OCR A A Level Physics | Hooke's law solutions

Matching solutions and teaching prompts for the class review | 38 marks

Specification points covered

- 3.4.1(a) Describe tensile and compressive deformation, extension and compression.
- 3.4.1(b) State and apply Hooke's law.
- 3.4.1(c) Define the force constant k of a spring or wire using F = kx.
- 3.4.1(d)(i) Interpret force-extension or force-compression graphs for springs and wires.
- 3.4.1(d)(ii) Use procedures to investigate force-extension characteristics.
- 3.4.2(a)-(b) Use the area under a force-deformation graph and calculate elastic potential energy.

Class review
Insist on directly proportional, not simply proportional, when pupils state Hooke's law. Keep force sharing in parallel separate from deformation adding in series.

1. Language and Hooke's law [4 marks]

The Asteria Observatory uses sealed spring cartridges to protect a giant segmented telescope mirror from vibrations.

(a) A loaded cartridge becomes shorter. State the type of deformation taking place. **[1 mark]**

(b) State Hooke's law, including the condition needed for it to apply. **[2 marks]**

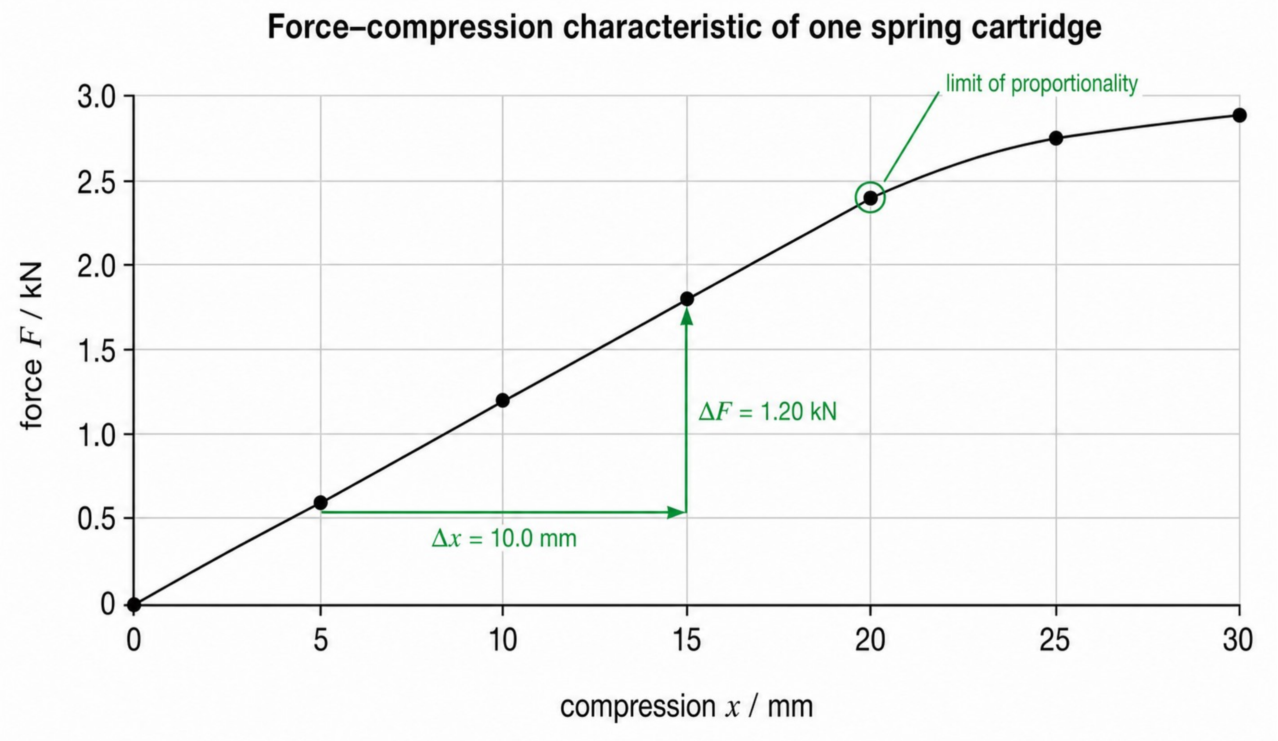
(c) Define the force constant k and state its SI unit. **[1 mark]**

Answer / mark points: (a) Compressive deformation, or compression. [1]
(b) The compression or extension is directly proportional to the applied force [1], provided the limit of proportionality is not exceeded. [1]
(c) k is force per unit extension or compression, k = F/x. Its SI unit is N m^{-1}. [1]

Teacher prompt / common error: A quantity can increase, decrease or stay the same. A spring does not 'compress the force'; its compression increases as the applied force increases within the linear region.

2. Calibrating one spring cartridge [6 marks]

A technician records the force-compression characteristic of one cartridge. Use the straight-line region when calculating the force constant.



(a) Use the graph to calculate the force constant of the cartridge. Give your answer in N m^{-1}. **[2 marks]**

(b) State the compression and force at the limit of proportionality. **[1 mark]**

(c) Describe how the graph shows that the cartridge no longer obeys Hooke's law beyond this point. **[2 marks]**

(d) Calculate the elastic potential energy stored when the compression is 20.0 mm. **[1 mark]**

Answer / mark points: (a) $k = \Delta F/\Delta x$ [1] $= (1.80 - 0.60) \times 10^3 / ((15.0 - 5.0) \times 10^{-3}) = 1.20 \times 10^5$ N m^{-1}. [1]
(b) Limit of proportionality: compression = 20.0 mm and force = 2.40 kN. [1]
(c) Beyond 20 mm, the graph is curved and its gradient decreases [1], so force is no longer directly proportional to compression. [1]
(d) E = area under the straight-line graph = $\frac{1}{2}Fx = \frac{1}{2} \times 2400 \times 0.0200 = 24.0$ J. [1]

Teacher prompt / common error: A straight line through the origin shows direct proportionality. Do not use a single point from the curved region to calculate the force constant.

3. One cartridge under load [3 marks]

During a bench test, a compressive force of 1.50 kN acts on one cartridge. Its force constant is 120 kN m⁻¹.

(a) Calculate the compression of the cartridge in mm. **[1 mark]**

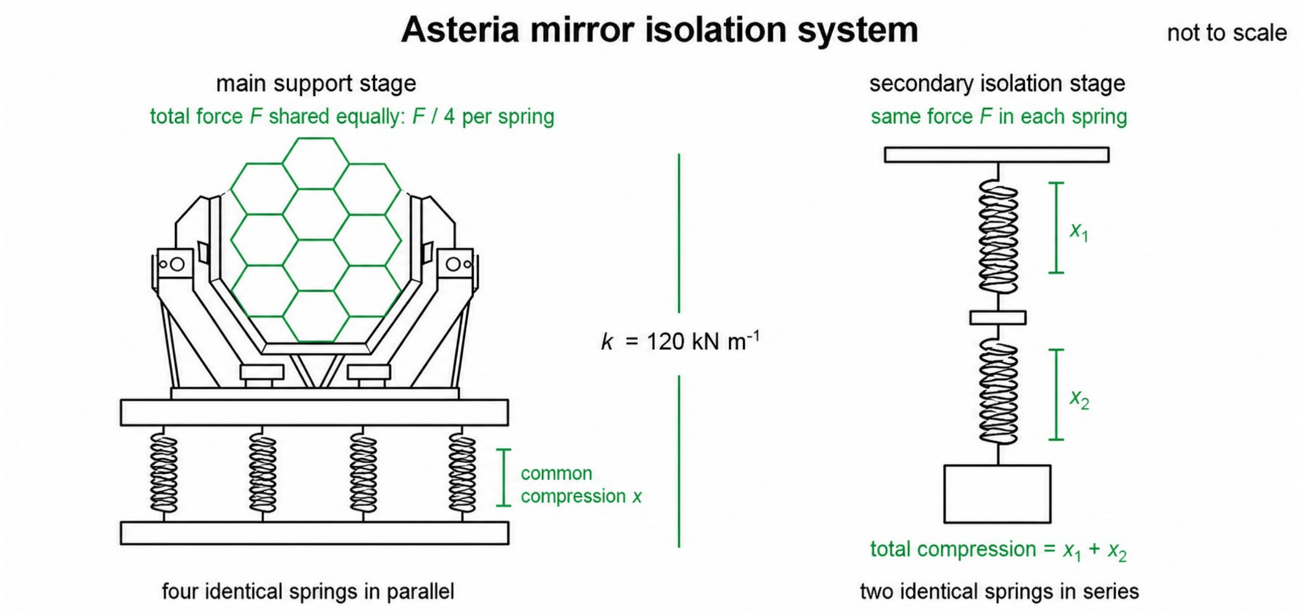
(b) Calculate the elastic potential energy stored in the cartridge. **[2 marks]**

Answer / mark points: (a) $x = F/k = 1500 / 120\,000 = 0.01250$ m = 12.5 mm. [1]
(b) $E = \frac{1}{2}Fx = \frac{1}{2} \times 1500 \times 0.0125$ [1] = 9.375 J ≈ 9.38 J. [1]

Teacher prompt / common error: The compression is below 20 mm, so the cartridge is within the proportional region and the elastic-energy expression is valid.

4. The main support stage: springs in parallel [5 marks]

A 650 kg mirror cradle is supported by four identical spring cartridges in parallel. The load is shared equally and each cartridge has force constant 120 kN m⁻¹. Use g = 9.81 N kg⁻¹.



(a) Calculate the weight of the mirror cradle. **[1 mark]**

(b) Calculate the force supported by each cartridge. **[1 mark]**

(c) Calculate the compression of the support stage in mm. **[1 mark]**

(d) Calculate the equivalent force constant of the four-cartridge support stage. **[1 mark]**

(e) Calculate the total elastic potential energy stored in the support stage. **[1 mark]**

Answer / mark points: (a) $W = mg = 650 \times 9.81 = 6.377$ kN. [1]
(b) Force per cartridge = 6376.5 / 4 = 1.594 kN. [1]
(c) $x = F/k = 1594.1 / 120\,000 = 13.3$ mm. [1]
(d) Equivalent $k = 4k = 480$ kN m⁻¹. [1]
(e) $E = \frac{1}{2}F_{total}x = \frac{1}{2} \times 6376.5 \times 0.01328 = 42.4$ J. [1]

Teacher prompt / common error: Parallel springs have the same compression. The total load is shared, so the equivalent force constant is the sum of the individual force constants.

5. The secondary stage: springs in series [5 marks]

A secondary isolation stage uses two identical cartridges connected in series. Each cartridge has force constant 120 kN m^{-1} and the stage supports a load of 1.80 kN.

(a) State the force acting on each cartridge. **[1 mark]**

(b) Calculate the compression of each cartridge. **[1 mark]**

(c) Calculate the total compression of the two-cartridge stage. **[1 mark]**

(d) Calculate the equivalent force constant of the stage. **[1 mark]**

(e) Calculate the total elastic potential energy stored in the stage. **[1 mark]**

Answer / mark points: (a) The same 1.80 kN force acts through each cartridge. [1]
(b) $x = 1800 / 120\,000 = 15.0$ mm for each cartridge. [1]
(c) $x_{total} = 15.0 + 15.0 = 30.0$ mm. [1]
(d) Equivalent $k = F/x_{total} = 1800 / 0.030 = 60.0$ kN m^{-1}. [1]
(e) $E = \frac{1}{2}Fx_{total} = \frac{1}{2} \times 1800 \times 0.030 = 27.0$ J. [1]

Teacher prompt / common error: Series springs carry the same force. Their individual compressions add, so two identical series springs are softer than one spring.

6. Unequal springs in series [4 marks]

A redesigned stage places a 160 kN m^{-1} cartridge and a 240 kN m^{-1} cartridge in series. The same compressive force of 2.40 kN acts through both cartridges.

(a) Calculate the compression of the 160 kN m^{-1} cartridge. **[1 mark]**

(b) Calculate the compression of the 240 kN m^{-1} cartridge. **[1 mark]**

(c) Calculate the total compression of the stage. **[1 mark]**

(d) Calculate the equivalent force constant of the stage. **[1 mark]**

Answer / mark points: (a) $x_1 = 2400 / 160\,000 = 15.0$ mm. [1]
(b) $x_2 = 2400 / 240\,000 = 10.0$ mm. [1]
(c) $x_{total} = 15.0 + 10.0 = 25.0$ mm. [1]
(d) Equivalent $k = 2400 / 0.025 = 96.0$ kN m^{-1}. [1]

Teacher prompt / common error: The lower-k cartridge compresses more under the same force. Do not share the force in proportion to the force constants when the springs are in series.

7. Choosing a transport support [4 marks]

A 720 kg transport pod must be supported by six identical 120 kN m^{-1} cartridges. Arrangement A uses all six in parallel. Arrangement B uses three parallel branches, with two cartridges in series in each branch. The maximum permitted compression is 12.0 mm. Use $g = 9.81$ N kg^{-1}.

(a) Calculate the compression of arrangement A. **[1 mark]**

(b) Calculate the compression of arrangement B. **[2 marks]**

(c) Recommend an arrangement. Justify your answer using the calculated compressions. **[1 mark]**

Answer / mark points: (a) Arrangement A: equivalent $k = 6k = 720$ kN m^{-1}, so $x = 7063.2 / 720\,000 = 9.81$ mm. [1]
(b) In each series pair, $k = 60.0$ kN m^{-1}; three such branches in parallel give an equivalent k of 180 kN m^{-1}. [1] Therefore $x = 7063.2 / 180\,000 = 39.24$ mm. [1]
(c) Choose arrangement A: 9.81 mm is below the 12.0 mm limit, whereas 39.24 mm is above it. [1]

Teacher prompt / common error: Establish one series branch first, then add the parallel branch force constants. A physically plausible layout is not automatically stiff enough for the engineering constraint.

8. Energy during a vibration [3 marks]

Arrangement A has an equivalent force constant of 720 kN m^{-1}. During a vibration, its compression increases from 4.00 mm to 12.0 mm.

(a) Calculate the increase in elastic potential energy stored in the support. **[2 marks]**

(b) State what this energy change represents during the compression of the support. **[1 mark]**

Answer / mark points: (a) $\Delta E = \frac{1}{2}k(x_2^2 - x_1^2)$ [1] $= \frac{1}{2} \times 720\,000 \times (0.0120^2 - 0.00400^2) = 46.08$ J. [1]
(b) It is the work done on the support and transferred into its elastic potential energy store during the additional compression. [1]

Teacher prompt / common error: Calculate the difference between the two elastic-energy values. Using $\frac{1}{2}k(\Delta x)^2$ would give the wrong result because elastic energy depends on x^2.

9. Testing a replacement cartridge [4 marks]

A replacement cartridge is tested using a set of known masses and a millimetre scale.

(a) Describe how the technician should obtain reliable force-compression data and use the results to determine both the force constant and the limit of proportionality. **[4 marks]**

Answer / mark points: (a) Measure the unloaded length or set a zero reference, then add known masses in suitable increments and allow the cartridge to settle. Convert each mass to force using $F = mg$ and measure the corresponding compression. [1]
Read the scale at eye level to avoid parallax and repeat readings, using mean values where appropriate. [1]
Plot force F against compression x. The gradient of the straight-line region gives the force constant k. [1]
The limit of proportionality is the point at which the graph first departs systematically from a straight line through the origin. [1]

Teacher prompt / common error: A conclusion should refer to the shape of the graph, not merely say that the values are 'close'. A safe investigation also avoids loading beyond the cartridge's rated range.

Review emphasis

Full-credit responses distinguish compressive deformation from force, use directly proportional precisely, and state whether force is shared or deformation is added before combining springs.