OCR A A Level Ideal Gases 2 solutions

Matching solutions and teaching prompts for the class review

Specification points covered

- 5.1.4(b) State assumptions of the kinetic model of gases: a large number of molecules in random motion; negligible molecular volume; perfectly elastic collisions; negligible forces except during collisions.
- 5.1.4(c) Explain the pressure of a gas in terms of the kinetic model and Newton's laws.
- 5.1.4(e) Use $pV = \frac{1}{3}Nmc^2$ and explain its origin qualitatively; a derivation is not required.
- 5.1.4(f) Define and interpret root-mean-square (r.m.s.) speed and the Maxwell-Boltzmann distribution.
- 5.1.4(g) State and use the Boltzmann constant $k = R/N_A$.
- 5.1.4(h) Use $pV = NkT$ and derive $\frac{1}{2}mc^2 = \frac{3}{2}kT$ from the kinetic-theory equations.
- 5.1.4(i) Describe the internal energy of an ideal gas as kinetic only and dependent on temperature.

Teaching focus

Keep particle and molar quantities separate: use m with k, and molar mass M with R.
Train precise language: speed increases or decreases; a distribution becomes broader; area represents particle number.

1. Kinetic model and pressure

1. Engineers model the neon in the analyser as an ideal gas. State any two assumptions of the kinetic model of gases. [2 marks]

Answer / mark points: Any two: a very large number of molecules are in continuous random motion; the molecules occupy negligible volume compared with the gas; collisions are perfectly elastic; intermolecular forces are negligible except during collisions.

Teacher prompt / common error: Accept the OCR wording or an unambiguous equivalent. Do not accept 'no collisions' or 'particles have no volume' without the comparison that their volume is negligible relative to the gas volume.

2. Explain how the moving neon atoms produce a pressure on the wall of the analyser cell. Refer to momentum and Newton's laws. [3 marks]

Answer / mark points: Atoms collide with the wall and rebound, so their momentum changes. By Newton's second law, the rate of change of momentum requires a force. The atoms exert an equal and opposite force on the wall by Newton's third law; the normal force per unit area is the gas pressure.

Teacher prompt / common error: Insist on a momentum change at the wall and a force on the wall. 'Particles hit the wall' alone is incomplete. Pressure is force per unit area, not the collision rate itself.

2. From gas pressure to molecular energy

3. In $pV = \frac{1}{3}Nmc^2$, state precisely what c^2 represents. [1 mark]

Answer / mark points: c^2 is the mean of the squares of all the molecular speeds. It is not the square of the mean speed.	
Teacher prompt / common error: This distinction is essential for r.m.s. work: mean(c^2) is not generally [mean(c)]^2.	

4. The gas also obeys pV = NkT. Starting with the two expressions for pV, show that the mean kinetic energy of one atom is $\frac{1}{2}mc^2 = \frac{3}{2}kT$. [2 marks]

Answer / mark points: Equate the two expressions: $\frac{1}{3}Nmc^2 = NkT$. Cancel N to give $\frac{1}{3}mc^2 = kT$. Multiplying both sides by $\frac{3}{2}$ gives $\frac{1}{2}mc^2 = \frac{3}{2}kT$.	
Teacher prompt / common error: Award one mark for equating the two pV expressions and cancelling N, and one for the correct rearrangement to the required energy equation.	

5. Calculate the mean kinetic energy of one neon atom at 460 K. [1 mark]

Answer / mark points: Mean kinetic energy = $\frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 460 = 9.52 \times 10^{-21}$ J.	
Teacher prompt / common error: This is the average translational kinetic energy of one atom. The answer is an energy in joules, not a speed.	

6. The analyser cell contains 2.40×10^{21} neon atoms at 460 K. Calculate the total internal energy of the gas. [1 mark]

Answer / mark points: $U = N(\frac{3}{2}kT) = 2.40 \times 10^{21} \times 9.522 \times 10^{-21} = 22.9$ J.	
Teacher prompt / common error: Multiply the per-particle energy by N. A common error is to insert N into the one-particle equation and then multiply by N again.	

7. A second cell contains an ideal monatomic gas at pressure 2.80×10^4 Pa in a volume of 6.00×10^{-4} m^3. Use the kinetic-theory equations to calculate its internal energy. [1 mark]

Answer / mark points: Since NkT = pV, $U = \frac{3}{2}NkT = \frac{3}{2}pV = \frac{3}{2} \times 2.80 \times 10^4 \times 6.00 \times 10^{-4} = 25.2$ J.	
Teacher prompt / common error: The shortest route is $U = \frac{3}{2}pV$. Pupils should be able to obtain this from pV = NkT rather than treating it as an unrelated formula.	

8. Describe the internal energy of an ideal gas and explain why, for a fixed number of atoms, it depends only on absolute temperature. [2 marks]

Answer / mark points: The internal energy is the sum of the randomly distributed kinetic energies of the particles. An ideal gas has no intermolecular potential-energy contribution, so at fixed N its internal energy is proportional to the absolute temperature.	
Teacher prompt / common error: Use 'sum of randomly distributed kinetic energies'. An ideal gas has no intermolecular potential-energy term; do not say that each particle has identical kinetic energy.	

9. A control system keeps the pressure and volume of a test cell constant while 20% of its atoms escape. The initial temperature is 300 K. Calculate the new temperature. State what happens to the mean kinetic energy per atom and to the total internal energy of the remaining gas. [2 marks]

Answer / mark points: At constant p and V, NkT is constant. $N_2 = 0.80N_1$, so $T_2 = 300/0.80 = 375$ K. The mean kinetic energy per atom increases by 25%, but $U = \frac{3}{2}NkT = \frac{3}{2}pV$, so the total internal energy remains constant.	
Teacher prompt / common error: Fewer particles share the energy at fixed pV. Average energy rises, but total U stays constant. This is deliberately different from a sealed, rigid vessel.	

Continue your physics journey

Videos, practice and support for your next step

www.physicsuk.co.uk

3. Root-mean-square speed

10. Describe the three mathematical operations used to calculate the r.m.s. speed from a set of molecular speeds. [1 mark]

Answer / mark points: Square every molecular speed, find the mean of those squared speeds, then take the square root.

Teacher prompt / common error: The order matters: square, mean, root. The final quantity has the dimensions of speed.

11. A simulation records five molecular speeds: 240, 310, 390, 470 and 560 m s^{-1}. Calculate the r.m.s. speed. [2 marks]

Answer / mark points: r.m.s. speed = $\sqrt{[(240^2 + 310^2 + 390^2 + 470^2 + 560^2)/5]}$ = 4.10×10^2 m s^{-1}.

Teacher prompt / common error: Do not accept the arithmetic mean. Keeping the squares visible makes the method markable and exposes calculator-entry errors.

12. Calculate the arithmetic mean speed for the data in question 11 and compare it with the r.m.s. speed. [1 mark]

Answer / mark points: Arithmetic mean = (240 + 310 + 390 + 470 + 560)/5 = 394 m s^{-1}. It is 16 m s^{-1} smaller than the r.m.s. speed.

Teacher prompt / common error: The r.m.s. speed must be at least as large as the arithmetic mean for non-negative speeds; equality occurs only when every speed is identical.

13. The mass of one neon atom is 3.35×10^{-26} kg. Calculate the r.m.s. speed of the neon atoms at 460 K. [1 mark]

Answer / mark points: r.m.s. speed = $\sqrt{(3kT/m)}$ = $\sqrt{[(3 \times 1.38 \times 10^{-23} \times 460)/(3.35 \times 10^{-26})]}$ = 754 m s^{-1}.

Teacher prompt / common error: Use the mass of one atom with k. Using molar mass with k mixes particle and molar quantities.

14. A xenon atom has mass 2.18×10^{-25} kg. Both gases are at 460 K. Calculate the ratio of the neon r.m.s. speed to the xenon r.m.s. speed without calculating either speed separately. [1 mark]

Answer / mark points: speed ratio = $\sqrt{(m_x\text{enon}/m_n\text{eon})}$ = $\sqrt{[(2.18 \times 10^{-25})/(3.35 \times 10^{-26})]}$ = 2.55.

Teacher prompt / common error: At equal temperature, r.m.s. speed is proportional to $1/\sqrt{m}$. Put the heavier mass in the numerator when forming neon speed divided by xenon speed.

15. A time-of-flight detector measures an r.m.s. speed of 790 m s^{-1} for a monatomic gas at 500 K. Calculate its molar mass and use the data table to identify the gas. [2 marks]

Answer / mark points: M = 3RT/(r.m.s. speed)^2 = $(3 \times 8.31 \times 500)/(790^2)$ = 1.997×10^{-2} kg mol^{-1} = 20.0 g mol^{-1}. The gas is neon.

Teacher prompt / common error: Use R with molar mass. Convert the calculated kg mol^{-1} value to g mol^{-1} before matching it to the table.

16. A detector cell contains 4.50×10^{20} neon atoms, each of mass 3.35×10^{-26} kg. Their r.m.s. speed is 620 m s^{-1} and the cell volume is 2.40×10^{-4} m^3. Calculate the gas pressure. [1 mark]

Answer / mark points: p = Nm(r.m.s. speed)^2/(3V) = $[4.50 \times 10^{20} \times 3.35 \times 10^{-26} \times 620^2]/[3 \times 2.40 \times 10^{-4}]$ = 8.05×10^3 Pa.

Teacher prompt / common error: Because (r.m.s. speed)^2 equals the mean square speed, it can be substituted directly for c^2 in the pressure equation.

17. Faulty software uses the arithmetic mean speed in $pV = ⅓Nmc^2$. Explain why this gives a pressure that is too small. [1 mark]

Answer / mark points: The arithmetic mean speed is smaller than the r.m.s. speed. Squaring that smaller value makes the substituted c^2 too small, so the calculated pressure is too small.

Teacher prompt / common error: The error is systematic because mean speed is below r.m.s. speed. Avoid saying that the molecules themselves slow down; only the software's representative value is wrong.

Continue your physics journey

Videos, practice and support for your next step

www.physicsuk.co.uk

4. Reading a Maxwell-Boltzmann distribution

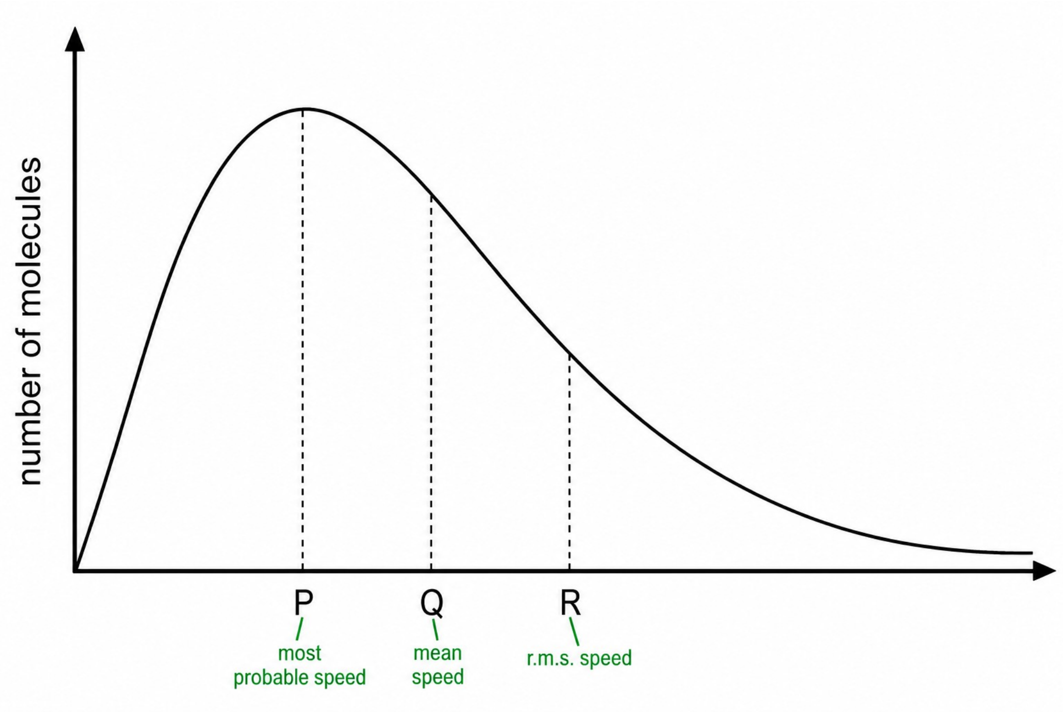

Figure 1: most probable, mean and r.m.s. speeds

18. Figure 1 shows a Maxwell-Boltzmann distribution for one gas. Identify the speeds marked P, Q and R. [3 marks]

Answer / mark points: P is the most probable speed, Q is the mean speed and R is the r.m.s. speed. Therefore most probable speed < mean speed < r.m.s. speed.

Teacher prompt / common error: For a molecular-speed distribution, the ordering is most probable < mean < r.m.s. The right-hand tail pulls the mean and r.m.s. to the right of the peak.

19. Explain why the curve contains very few molecules at speeds close to zero and at extremely high speeds. [1 mark]

Answer / mark points: Almost no molecules have exactly zero kinetic energy, while extremely high energies are statistically very unlikely, so the number at either extreme tends towards zero.

Teacher prompt / common error: The curve tends to zero at both ends. Do not say no molecule can ever have a very high speed; the probability becomes very small.

20. State what the area under a Maxwell-Boltzmann distribution represents. [1 mark]

Answer / mark points: The area represents the total number of molecules in the sample. For a fixed sample, the area is unchanged when temperature changes.

Teacher prompt / common error: Area is particle number, not mean kinetic energy. For the same fixed sample, heating redistributes speeds but does not change the area.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

5. Heating a fixed sample

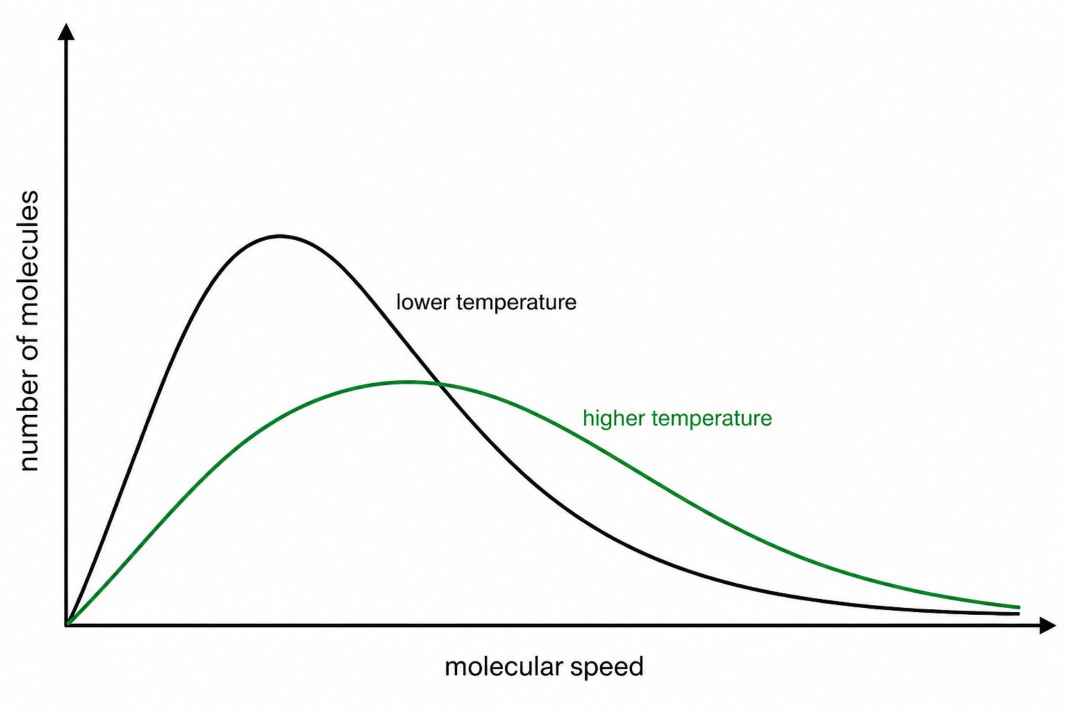

Figure 2: higher-temperature solution curve

21. Figure 2 shows the distribution for a fixed sample at a lower temperature. On the figure, sketch the distribution after the gas is heated. [2 marks]

Answer / mark points: The higher-temperature curve starts at the origin, has a lower and broader peak shifted to the right, and encloses the same area as the original curve.

Teacher prompt / common error: For both marks, look for a lower, broader curve with its peak to the right and a qualitatively unchanged area. The green solution overlay shows the intended relationship.

22. Describe how the most probable speed, the height of the peak and the width of the distribution change when the gas is heated. Use quantity-based language. [2 marks]

Answer / mark points: The most probable speed increases; the peak height decreases; the distribution becomes broader. The area stays the same because the number of molecules is fixed.

Teacher prompt / common error: Use 'speed increases', 'peak height decreases' and 'distribution broadens'. A speed does not accelerate; particles accelerate during collisions.

23. The detector counts only molecules faster than a threshold speed. Explain why heating the gas increases the fraction of molecules detected. [1 mark]

Answer / mark points: Heating increases the proportion of the distribution beyond the threshold speed, so a greater fraction of molecules has enough speed to be counted.

Teacher prompt / common error: The relevant idea is the greater area to the right of the threshold, not that every molecule moves faster than before.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

6. Apply and connect the ideas

24. Helium and xenon samples contain the same number of atoms at the same temperature. Compare their mean kinetic energies and their Maxwell-Boltzmann distributions. [2 marks]

Answer / mark points: The mean kinetic energy is the same because both gases have the same temperature. Helium is lighter, so its distribution is shifted to higher speeds and is broader with a lower peak than xenon's distribution for equal particle numbers.

Teacher prompt / common error: Equal temperature means equal mean kinetic energy, not equal molecular speed. The lighter helium atoms have the greater characteristic speeds.

25. Cells X and Y have the same volume and temperature. X contains 2N atoms of mass m; Y contains N atoms of mass 4m. Compare X with Y for: pressure, mean kinetic energy per atom, r.m.s. speed and total internal energy. Give a ratio for each quantity. [4 marks]

Answer / mark points: Pressure: $p_X/p_Y = 2$ because p is proportional to N at fixed V and T. Mean kinetic energy per atom: $E_X/E_Y = 1$ because T is the same. R.m.s. speed: $c_X/c_Y = \sqrt{(4m/m)} = 2$. Total internal energy: $U_X/U_Y = 2$ because $U = \frac{3}{2}NkT$.

Teacher prompt / common error: Separate particle number from particle mass. At fixed V and T, pressure and total internal energy depend on N; mean energy depends only on T; r.m.s. speed depends on T/m.

Language check

Say that mean kinetic energy increases with absolute temperature; do not say that energy itself accelerates.
On a distribution, say the most probable speed increases, the peak height decreases, the curve broadens and the area stays the same.

Continue your physics journey

Videos, practice and support for your next step

www.physicsuk.co.uk