OCR A A Level Physics | Kinetic and gravitational potential energy solutions

Matching solutions and teaching prompts for the class review | 40 marks

Specification points covered

- 3.3.2(a) Define and calculate kinetic energy, $E_k = \frac{1}{2}mv^2$, and derive it from first principles.
- 3.3.2(b) Define and calculate gravitational potential energy, $E_p = mgh$, and derive it from first principles.
- 3.3.2(c) Explain the exchange between gravitational potential energy and kinetic energy.

Class review
Model the energy account before substituting numbers. Insist that pupils say a quantity increases, decreases or stays the same, and distinguish energy transferred to another store from energy being destroyed.

1. Deriving the equations from first principles [7 marks]

The mission team checks that its energy model follows from earlier mechanics rather than treating the equations as facts to memorise.

(a) Starting with $W = Fs$, $F = ma$ and $v^2 = u^2 + 2as$, derive $\Delta E_k = \frac{1}{2}m(v^2 - u^2)$. Show every algebraic step. **[4 marks]**

(b) A payload is raised slowly through a vertical height Δh so that its speed stays constant. Starting with $W = Fs$ and the weight mg, derive $\Delta E_p = mg\Delta h$. Explain why the work done increases the gravitational potential energy store. **[3 marks]**

Answer / mark points: (a) From $v^2 = u^2 + 2as$, $s = (v^2 - u^2)/(2a)$. [1] Substitute $F = ma$ into $W = Fs$ to give $W = mas$. [1] Therefore $W = ma(v^2 - u^2)/(2a)$. [1] Cancelling a gives $W = \frac{1}{2}m(v^2 - u^2) = \Delta E_k$. [1]
(b) For a slow lift at constant speed, the upward force equals the weight: $F = mg$. [1] The displacement in the force direction is Δh, so $W = F\Delta h = mg\Delta h$. [1] Work done against the weight transfers energy to the gravitational potential energy store, hence $\Delta E_p = mg\Delta h$. [1]

Teacher prompt / common error: Do not accept a derivation that simply quotes the target equation. For E_p, constant speed is used so the lifting force equals mg; the result also assumes g is effectively uniform over Δh.

2. Calibration calculations [5 marks]

Before descent, three systems are checked close to the asteroid surface. Use $g = 0.24\ \mathrm{N\ kg^{-1}}$.

(a) A 24.0 kg mapping drone moves at $6.50\ \mathrm{m\ s^{-1}}$. Calculate its kinetic energy. **[2 marks]**

(b) A 27.0 kg inspection unit has kinetic energy 864 J. Calculate its speed. **[2 marks]**

(c) An 18.0 kg instrument is lifted vertically through 75.0 m. Calculate its increase in gravitational potential energy. **[1 mark]**

Answer / mark points: (a) $E_k = ½mv^2 = ½ \times 24.0 \times 6.50^2$ [1] = 507 J. [1] (b) $v = \sqrt{(2E_k/m)} = \sqrt{(2 \times 864 / 27.0)}$ [1] = $8\ \mathrm{m\ s^{-1}}$. [1] (c) $\Delta E_p = mg\Delta h = 18.0 \times 0.24 \times 75.0 = 324$ J. [1]
Teacher prompt / common error: Square the speed in the kinetic-energy equation. For speed, take the positive square root because speed is a magnitude.

3. A complete energy transfer [4 marks]

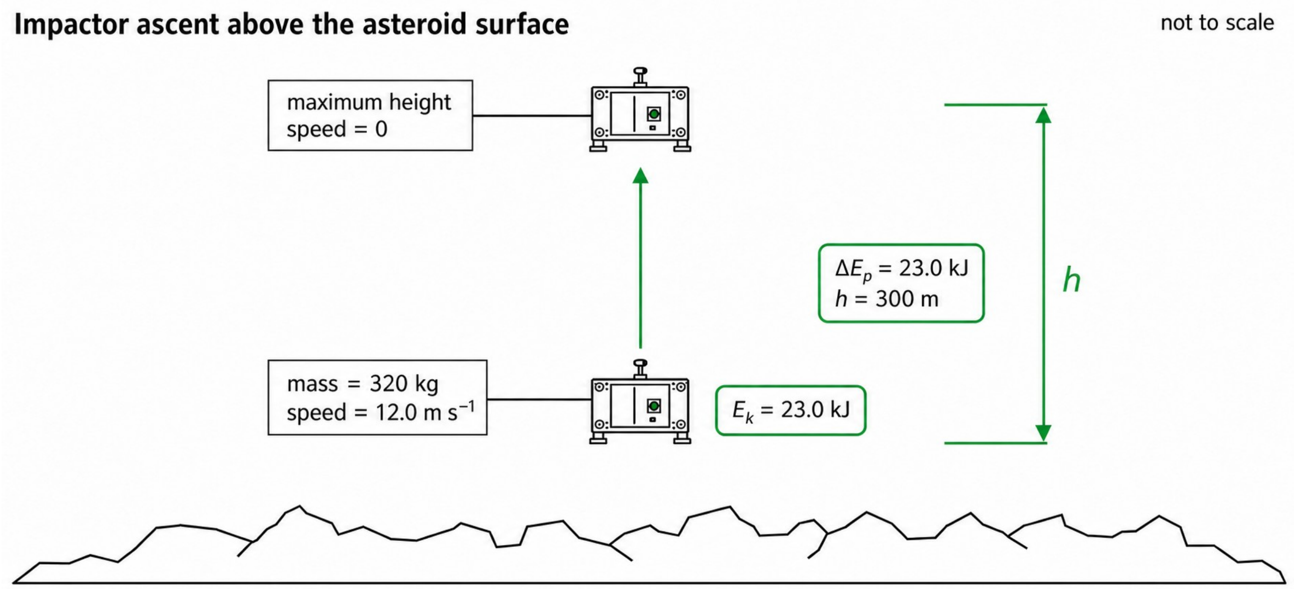
A 320 kg kinetic impactor fires its thrusters close to the asteroid surface until it is moving vertically upwards at $12.0\ \mathrm{m\ s^{-1}}$. Its engines then switch off. Resistive forces are negligible and $g = 0.24\ \mathrm{N\ kg^{-1}}$ is uniform.

(a) Calculate the impactor's kinetic energy when the engines switch off. **[1 mark]**

(b) Calculate the maximum height reached above the switch-off point. **[2 marks]**

(c) Describe the exchange between kinetic energy and gravitational potential energy as the impactor rises. **[1 mark]**

Impactor ascent above the asteroid surface



Near the surface, take $g = 0.24\ \mathrm{N\ kg^{-1}}$ and ignore resistive forces.

Answer / mark points: (a) $E_k = ½ \times 320 \times 12.0^2 = 23.0$ kJ. [1] (b) Complete transfer: $mg\Delta h = E_k$. [1] $\Delta h = 23040/(320 \times 0.24) = 300$ m. [1] (c) As the impactor rises, its kinetic energy decreases and its gravitational potential energy increases by the same amount; at maximum height its speed and kinetic energy are zero. [1]
Teacher prompt / common error: Use 'kinetic energy decreases', not 'velocity is decelerating'. In the stated model, no energy is transferred to thermal stores, so the exchange is complete.

4. Does mass affect the maximum height? [3 marks]

Two sample hoppers have different masses. Each leaves the same point at 9.60 m s^{-1}. Resistive forces are negligible and g = 0.24 N kg^{-1}.

(a) State which hopper reaches the greater maximum height. **[1 mark]**

(b) Use the kinetic and gravitational potential energy equations to show that mass cancels, then calculate the maximum height. **[2 marks]**

Answer / mark points: (a) The hoppers reach the same maximum height. [1]
(b) $\frac{1}{2}mv^2 = mg\Delta h$, so $\frac{1}{2}v^2 = g\Delta h$ after cancelling m. $\Delta h = v^2/(2g)$ [1] $= 9.60^2/(2 \times 0.24) = 192$ m. [1]

Teacher prompt / common error: A more massive hopper has more kinetic energy at the same speed, but it also needs proportionally more energy for each metre of height gained; these mass factors cancel.

5. A tethered canister: partial transfer while rising [5 marks]

A 48.0 kg sample canister leaves the surface vertically at 10.0 m s^{-1}. A magnetic brake on its tether transfers energy to thermal stores. At the highest point, 72% of the initial kinetic energy has increased the gravitational potential energy store. Use g = 0.24 N kg^{-1}.

(a) Calculate the initial kinetic energy of the canister. **[1 mark]**

(b) Calculate the increase in gravitational potential energy. **[1 mark]**

(c) Calculate the maximum height reached. **[2 marks]**

(d) Calculate the energy transferred to thermal stores by the brake. **[1 mark]**

Answer / mark points: (a) $E_k = \frac{1}{2} \times 48.0 \times 10.0^2 = 2400$ J. [1]
(b) $\Delta E_p = 0.72 \times 2400 = 1728$ J. [1]
(c) $\Delta h = \Delta E_p/(mg)$ [1] $= 1728/(48.0 \times 0.24) = 150$ m. [1]
(d) Thermal transfer = 2400 − 1728 = 672 J. [1]

Teacher prompt / common error: At the highest point the initial kinetic energy has not vanished: 72% is in the gravitational store and 28% has been transferred to thermal stores by the brake.

6. A landing package: partial transfer while falling [5 marks]

A 90.0 kg seismometer package starts from rest 160 m above its landing site. At touchdown, 65% of the decrease in gravitational potential energy is in the package's kinetic energy store. The remainder has been transferred to thermal and internal energy stores. Use $g = 0.24\ \text{N kg}^{-1}$.

(a) Calculate the decrease in gravitational potential energy. **[1 mark]**

(b) Calculate the kinetic energy at touchdown. **[1 mark]**

(c) Calculate the touchdown speed. **[2 marks]**

(d) Calculate the energy transferred to thermal and internal energy stores. **[1 mark]**

Answer / mark points: (a) Decrease in $E_p = mg\Delta h = 90.0 \times 0.24 \times 160 = 3456$ J. [1]
(b) $E_k = 0.65 \times 3456 = 2246.4$ J. [1]
(c) $v = \sqrt{(2E_k/m)}$ [1] $= \sqrt{(2 \times 2246.4/90.0)} = 7.07\ \text{m s}^{-1}$. [1]
(d) Other transfers $= 3456 - 2246.4 = 1209.6\ \text{J} \approx 1.21$ kJ. [1]

Teacher prompt / common error: Do not set all of the lost gravitational potential energy equal to kinetic energy when the question explicitly gives a 65% transfer.

7. Tracking an ejecta fragment [4 marks]

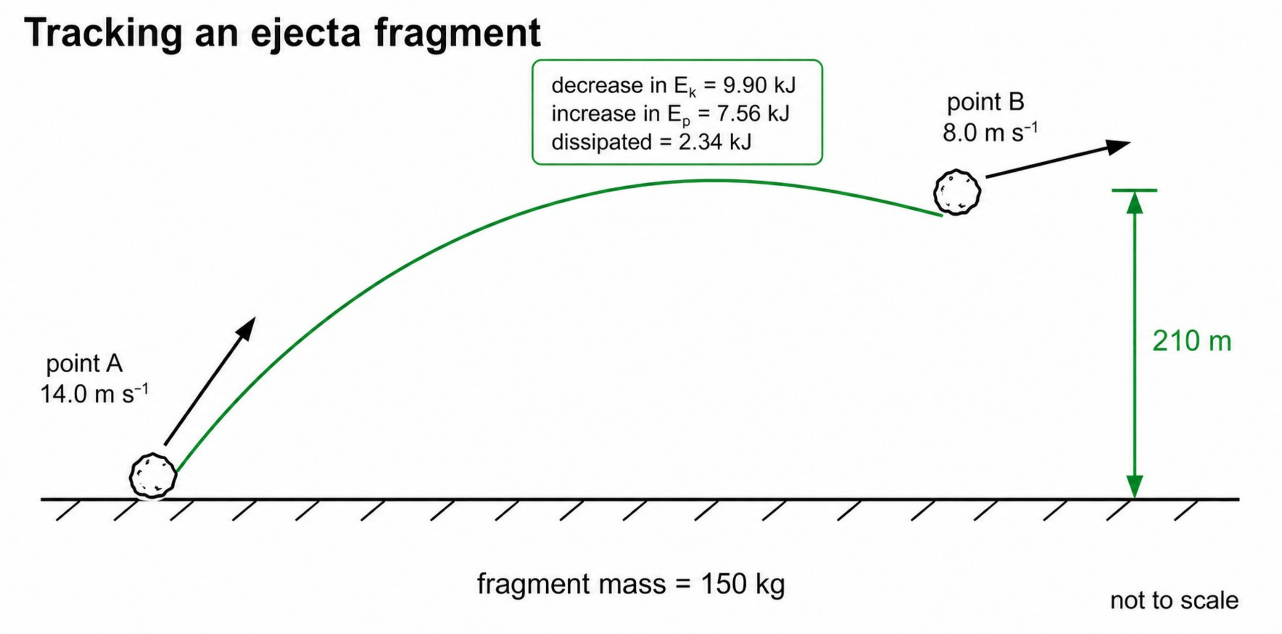
A 150 kg fragment leaves an impact crater at point A with speed $14.0\ \text{m s}^{-1}$. At point B it is 210 m higher and moving at $8.0\ \text{m s}^{-1}$. Use $g = 0.24\ \text{N kg}^{-1}$.

(a) Calculate the decrease in kinetic energy from A to B. **[1 mark]**

(b) Calculate the increase in gravitational potential energy from A to B. **[1 mark]**

(c) Calculate the energy transferred to other stores between A and B. **[1 mark]**

(d) Explain why the calculation shows that the transfer from kinetic energy to gravitational potential energy is only partial. **[1 mark]**



Answer / mark points: (a) Decrease in $E_k = \frac{1}{2} \times 150 \times (14.0^2 - 8.0^2) = 9.90$ kJ. [1]
(b) Increase in $E_p = 150 \times 0.24 \times 210 = 7.56$ kJ. [1]
(c) Energy transferred to other stores $= 9.90 - 7.56 = 2.34$ kJ. [1]
(d) The increase in E_p is smaller than the decrease in E_k, so only part of the kinetic-energy decrease is transferred to the gravitational store; the remaining 2.34 kJ is transferred to other stores. [1]

Teacher prompt / common error: Compare changes, not the final values of the stores. State clearly which quantity decreases and which increases.

8. Two-stage sample-return launch [7 marks]

A launcher transfers 30.0 kJ from a battery's chemical energy store. In stage 1, 60% becomes kinetic energy of a 120 kg sample capsule. In stage 2, 80% of that kinetic energy increases the capsule's gravitational potential energy. Use g = 0.24 N kg⁻¹.

(a) Calculate the kinetic energy of the capsule immediately after launch. **[1 mark]**

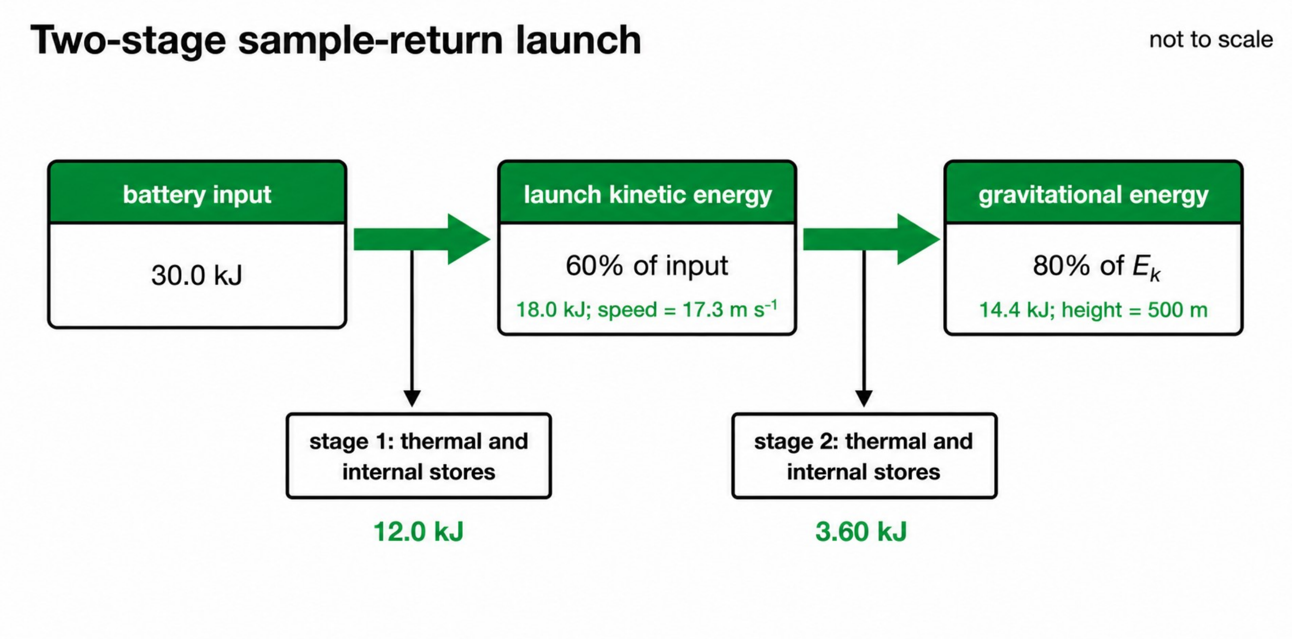
(b) Calculate the launch speed. **[2 marks]**

(c) Calculate the increase in gravitational potential energy during stage 2. **[1 mark]**

(d) Calculate the height gained during stage 2. **[1 mark]**

(e) Calculate the total energy transferred to thermal and internal energy stores across both stages. **[1 mark]**

(f) Calculate the percentage of the original 30.0 kJ that becomes gravitational potential energy. **[1 mark]**



Answer / mark points: (a) E_k = 0.60 × 30.0 = 18.0 kJ. [1]
(b) v = √(2E_k/m) [1] = √(2 × 18000/120) = 17.3 m s^{-1}. [1]
(c) ΔE_p = 0.80 × 18.0 = 14.4 kJ. [1]
(d) Δh = 14400/(120 × 0.24) = 500 m. [1]
(e) Stage 1 transfer = 12.0 kJ; stage 2 transfer = 3.60 kJ; total = 15.6 kJ. [1]
(f) Percentage = (14.4/30.0) × 100 = 48%. [1]

Teacher prompt / common error: For successive stages, multiply the fractions: 0.60 × 0.80 = 0.48. Do not add 60% and 80%, because the second percentage acts only on the kinetic energy produced in stage 1.

Review emphasis

Full-credit energy accounts name the store that decreases, the store that increases and any energy transferred to thermal or internal stores. The total energy before and after must agree.