Hooke’s law and elastic energy solutions

AQA AS 7407 and first-year A Level 7408 | OCR A AS H156 and A Level H556

Matching solutions and teaching prompts for the class review | 40 marks

Specification points covered

- AQA 7407 / 7408: 3.4.2.1 selected Hooke’s law, deformation graphs and elastic energy; 3.4.1.8 selected kinetic and gravitational transfers.
- OCR A H156 / H556: 3.4.1(a)–(d)(ii), 3.4.2(a)–(b); 3.3.2(a)–(b) selected energy calculations. No SHM equations are required.

The Asteria mirror support tests

Model the stated cartridges and stages as having negligible moving mass. Guides carry no vertical load. Use blank space for calculations and ruled lines for explanations.

1. Language and Hooke’s law [4 marks]

The Asteria Observatory uses spring cartridges to protect a segmented telescope mirror.

(a) A loaded cartridge becomes shorter. State the type of deformation. [1 mark]

(b) State Hooke’s law, including the condition for it to apply. [2 marks]

(c) Define the force constant k and state its SI unit. [1 mark]

Answer / mark points: (a) Compressive deformation, or compression. [1] (b) Extension or compression is directly proportional to applied force [1], provided the limit of proportionality is not exceeded. [1] (c) Force per unit extension or compression, $k = F/x$, measured in N m^{-1}. Require the definition and unit for this complete statement. [1]

Teacher prompt / common error: Use directly proportional, not merely increases with. The limit of proportionality and the elastic limit are different ideas; non-linear behaviour need not imply permanent deformation.

2. One cartridge under load [2 marks]

During a bench test, a compressive force of 1.50 kN acts on one cartridge of force constant 120 kN m^{-1}. It remains in its linear elastic range.

(a) Calculate the compression in mm. [1 mark]

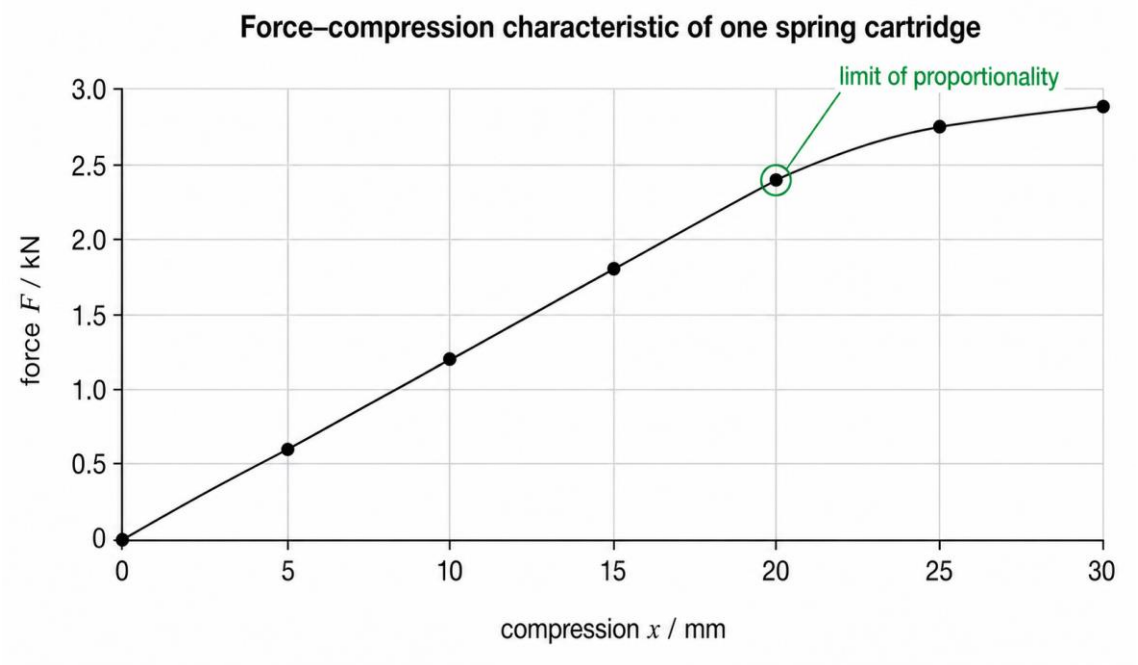
(b) Calculate the elastic energy stored. [1 mark]

Answer / mark points: (a) $x = F/k = 1500/120000 = 0.01250$ m = 12.5 mm. [1] (b) $E_{el} = \frac{1}{2}Fx = \frac{1}{2} \times 1500 \times 0.01250 = 9.375$ J, approximately 9.38 J. [1]

Teacher prompt / common error: Compression is a change in length, not the loaded length. Convert both kN and mm consistently. The second calculation is one direct outcome after part(a).

3. Calibrating a cartridge [6 marks]

The force–compression graph describes one cartridge. Use its straight-line region when finding k.



(a) Use two points on the straight-line region to calculate k in N m^{-1}. Show the changes in force and compression. [2 marks]

(b) State the compression and force at the limit of proportionality. [1 mark]

(c) Describe how the graph shows that Hooke's law is no longer obeyed beyond that point. [2 marks]

(d) Calculate the elastic energy stored at a compression of 20.0 mm. [1 mark]

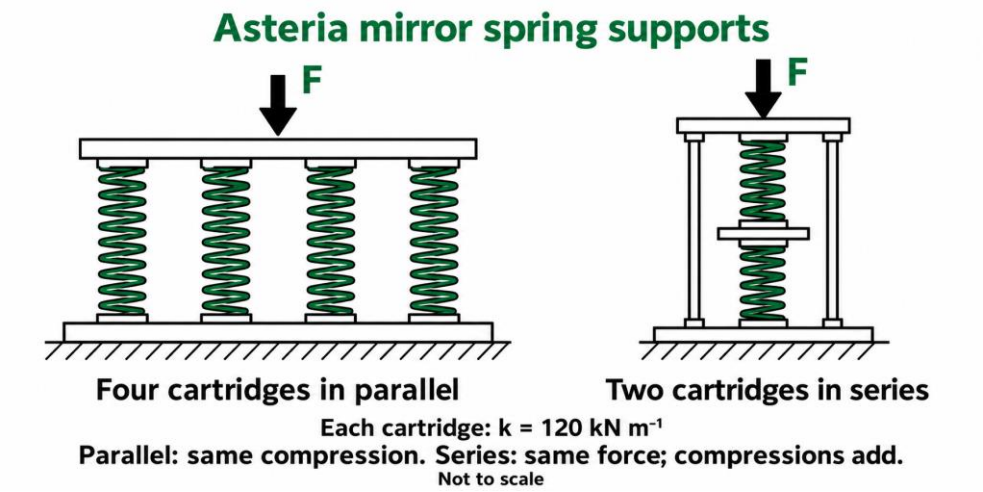
Answer / mark points: (a) $k = \Delta F/\Delta x$. Taking (5.0 mm, 0.60 kN) and (15.0 mm, 1.80 kN): $\Delta F = 1200$ N and $\Delta x = 0.0100$ m. [1] $k = 1200/0.0100 = 1.20 \times 10^5$ N m^{-1}. [1] Accept other well-separated points in the straight-line region. (b) Compression 20.0 mm and force 2.40 kN. [1] (c) The graph curves and its gradient decreases beyond 20.0 mm. [1] Force is no longer directly proportional to compression. [1] (d) E_{el} = area under the linear section = $\frac{1}{2}Fx = \frac{1}{2} \times 2400 \times 0.0200 = 24.0$ J. [1]

Teacher prompt / common error: A straight line through the origin shows direct proportionality. Do not call the last point on the straight line an elastic limit without unloading evidence. Graph area units are kN mm, equal to J here.

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4. Four supports in parallel [4 marks]

A rigid plate keeps four identical cartridges at the same compression. They share the weight of a 650 kg mirror cradle equally. Each has k = 120 kN m^{-1}. Use g = 9.81 N kg^{-1}.



(a) Calculate the cradle's weight. [1 mark]

(b) Calculate the force carried by each cartridge. [1 mark]

(c) Use F = kx for one cartridge to calculate the common compression in mm. [1 mark]

(d) Use K = total force/common compression to calculate the force constant of the whole stage. [1 mark]

Answer / mark points: (a) W = mg = 650 × 9.81 = 6376.5 N, approximately 6.38 kN. [1] (b) Force per cartridge = 6376.5/4 = 1594.125 N, approximately 1.59 kN. [1] (c) x = 1594.125/120000 = 0.013284375 m, approximately 13.3 mm. [1] (d) K = 6376.5/0.013284375 = 480000 N m^{-1} = 480 kN m^{-1}. [1]

Teacher prompt / common error: The parallel result is obtained from equal compression and force sharing, not specialist formula recall. Each 13.3 mm compression is below the calibrated 20.0 mm proportional limit.

5. Two supports in series [5 marks]

A separate guided stage has two identical cartridges stacked end to end with a negligible-mass spacer. The same 1.80 kN compressive force passes through both. Each has k = 120 kN m^{-1}. The guide carries no vertical load.

(a) State the compressive force acting on each cartridge. [1 mark]

(b) Calculate the compression of each cartridge. [1 mark]

(c) Add the compressions to find the stage's total compression. Decide whether it meets a maximum total compression of 28.0 mm. [2 marks]

(d) Use K = stage force/total compression to calculate the stage's force constant. [1 mark]

Answer / mark points: (a) Each cartridge carries 1.80 kN. [1] (b) Each compression x = 1800/120000 = 0.0150 m = 15.0 mm. [1] (c) Total compression = 15.0 + 15.0 = 30.0 mm. [1] It exceeds 28.0 mm, so the design limit is not met. [1] (d) K = 1800/0.0300 = 60000 N m^{-1} = 60.0 kN m^{-1}. [1]

Teacher prompt / common error: Series cartridges share a force; their compressions add. Each is compressed 15.0 mm, within its own 20.0 mm proportional range, even though the total stage compression is 30.0 mm. The separate 28.0 mm design limit is still exceeded.

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6. Horizontal release test [5 marks]

In a guarded test, one cartridge of $k = 120$ kN m^{-1} is compressed 12.0 mm against an 18.0 kg calibration sled initially at rest. At separation the cartridge is uncompressed and its moving mass has negligible kinetic energy. During release 2.16 J is dissipated; all remaining released elastic energy becomes sled kinetic energy. After separation it coasts horizontally into a receiver with negligible resistance.

(a) Calculate the initial elastic energy. [1 mark]

(b) Calculate the sled's kinetic energy at separation. [1 mark]

(c) Calculate the sled's speed as it enters the receiver. [2 marks]

(d) The receiver accepts entry speeds no greater than 0.900 m s^{-1}. Decide whether this test meets the limit. [1 mark]

Answer / mark points: (a) $E_{el} = ½kx^2 = ½ \times 120000 \times 0.0120^2 = 8.64$ J. [1] (b) $E_k = 8.64 - 2.16 = 6.48$ J. [1] (c) $v = \sqrt{(2E_k/m)}$ [1] $= \sqrt{(2 \times 6.48/18.0)} = 0.848528\ldots$ m s^{-1}, approximately 0.849 m s^{-1}. [1] (d) Yes:0.849 m s^{-1} is less than 0.900 m s^{-1}. [1]

Teacher prompt / common error: The limit applies at receiver entry, not automatically to the maximum speed during spring contact. There is no gravitational change on this horizontal track. Account for the 2.16 J thermal transfer before using the kinetic-energy equation.

7. Vertical release test [6 marks]

A separate enclosed test uses one cartridge of $k = 120$ kN m^{-1} compressed 15.0 mm. It releases a total moving mass of 1.50 kg vertically from rest. Heights are measured from the mass's initial centre-of-mass position. It separates when the cartridge is uncompressed. Contact dissipates 1.50 J; there are no subsequent losses. At maximum height the mass is at rest. Use $g = 9.81$ N kg^{-1}.

(a) Calculate the initial elastic energy. [1 mark]

(b) Determine the maximum height and assess a designer's claim that the mass rises through 0.850 m. [3 marks]

(c) Calculate the mass's speed when it is 0.500 m above its starting position. [2 marks]

Answer / mark points: (a) $E_{el} = ½ \times 120000 \times 0.0150^2 = 13.5$ J. [1] (b) At the top $E_k = 0$ and the spring is uncompressed, so $mg\Delta h = 13.5 - 1.50 = 12.0$ J. [1] $\Delta h = 12.0/(1.50 \times 9.81) = 0.815494\ldots$ m, approximately 0.815 m. [1] The 0.850 m claim is not supported. [1] (c) At this height the spring is uncompressed and the contact loss is complete: $E_k = 12.0 - 1.50 \times 9.81 \times 0.500 = 4.6425$ J. [1] $v = \sqrt{(2 \times 4.6425/1.50)} = 2.487971\ldots$ m s^{-1}, approximately 2.49 m s^{-1}. [1]

Teacher prompt / common error: Use the change in centre-of-mass height, not the spring's compression as the whole rise. At 0.500 m there is still kinetic energy; at the maximum height there is not. The contact loss has already happened in both accounts.

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8. Additional compression in transit [3 marks]

A transport fixture uses six cartridges in parallel and has a measured total force constant of 720 kN m^{-1}. During a disturbance its compression increases from 4.00 mm to 12.0 mm, entirely within the linear elastic range.

(a) Calculate the increase in elastic energy stored in the fixture. [2 marks]

(b) State what this increase represents during the additional compression. [1 mark]

Answer / mark points: (a) $\Delta E_{el} = \frac{1}{2}K(x_2^2 - x_1^2)$. [1] $\Delta E_{el} = \frac{1}{2} \times 720000 \times (0.0120^2 - 0.00400^2) = 46.08$ J, approximately 46.1 J. [1] (b) Work done on the springs and transferred into their elastic energy store during the additional compression. [1]

Teacher prompt / common error: Subtract the two stored energies. Using $\frac{1}{2}K(\Delta x)^2$ incorrectly treats the springs as initially uncompressed. This question does not require an SHM model.

9. Checking the spring model [5 marks]

For a classroom-scale test, a low-force training cartridge is mounted vertically between a fixed base and a negligible-mass guided top plate. The guide has negligible friction. A millimetre ruler, balance and small known masses are available. Keep the supported load below 5 N and secure it against falling; this is not the full-scale 120 kN m^{-1} cartridge.

(a) Describe how to obtain reliable force–compression data and determine the force constant and limit of proportionality. [4 marks]

(b) Explain how to check whether permanent deformation has occurred after loading. [1 mark]

Answer / mark points: (a) Record the unloaded length/zero reference; add known masses in suitable increments, allow settling, use F = mg and measure compression relative to the zero. [1] Read the scale at eye level, repeat each reading and use means where appropriate. [1] Plot force F against compression x; the gradient of the straight-line region gives k, with consistent SI units. [1] The limit of proportionality is where the graph first departs systematically from the straight line through the origin. [1] (b) Remove the applied load and compare with the original unloaded length: a remaining change in length/compression indicates permanent deformation. [1]

Teacher prompt / common error: Stable guided compression differs from hanging a spring in tension. Follow the stated 5 N limit. If the graph remains straight through the permitted range, the proportional limit has not been reached;5 N is not automatically that limit. Distinguish a non-linear loading curve from failure to return to the unloaded length.

Review: identify the deformation, check the linear range and write a complete energy account before choosing equations. A combination's total deformation is not the deformation of each cartridge.

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