Mass defect and binding energy solutions

Checking the reports from a nuclear-mass laboratory

Matching answers and examiner-style commentary

OCR A H556 | A-level physics | Module 6 | 37 marks

Specification points covered

6.4.4(d): mass defect and binding energy.

6.4.4(f): binding-energy calculations using supplied nuclear masses and $E = \Delta mc^2$.

6.4.4(b): energy released or absorbed in simple nuclear reactions. Supporting: 6.4.4(a), (k).

Use $m_p = 1.00728$ u, $m_n = 1.00866$ u; 1 u $c^2 = 931.5$ MeV; 1 MeV $= 1.602 \times 10^{-13}$ J. For kg calculations use 1 u $= 1.66054 \times 10^{-27}$ kg and $c = 2.998 \times 10^8$ m s^{-1}. Small conversion differences reflect rounded constants. All stated nuclear masses exclude electrons.

1 Say precisely what binding energy means

A research team checks mass measurements before using nuclear reactions in an instrument energy standard. Supplied masses are rounded data; keep guard digits until the final answer.

Figure 1 compares the same numbers of protons and neutrons before and after binding. It does not compare two different nuclides.

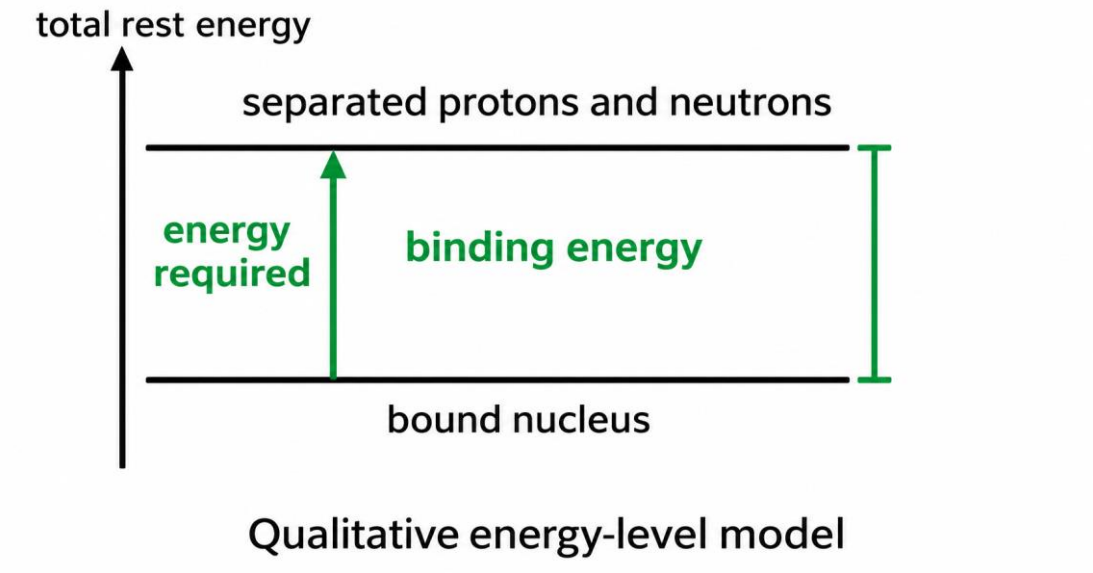


Figure 1 Separation requires energy; assembly releases the same energy difference.

1(a) Define the nuclear binding energy of a nucleus. Your definition must identify the starting system and the separated final particles. **[2 marks]**

Nuclear binding energy is the minimum energy required to separate a nucleus completely into its constituent protons and neutrons at infinite separation, so that their mutual interactions are negligible.

It is not a force, and “energy that holds the nucleus together” does not specify the required energy change.

Marking guidance 1: minimum energy required for complete separation. 1: into the constituent protons and neutrons at infinite separation.

1(b) Use Figure 1 to explain whether forming a bound nucleus from the same separated protons and neutrons releases or requires energy. **[2 marks]**

The bound nucleus is at lower total rest energy than the same separated protons and neutrons. Forming it releases the difference to the surroundings, for example as radiation or kinetic energy.

Separating that nucleus reverses the energy change and requires an input of the same magnitude.

Marking guidance 1: bound system has lower total rest energy. 1: formation releases the difference.

2 Audit a helium mass measurement

2(a) A helium-4 nucleus has Z = 2 and measured nuclear mass 4.00151 u. Calculate the total mass of its constituent free protons and neutrons. **[1 mark]**

Helium-4 contains two protons and two neutrons.

M_{free} = 2(1.00728) + 2(1.00866) = 4.03188 u. This is the separated-particle reference, not the measured bound mass.

Marking guidance 1: 4.03188 u.

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2(b) Calculate the mass defect of this nucleus, stating which mass is subtracted from which. **[1 mark]**

The positive mass defect is free-nucleon mass minus bound nuclear mass.

$\Delta m = 4.03188 - 4.00151 = 0.03037$ u. Reversing this order would give the signed mass change for assembly, rather than the positive binding mass defect.

Marking guidance 1: +0.03037 u with correct subtraction.

2(c) Calculate the total binding energy of this helium-4 nucleus in MeV and in joules. **[2 marks]**

Step 1 Convert the mass defect to energy with the supplied u conversion.

$E_B = 0.03037 \times 931.5 = 28.289655$ MeV ≈ 28.3 MeV.

Step 2 Convert to joules: $E_B = 28.289655(1.602 \times 10^{-13}) = 4.531999 \times 10^{-12}$ J $\approx 4.53 \times 10^{-12}$ J.

This is the total binding energy of one nucleus, not a value per nucleon.

Marking guidance 1: about 28.3 MeV. 1: about 4.53×10^{-12} J with conversion.

2(d) Convert the helium mass defect to kilograms and check the binding energy using $E = \Delta mc^2$. **[2 marks]**

$\Delta m = 0.03037(1.66054 \times 10^{-27}) = 5.04306 \times 10^{-29}$ kg.

$E = (5.04306 \times 10^{-29})(2.998 \times 10^{8})^2 = 4.53271 \times 10^{-12}$ J $\approx 4.53 \times 10^{-12}$ J.

The two routes agree to three significant figures. Keep the factor c squared; multiplying by c alone is dimensionally incorrect.

Marking guidance 1: correct mass conversion. 1: energy consistent with 4.53×10^{-12} J.

3 Handle a different nucleus and a reverse problem

3(a) A carbon-12 nucleus has Z = 6 and nuclear mass 11.99671 u. Calculate its mass defect and total binding energy in MeV. **[2 marks]**

Step 1 There are six protons and six neutrons, so $M_{free} = 6(1.00728) + 6(1.00866) = 12.09564$ u.

Step 2 $\Delta m = 12.09564 - 11.99671 = 0.09893$ u.

Step 3 $E_B = 0.09893 \times 931.5 = 92.153295$ MeV ≈ 92.2 MeV.

Marking guidance 1: correct constituent mass and defect. 1: about 92.2 MeV.

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3(b) A model nucleus with A = 20 and Z = 10 has total binding energy 160.0 MeV. Use the supplied free-nucleon masses to calculate its nuclear mass in u. **[3 marks]**

Step 1 Count ten protons and ten neutrons: M_{free} = 10(1.00728) + 10(1.00866) = 20.15940 u.

Step 2 The binding mass defect is Δm = 160.0/931.5 = 0.171766 u.

Step 3 The bound nucleus has less mass: M = M_{free} − Δm = 19.987634 u, approximately 19.9876 u.

Retaining digits in the mass is useful because future mass-difference calculations depend on small differences between large values.

Marking guidance 1: free-nucleon total. 1: defect from energy. 1: correct subtraction giving about 19.9876 u.

3(c) A different instrument reports the mass of a neutral helium-4 atom as 4.00260 u. Use electron mass 0.0005486 u and neglect electronic binding energy to infer the nuclear mass. Explain why the atomic mass must not be inserted unchanged into the nuclear-mass formula. **[2 marks]**

A neutral helium atom contains two electrons. Remove their masses to infer the nuclear mass:

$M_{nucleus}$ = 4.00260 − 2(0.0005486) = 4.0015028 u ≈ 4.00150 u.

Using the atomic mass unchanged alongside free proton and neutron masses would wrongly include two electron masses in the bound system but none in the separated reference.

Marking guidance 1: subtracts both electron masses and obtains about 4.00150 u. 1: explains consistent mass accounting.

3(d) The laboratory software rounds every mass to the nearest 0.1 u before calculating the helium mass defect. Explain why keeping only those rounded masses can destroy the result. **[1 mark]**

Both 4.03188 u and 4.00151 u round to 4.0 u, so their small but important difference would be lost. Retain guard digits before subtracting nearly equal masses.

Marking guidance 1: small difference lost through premature rounding, applied to the data.

4 Use complete reaction mass balances

Table 1 Rounded nuclear masses

Nucleus	Carbon-12	Helium-4	Oxygen-16
Nuclear mass / u	11.99671	4.00151	15.99053

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4(a) The team models carbon-12 plus helium-4 forming oxygen-16 and one gamma photon. Write the nuclear equation and check the proton and nucleon totals. **[2 marks]**

$^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{16}_{8}\mathrm{O} + \gamma$.

A: 12 + 4 = 16. Z: 6 + 2 = 8. The photon has no proton or nucleon number.

Marking guidance 1: correct nuclear equation. 1: explicit checks of both totals.

4(b) Use the nuclear masses in Table 1 to calculate the rest-energy release Q for the reaction in 4(a). State whether Q is released or required. **[2 marks]**

Step 1 Add all initial and final rest masses. Initial = 11.99671 + 4.00151 = 15.99822 u. Final nuclear rest mass = 15.99053 u; a photon has zero rest mass.

Step 2 $\Delta m = M_{initial} - M_{final} = 0.00769$ u.

Q = 0.00769 × 931.5 = 7.163235 MeV ≈ 7.16 MeV, released because the final rest mass is smaller.

Marking guidance 1: complete mass difference. 1: +7.16 MeV released.

4(c) In an ideal limiting case with negligible initial kinetic energy and negligible recoil energy, calculate the gamma-photon energy for this reaction in joules. **[1 mark]**

Under the stated limiting assumptions, the photon carries the rest-energy release.

$E_\gamma = 7.163235(1.602 \times 10^{-13}) = 1.14755 \times 10^{-12}$ J ≈ 1.15×10^{-12} J. Real recoil would leave slightly less energy for the photon.

Marking guidance 1: approximately 1.15×10^{-12} J.

4(d) In another run, the colliding nuclei have total initial kinetic energy 0.400 MeV and the oxygen nucleus leaves with kinetic energy 0.060 MeV. Calculate the gamma-photon energy. **[2 marks]**

Total energy available above final rest energies is $Q + K_{initial} = 7.163235 + 0.400 = 7.563235$ MeV.

Subtract the final oxygen kinetic energy: $E_\gamma = 7.563235 - 0.060 = 7.503235$ MeV ≈ 7.50 MeV.

The final oxygen kinetic energy is an output sharing the available energy, not an extra input.

Marking guidance 1: adds initial kinetic energy to Q. 1: subtracts final recoil kinetic energy, giving about 7.50 MeV.

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5 Recognise an energy input and a measurement limit

5(a) For another nuclear reaction, the sum of final rest masses exceeds the sum of initial rest masses by 1.00×10^{-29} kg. Calculate Q in MeV, taking $Q = (M_{initial} - M_{final})c^2$. **[2 marks]**

The mass difference $M_{initial} - M_{final}$ is negative: -1.00×10^{-29} kg.

$Q = (-1.00 \times 10^{-29})(2.998 \times 10^{8})^2 = -8.98800 \times 10^{-13}$ J.

$Q = (-8.98800 \times 10^{-13})/(1.602 \times 10^{-13}) = -5.61049$ MeV ≈ -5.61 MeV. Energy must be supplied to increase the rest energy.

Marking guidance 1: correct signed mass-energy change in J. 1: −5.61 MeV with energy-input interpretation.

5(b) For the reaction in 5(a), the particles initially have total kinetic energy 6.00 MeV. In a simple total-energy budget, calculate the total kinetic energy remaining in the products. Do not infer a laboratory threshold from this energy-only model. **[1 mark]**

$K_{final,total} = K_{initial,total} + Q = 6.00 - 5.61049 = 0.389514$ MeV ≈ 0.390 MeV.

This is an energy budget. A detailed laboratory threshold can also depend on momentum and recoil; those are not being calculated here.

Marking guidance 1: approximately 0.390 MeV.

5(c) A proposed reaction releases 2.80 MeV per event. Calculate the decrease in the system's total rest mass per event in kg. **[2 marks]**

Step 1 Convert energy: $Q = 2.80(1.602 \times 10^{-13}) = 4.4856 \times 10^{-13}$ J.

Step 2 $\Delta m = Q/c^2 = (4.4856 \times 10^{-13})/(2.998 \times 10^{8})^2 = 4.99066 \times 10^{-30}$ kg $\approx 4.99 \times 10^{-30}$ kg.

This is the decrease of the entire reaction system's rest mass, not necessarily the mass of a single product.

Marking guidance 1: energy converted to J. 1: approximately 4.99×10^{-30} kg.

5(d) Two measured total masses each have absolute uncertainty ±0.000005 u. Their difference is 0.000100 u. Use the sum of the absolute uncertainties to estimate the percentage uncertainty in the difference. **[2 marks]**

For the stated worst-case estimate, uncertainty in the difference = 0.000005 + 0.000005 = 0.000010 u.

Percentage uncertainty = (0.000010/0.000100) × 100 = 10%. Subtracting large masses can leave a small difference with a substantial percentage uncertainty.

Marking guidance 1: absolute difference uncertainty 0.000010 u. 1: 10%.

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6 Approve or reject the report's conclusions

6(a) The report states "helium's positive binding energy means the nucleus contains 28.3 MeV more rest energy than its separated nucleons". Correct this statement using the masses and the energy-level model. **[2 marks]**

The bound helium nucleus has 28.3 MeV less total rest energy than its separated protons and neutrons. Its lower mass is the mass defect.

Positive binding energy is the energy required to undo that binding and separate the nucleons; assembly releases this amount.

Marking guidance 1: lower rest energy/mass of bound system. 1: positive binding energy interpreted as separation input.

6(b) The report compares carbon-12's total binding energy with helium-4's and calls the larger total proof that each carbon nucleon is more tightly bound. Explain what additional calculation is needed before making that comparison. **[1 mark]**

The nuclei contain different numbers of nucleons. Divide each total binding energy by its own nucleon number A, then compare binding energy per nucleon. A larger total alone can simply reflect more nucleons.

Marking guidance 1: normalise each total by its own A.

6(c) A reaction releases energy although the total number of nucleons is unchanged. Explain how that can happen without violating mass–energy conservation. **[2 marks]**

The same nucleons can form products with lower combined rest energy and hence smaller total rest mass. The decrease in rest energy appears as kinetic energy and/or radiation.

Nucleon number is conserved; total mass–energy is conserved. Rest mass alone need not have the same summed value before and after.

Marking guidance 1: lower product rest mass with released kinetic/photon energy. 1: total mass–energy, rather than rest mass alone, is conserved.

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