Mass, energy and antiparticles solutions

A gamma-converter instrument for a lunar probe

Matching answers and examiner-style commentary

OCR A H556 | A-level physics | Module 6 | 37 marks

Specification points covered

6.4.4(a): mass–energy equivalence, $E = mc^2$, including eV and MeV.

6.4.4(c): pair production and particle–antiparticle annihilation.

Prerequisites supplied or previously taught: photon energy $E = hf = hc/\lambda$ and conservation of energy/momentum.

Use $c = 3.00 \times 10^8$ m s^{-1}, $h = 6.63 \times 10^{-34}$ J s, $m_e = 9.11 \times 10^{-31}$ kg and 1 eV = 1.60×10^{-19} J. Retain unrounded values: these constants give $m_e c^2 = 0.5124375$ MeV. Ignore recoil energy only where the question says so.

1 Start with a reliable energy scale

A lunar-probe instrument converts sufficiently energetic gamma photons into electron–positron pairs. A separate calibration module uses annihilation photons. These are simplified instrument models.

A pair consists of one electron and one positron. The positron has the same rest mass as the electron and the opposite electric charge.

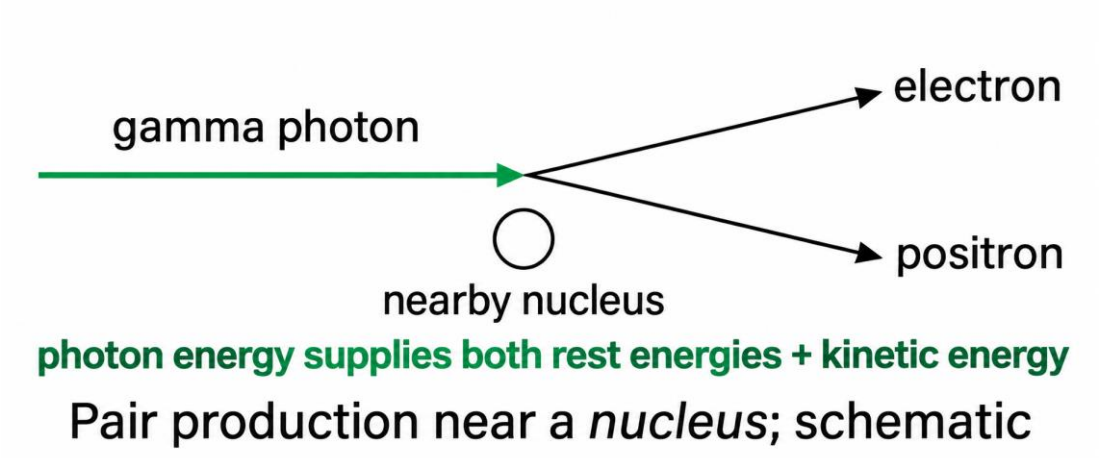


Figure 1 The photon energy supplies both rest energies and any kinetic energy.

1(a) Convert a gamma-photon energy of 1.60 MeV to joules. **[1 mark]**

1 MeV = 10^6 eV = 1.60×10^{-13} J.

E = $1.60 \times 1.60 \times 10^{-13} = 2.56 \times 10^{-13}$ J.

Marking guidance 1: 2.56×10^{-13} J.

1(b) Calculate the rest energy of one electron in joules and in MeV using the supplied constants. **[2 marks]**

Rest energy is $E_0 = m_e c^2$.

$E_0 = (9.11 \times 10^{-31})(3.00 \times 10^8)^2 = 8.199 \times 10^{-14}$ J $\approx 8.20 \times 10^{-14}$ J.

In MeV, divide by 1.60×10^{-13} J MeV^{-1}: $E_0 = 0.5124375$ MeV ≈ 0.512 MeV. The slight difference from a data-booklet value of about 0.511 MeV is due to the rounded constants supplied here.

Marking guidance 1: correct mc^2 calculation in J. 1: correct conversion to about 0.512 MeV.

2 Decide which photons can produce a pair

2(a) Calculate the minimum photon energy required to supply the rest energies of an electron–positron pair. Ignore the small recoil-energy correction for a massive nearby nucleus. **[1 mark]**

Both particles must be created, so the rest-energy requirement is $2m_e c^2$.

$E_{min} = 2(0.5124375) = 1.024875$ MeV ≈ 1.02 MeV, or 1.6398×10^{-13} J.

Marking guidance 1: twice the electron rest energy, approximately 1.02 MeV.

2(b) The converter receives photons of energies 0.900, 1.10 and 1.60 MeV. Identify which have enough energy for pair production near a nucleus, and explain why the test uses photon energy rather than beam brightness. **[2 marks]**

The 1.10 and 1.60 MeV photons exceed the approximately 1.025 MeV threshold; 0.900 MeV photons do not.

Each event must supply two rest energies from one photon. A brighter beam of sub-threshold photons gives more insufficient-energy photons, not a larger energy for each photon in this single-photon process.

Marking guidance 1: correct classification of all three energies. 1: per-photon energy versus brightness distinction.

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2(c) Calculate the largest photon wavelength that can meet the rest-energy threshold in this model. **[3 marks]**

Step 1 Use the threshold in joules: $E_{min} = 1.6398 \times 10^{-13}$ J.

Step 2 Rearrange photon energy, $E = hc/\lambda$, to $\lambda_{max} = hc/E_{min}$.

Step 3 $\lambda_{max} = [(6.63 \times 10^{-34})(3.00 \times 10^{8})]/(1.6398 \times 10^{-13}) = 1.2130 \times 10^{-12}$ m $\approx 1.21 \times 10^{-12}$ m.

Longer wavelengths have lower photon energies, so this is a maximum wavelength, not a minimum.

Marking guidance 1: threshold energy in J. 1: correct rearrangement/substitution. 1: 1.21×10^{-12} m.

2(d) Explain why Figure 1 includes a nearby nucleus rather than showing a lone photon producing the pair in empty space. **[2 marks]**

Energy and momentum must both be conserved. A nearby nucleus can take recoil momentum, allowing the photon to produce the massive pair while the whole system conserves momentum.

A single photon cannot produce a lone electron–positron pair in otherwise empty space while satisfying both conservation laws. The nucleus need not be consumed or change its A and Z.

Marking guidance 1: simultaneous energy and momentum conservation. 1: nucleus can take recoil momentum.

3 Follow the energy after conversion

3(a) A 1.60 MeV photon makes a pair. Neglect nucleus recoil energy. Calculate the total kinetic energy initially available to the electron and positron. **[2 marks]**

Step 1 The photon energy first supplies the two rest energies, $2m_ec^2 = 1.024875$ MeV.

Step 2 The remaining energy is kinetic: $K_{total} = 1.60 - 1.024875 = 0.575125$ MeV ≈ 0.575 MeV.

Do not give each particle 0.575 MeV; this is their combined kinetic energy.

Marking guidance 1: subtracts both rest energies. 1: combined kinetic energy about 0.575 MeV.

3(b) In the same event the electron has kinetic energy 0.340 MeV. Calculate the positron's kinetic energy. **[1 mark]**

$K_{positron} = K_{total} - K_{electron} = 0.575125 - 0.340 = 0.235125$ MeV ≈ 0.235 MeV.

Energy conservation does not require the two kinetic energies to be equal.

Marking guidance 1: approximately 0.235 MeV.

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3(c) The tracks suggest a total pair kinetic energy of 0.900 MeV for a 1.60 MeV photon. Explain quantitatively why the event record is inconsistent with this model. **[2 marks]**

The minimum energy needed for the stated event would be 1.024875 + 0.900 = 1.924875 MeV, approximately 1.92 MeV.

This exceeds the incident 1.60 MeV. Including a positive nucleus recoil energy would increase the requirement, so it cannot repair the energy deficit. The stated record is inconsistent.

Marking guidance 1: combines kinetic energy and both rest energies. 1: correct numerical comparison/conclusion, including recoil not supplying missing energy.

3(d) Calculate the mass equivalent of the 0.575125 MeV kinetic energy from 3(a). Explain why this is not extra electron rest mass. **[2 marks]**

Convert the energy: $K = 0.575125(1.60 \times 10^{-13}) = 9.202 \times 10^{-14}$ J.

The mass equivalent is $K/c^2 = (9.202 \times 10^{-14})/(9.00 \times 10^{16}) = 1.0224 \times 10^{-30}$ kg $\approx 1.02 \times 10^{-30}$ kg.

This expresses the energy as a mass equivalent. The electron's invariant rest mass remains m_e; its kinetic energy does not create a different electron rest mass.

Marking guidance 1: correct conversion and division by c^2. 1: about 1.02×10^{-30} kg with the rest-mass distinction.

4 Calibrate with annihilation photons

4(a) An electron and a positron annihilate from rest in the laboratory frame, producing two photons. Calculate the energy of each photon in joules and MeV. **[2 marks]**

The initial total energy is $2m_ec^2 = 1.6398 \times 10^{-13}$ J. Initial total momentum is zero.

The two photons have equal energies, so each carries $m_ec^2 = 8.199 \times 10^{-14}$ J = 0.5124375 MeV $\approx$ 0.512 MeV. The total pair energy must be divided between the two photons.

Marking guidance 1: correct total energy shared between two photons. 1: each energy about 8.20×10^{-14} J or 0.512 MeV, with both requested units.

4(b) Explain the directions of the two photons in the laboratory frame for this annihilation from rest. **[2 marks]**

The initial total momentum is zero. Photons carry momentum, and the two equal-magnitude photon momenta must add vectorially to zero.

They therefore travel in opposite directions along one straight line, 180° apart. Two photons travelling together would give a non-zero final total momentum.

Marking guidance 1: zero initial total momentum with momentum conservation. 1: opposite directions, 180° apart.

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4(c) Calculate the wavelength of one calibration photon from this rest annihilation. **[2 marks]**

Use the energy of one photon, not the total energy of the pair.

$\lambda = hc/E = [(6.63 \times 10^{-34})(3.00 \times 10^{8})]/(8.199 \times 10^{-14}) = 2.426 \times 10^{-12}$ m $\approx 2.43 \times 10^{-12}$ m.

Marking guidance 1: uses single-photon energy in hc/E. 1: 2.43×10^{-12} m.

4(d) A pupil writes “the detector sees the two positrons produced in annihilation”. Correct the statement, identifying the initial and final particles. **[1 mark]**

Initially there is one electron and one positron. They annihilate, and the detectors record the two resulting gamma photons, not two positrons.

Marking guidance 1: correct initial pair and final two photons.

5 Use an unfamiliar calibration setting

5(a) A second calibration event begins with an electron and a positron having equal and opposite momenta and total kinetic energy 0.240 MeV in the laboratory frame. They annihilate into exactly two photons. Calculate the energy of each photon. **[2 marks]**

Step 1 The equal and opposite incident momenta make the total initial momentum zero, so the two final photons have equal energies.

Step 2 Add all initial energy: $E_{total} = 2(0.5124375) + 0.240 = 1.264875$ MeV.

Each photon has energy 1.264875/2 = 0.6324375 MeV ≈ 0.632 MeV. The initial kinetic energy must not be discarded.

Marking guidance 1: adds total rest and kinetic energies. 1: divides by two to obtain about 0.632 MeV.

5(b) In a different event the electron–positron system has non-zero total momentum in the laboratory frame. Is it valid to assume that the two detected photons must have equal energies in that frame? Explain. **[1 mark]**

No. Equal photon energies in opposite directions are required for zero total momentum, as in annihilation from rest. For a system with non-zero laboratory momentum, the photons must together carry that momentum and need not have equal laboratory energies.

Marking guidance 1: rejects the assumption with a momentum-based reason.

5(c) A converter absorbs incident gamma energy at a rate of 4.00×10^{-6} W, entirely from 1.60 MeV photons. Calculate the number of absorbed photons per second. **[2 marks]**

Each photon carries 2.56×10^{-13} J.

Number per second = power / energy per photon = $(4.00 \times 10^{-6})/(2.56 \times 10^{-13}) = 1.5625 \times 10^{7}$ s^{-1} ≈ 1.56×10^{7} s^{-1}. This does not assume every absorbed photon creates a pair.

Marking guidance 1: uses photon energy in J. 1: about 1.56×10^{7} photons s^{-1}.

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5(d) The instrument records 3.00×10^6 pair events per second in the setting of 5(c). Calculate the fraction of absorbed photons giving a recorded pair event. **[1 mark]**

Recorded fraction = $(3.00 \times 10^6)/(1.5625 \times 10^7) = 0.192 = 19.2\%$.

A fraction below one is consistent with some absorbed photons not yielding recorded pairs.

Marking guidance 1: 0.192 or 19.2%.

6 Review the instrument team's language

6(a) Explain why calculating E/c^2 for a photon does not give it a non-zero rest mass, and why $E = hf$ is the appropriate relationship for its energy. **[2 marks]**

A photon has zero rest mass and cannot be at rest. E/c^2 is a mass equivalent of its energy, not its invariant rest mass.

Its energy is determined by frequency, $E = hf$ (equivalently hc/λ), and it carries momentum. Applying the rest-energy equation $E_0 = m_0c^2$ as if it were the total photon energy would incorrectly give zero energy.

Marking guidance 1: zero photon rest mass distinguished from mass equivalent. 1: correct use of hf for photon energy.

6(b) For a full-energy event, the instrument records the pair's kinetic energy and later both annihilation photons after the pair has slowed to rest. Show that their combined energies account for the original 1.60 MeV photon, neglecting recoil and other losses. **[2 marks]**

The initial pair kinetic energy is 0.575125 MeV. Slowing the particles transfers this energy to the instrument.

The two later rest-annihilation photons carry $2 \times 0.5124375 = 1.024875$ MeV.

Total recorded/transferred energy = $0.575125 + 1.024875 = 1.600000$ MeV. Counting the incident photon as an additional output would double-count the same energy.

Marking guidance 1: includes kinetic-energy loss plus both photon energies. 1: total 1.60 MeV with no double counting.

6(c) Summarise the difference between pair production and annihilation, using the direction of energy transfer and conservation of electric charge. **[2 marks]**

In pair production, photon energy creates an electron and a positron and can provide kinetic energy. Their charges $-e$ and $+e$ sum to the initial photon charge of zero.

In annihilation, an electron and positron disappear as particles and their rest energy and any kinetic energy become photon energy. The initial total charge and final photon charge are both zero. Energy is conserved in both directions; it is not created.

Marking guidance 1: correct contrasting processes/energy transfer. 1: zero total charge conserved in each.

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