Turning back before contact/upper-bound distinction; relevant limitation of the inferred radius.

4 Use a nuclear-size relationship

4(a) Use $R = r_0A^{1/3}$ to estimate the radius of a tin-120 nucleus. Give the result in fm and in m. **[1 mark]**

$R = 1.20 \times 120^{1/3} = 5.9189$ fm = 5.92×10^{-15} m. The cube root is applied to the nucleon number, not to r_0.

Marking guidance About 5.92 fm = 5.92×10^{-15} m.

4(b) In the same model, a nucleus has radius 6.00 fm. Infer its nucleon number. **[1 mark]**

$R/r_0 = A^{1/3}$, so $A = (R/r_0)^3 = (6.00/1.20)^3 = 125$.

Marking guidance 125.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

4(c) Compare the radii of nuclei with A = 216 and A = 27. State the ratio R_{216}/R_{27}. **[1 mark]**

$R_{216}/R_{27} = (216/27)^{1/3} = 8^{1/3} = 2$. Their masses are in an 8:1 ratio in the simple nucleon-mass model, but their radii are in a 2:1 ratio.

Marking guidance 2.

4(d) The spherical mean-atom model uses radius 1.50×10^{-10} m for a tin atom. Calculate the ratio of atomic radius to nuclear radius for tin-120 and explain what this indicates about relative sizes. **[2 marks]**

Atomic/nuclear radius = $(1.50 \times 10^{-10})/(5.9189 \times 10^{-15}) = 2.53 \times 10^{4}$.

The atom's radius is about 25 000 times the nuclear radius. This is a radius comparison; the volume ratio is much larger because volume depends on the cube of radius.

Marking guidance About 2.53×10^{4}; correct radius/volume distinction.

5 Check the density model

Table 1 Rounded candidate radius records

A	R / fm
27	3.60
64	4.80
125	6.00

5(a) Treat the tin-120 nucleus as a sphere and its mass as A times the given nucleon mass. Calculate its mean density. **[3 marks]**

Step 1 Find the mass of the chosen nucleus

$M = 120 \times 1.67 \times 10^{-27} = 2.004 \times 10^{-25}$ kg.

Step 2 Find its spherical volume using the radius in metres

$V = (4/3)\pi R^3 = (4/3)\pi(5.9189 \times 10^{-15})^3 \approx 8.69 \times 10^{-43}$ m^3.

Step 3 Divide mass by volume

$\rho = M/V = (2.004 \times 10^{-25})/(8.69 \times 10^{-43}) \approx 2.31 \times 10^{17}$ kg m^{-3}.

Marking guidance Nuclear mass; spherical volume in m^3; density about 2.31×10^{17} kg m^{-3}.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

5(b) Use $R = r_0A^{1/3}$ and $M \approx Am_n$ to show why the simple model predicts approximately the same mean nuclear density for different A. **[2 marks]**

$\rho = M/V = Am_n/[(4/3)\pi(r_0A^{1/3})^3]$.

Since $(A^{1/3})^3 = A$, the A factors cancel: $\rho = 3m_n/(4\pi r_0^3)$. The result is independent of A if the same r_0 and nucleon-mass approximation apply.

Marking guidance Correct mass/volume substitution; cancellation and independent-of-A conclusion.

5(c) The rounded radius records in Table 1 are to be compared with $R \propto A^{1/3}$, rather than $R \propto A$. Choose straight-line graph axes and calculate the expected gradient from the table. **[2 marks]**

Plot R/fm vertically against $A^{1/3}$ horizontally. The horizontal coordinates are 3, 4 and 5.

The gradient is $(6.00 - 3.60)/(5 - 3) = 1.20$ fm. A line through these records is consistent with $R = r_0A^{1/3}$; the slope corresponds to r_0.

Marking guidance Appropriate transformed axes; gradient 1.20 fm.

5(d) A radius reading is 2.0% too large while the assumed mass is correct. Calculate the factor by which the density inferred from that reading differs from the correct density. **[2 marks]**

At fixed mass, $\rho \propto 1/R^3$. A radius factor of 1.020 therefore gives density factor $1/(1.020)^3 = 0.9423$.

The inferred density is about 5.8% too small, not 2.0% too large.

Marking guidance Inverse-cube relationship; factor about 0.942 or decrease about 5.8%.

6 Interpret the model limits

6(a) Describe the strong nuclear force between two nucleons at separations of about 1 fm and below about 0.5 fm. State its approximate attractive range. **[2 marks]**

At about 1 fm the strong nuclear force is attractive. At separations below about 0.5 fm it is repulsive.

It is short range: the attractive interaction extends to about 3 fm, rather than across an entire atom.

Marking guidance Attractive at about 1 fm with range about 3 fm; repulsive below about 0.5 fm.

6(b) Using the same tin-120 mass and the stated mean-atom radius, calculate the mean density of the spherical atom model. Compare it with the nuclear density and interpret the difference. **[2 marks]**

The spherical mean-atom volume is $(4/3)\pi(1.50 \times 10^{-10})^3 = 1.414 \times 10^{-29}$ m^3.

Mean density $\approx (2.004 \times 10^{-25})/(1.414 \times 10^{-29}) = 1.42 \times 10^4$ kg m^{-3}. The nuclear density is about 1.63×10^{13} times greater.

Most atomic mass occupies the much smaller nuclear volume. This simple mean-atom calculation is not a measurement of the bulk density of a real tin film.

Marking guidance About 1.42×10^4 kg m^{-3}; correct concentration-of-mass comparison.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

6(c) Correct both claims in the instrument report: “Closest approach proves a sharp nuclear radius” and “A nearly constant mean density proves the density is identical at every point inside and outside a nucleus.” **[2 marks]**

Closest approach is model-dependent and does not establish contact with a sharp surface; it can only bound the radius in the stated approach model.

A mean density is mass divided by a chosen volume. Similar means for different nuclei do not establish an identical local density at every position; a real density can fall near the nuclear edge.

Marking guidance Correct limitation for closest approach; mean density distinguished from a local density profile.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk