physics UK
Smarter practice. Stronger understanding

SOLUTIONS
ATTEMPT | CHECK | CORRECT | IMPROVE

OCR A A Level Gravitational Fields 3 solutions

Matching solutions and teaching prompts for the class review

Specification points covered

- 5.4.4(a) Define gravitational potential at a point as work done per unit mass in bringing a small test mass from infinity to the point.
- 5.4.4(b) Use $V_g = -GM/r$ and relate changes in potential to energy.
- 5.4.4(c) Interpret force-distance graphs for point or spherical masses and relate the area under a curve to work done.
- 5.4.4(d) Define gravitational potential energy $E_g = mV_g = -GMm/r$.
- 5.4.4(e) Define escape velocity and apply it to planetary and atomic-scale examples.

Teaching focus

Keep the distinction clear: gravitational potential is work done per unit mass, whereas gravitational field strength is force per unit mass.
Use signed quantities. A potential can increase while remaining negative; it becomes less negative as the distance from Asteria increases.
For the minimum escape speed, total energy is zero at launch and at infinity. The mass of the escaping object cancels.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

1. Potential and potential energy

1. Define the gravitational potential at a point in a gravitational field. **[2 marks]**

Answer / mark points: The work done per unit mass in bringing a small test mass from infinity to the point, with no change in kinetic energy.
Teacher prompt / common error: Require both ‘work done per unit mass’ and a journey from infinity. Gravitational potential is not force per unit mass; that is field strength.

2. Calculate the gravitational potential V_g at the surface of Asteria. **[2 marks]**

Answer / mark points: $V_g = -GM/r = -(6.67 \times 10^{-11} \times 2.40 \times 10^{17}) / (2.00 \times 10^{3}) = -8.00 \times 10^{3}$ J kg^{-1}.
Teacher prompt / common error: The negative sign is physically meaningful. A common error is to calculate gravitational field strength, which has different units.

3. Calculate the gravitational potential energy E_g of the 18.0 kg sample canister when it is at Asteria’s surface. **[2 marks]**

Answer / mark points: $E_g = mV_g = 18.0 \times (-8.00 \times 10^{3}) = -1.44 \times 10^{5}$ J.
Teacher prompt / common error: Potential energy is negative at a finite distance when zero is defined at infinity. Do not award a positive value.

Continue your physics journey

Videos, practice and support for your next step

www.physicsuk.co.uk

2. Raising the sample canister

4. The canister is raised slowly to a distance of 6.00 km from Asteria's centre. Calculate (a) the gravitational potential at this distance and (b) the change in gravitational potential from the surface to this distance. Include the sign of the change. **[3 marks]**

Answer / mark points: (a) $V_g = -GM/r = -(6.67 \times 10^{-11} \times 2.40 \times 10^{17}) / (6.00 \times 10^{3}) = -2.67 \times 10^{3}$ J kg^{-1}. (b) $\Delta V_g = V_{final} - V_{initial} = (-2.67 \times 10^{3}) - (-8.00 \times 10^{3}) = +5.34 \times 10^{3}$ J kg^{-1}. The potential increases and becomes less negative.

Teacher prompt / common error: Check that pupils subtract signed values correctly. An increase from -8.00×10^{3} to -2.67×10^{3} is positive, not negative.

5. Calculate the minimum energy supplied by the transfer actuator to move the 18.0 kg canister slowly from the surface to a distance of 6.00 km from Asteria's centre. **[2 marks]**

Answer / mark points: $\Delta E_g = m\Delta V_g = 18.0 \times 5.34 \times 10^{3} = 9.60 \times 10^{4}$ J. This is the minimum energy because the canister is transferred slowly, so its kinetic energy does not increase.

Teacher prompt / common error: A slow transfer implies no change in kinetic energy. The actuator's work is the increase in gravitational potential energy.

6. Explain why the gravitational potential at both the surface and 6.00 km from Asteria's centre is negative, even though the potential increases during the transfer. **[2 marks]**

Answer / mark points: The reference potential is defined as zero at infinity. At any finite distance, external work must be done to move a mass to infinity against Asteria's attraction, so the potential is below this zero reference and is negative. Moving outwards increases the potential because it becomes less negative.

Teacher prompt / common error: Use 'increases' and 'becomes less negative'. Avoid saying that a negative quantity 'decreases' when its numerical value rises from −8000 to −2667.

Continue your physics journey

Videos, practice and support for your next step

www.physicsuk.co.uk

3. Work from a force-distance graph

7. Figure 1 shows the force required to hold a 1.00 kg test package stationary at different distances from Asteria's centre. State the force required when r = 4.00 km. [1 mark]

Answer / mark points: 1.00 N.
Teacher prompt / common error: The plotted point at 4.00 km is exactly 1.00 N. The vertical axis is force magnitude, not potential or field strength.

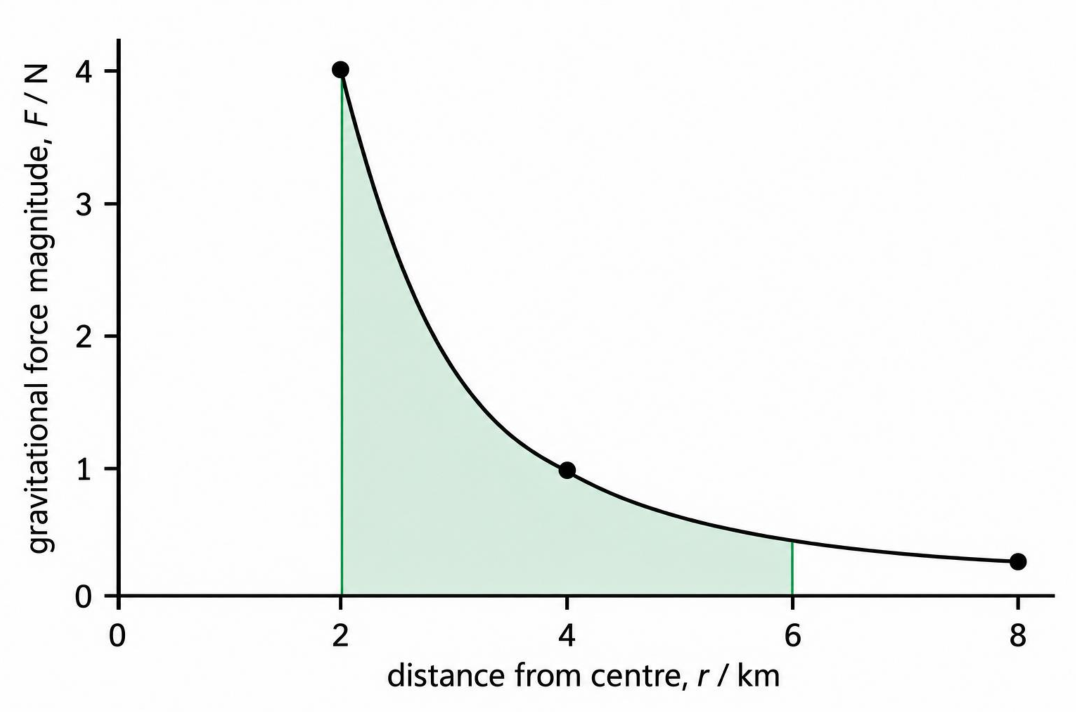

Figure 1 solution: the green area from 2.00 km to 6.00 km is the external work for the slow outward transfer

8. State what the area under the curve in Figure 1 between r = 2.00 km and r = 6.00 km represents. Relate it to the change in gravitational potential energy of the 1.00 kg test package. [2 marks]

Answer / mark points: It is the work done by the external force to move the 1.00 kg package slowly from 2.00 km to 6.00 km. For a slow transfer, this work is equal to the increase in the package's gravitational potential energy. A numerical area found in N km must be multiplied by 1000 to give J.
Teacher prompt / common error: Area under an F–r graph has units of work only after the horizontal distance is expressed in metres. The area represents external work for the stated slow outward movement.

9. Using Figure 1, state which transfer needs more work: from r = 2.00 km to r = 4.00 km, or from r = 4.00 km to r = 6.00 km. Explain your answer using the graph. [2 marks]

Answer / mark points: The transfer from 2.00 km to 4.00 km needs more work. The two intervals have equal width on the horizontal axis, but the curve is higher between 2.00 km and 4.00 km, so the area under the curve is larger.
Teacher prompt / common error: Pupils need not calculate an area. Equal horizontal widths mean the interval with the higher curve has the larger area.

10. The transfer actuator moves the test package outwards at constant speed. State the direction of the actuator force. Explain why the actuator supplies energy even though the package's kinetic energy does not change. [2 marks]

Answer / mark points: The actuator force is outwards, opposite to the inward gravitational force. Its positive work increases the package's gravitational potential energy. The speed is constant, so the kinetic energy does not change; the supplied energy is not used to increase kinetic energy.
Teacher prompt / common error: Constant speed means zero acceleration and no change in kinetic energy; it does not mean that no forces or no energy transfer are present.

Continue your physics journey

Videos, practice and support for your next step

www.physicsuk.co.uk

4. Leaving Asteria

11. Define the escape velocity from the surface of a spherical body. [2 marks]

Answer / mark points: The minimum speed at the surface needed for an object to reach infinity with zero speed, without further propulsion. Accept an equivalent definition that states no subsequent force other than gravity acts.

Teacher prompt / common error: 'Minimum speed' and 'reaches infinity with zero speed' are both essential. The object need not travel at a constant speed.

12. At the minimum escape speed from the surface of a body of mass M and radius R, the total energy of an object is zero. Use $E_k = \frac{1}{2}mv^2$ and $E_g = -GMm/R$ to show that $v_{esc} = \sqrt{(2GM/R)}$. [2 marks]

Answer / mark points: At escape, $E_{total} = 0$: $\frac{1}{2}mv^2 - GMm/R = 0$. Therefore $\frac{1}{2}mv^2 = GMm/R$. Cancelling m and rearranging gives $v^2 = 2GM/R$, so $v_{esc} = \sqrt{(2GM/R)}$.

Teacher prompt / common error: At the minimum escape speed the total energy is zero. The test object mass cancels, so escape velocity does not depend on the canister mass.

13. Calculate the escape velocity from Asteria's surface. [2 marks]

Answer / mark points: $v_{esc} = \sqrt{(2GM/R)} = \sqrt{((2 \times 6.67 \times 10^{-11} \times 2.40 \times 10^{17}) / (2.00 \times 10^3))} = 1.27 \times 10^2$ m s^{-1}, or 127 m s^{-1} to three significant figures.

Teacher prompt / common error: Radius is measured from Asteria's centre and must be in metres. Retain a few guard digits before rounding to 127 m s^{-1}.

14. A launcher releases a canister radially outwards from Asteria's surface at 180 m s^{-1}. Neglecting drag and further propulsion, calculate its speed when it is a very large distance from Asteria. State whether it escapes. [3 marks]

Answer / mark points: At infinity, gravitational potential energy is zero. Conservation of energy gives $\frac{1}{2}mu^2 - GMm/R = \frac{1}{2}mv_\infty^2$. Therefore $v_\infty = \sqrt{(u^2 - v_{esc}^2)} = \sqrt{(180^2 - 126.5^2)} = 128$ m s^{-1}. The canister escapes because its launch speed is greater than the escape velocity.

Teacher prompt / common error: A speed at infinity is real only because the launch speed exceeds the escape speed. If it did not, the expression under the square root would be negative and the craft would return.

5. Atomic-scale escape

15. A small leak releases helium atoms from the return craft. One atom moves directly away from Asteria at 140 m s^{-1}; another moves directly away at 90.0 m s^{-1}. State the likely fate of each atom, neglecting collisions. Explain why a collection of gas atoms with a range of speeds can gradually be depleted from Asteria. [3 marks]

Answer / mark points: The 140 m s^{-1} atom escapes because it is faster than the 127 m s^{-1} escape velocity. The 90.0 m s^{-1} atom remains gravitationally bound if it experiences no collision or other force. A population contains atoms with a range of speeds, so the faster atoms can escape; repeated loss of this fast tail depletes the gas over time.

Teacher prompt / common error: This is an atomic-scale application of escape speed. Faster atoms can be lost, whereas slower atoms remain bound unless later collisions change their speed or direction.

Language and model check

Say that a quantity increases, decreases or remains constant. Here gravitational potential and gravitational potential energy increase during an outward transfer because they become less negative.

Say 'the gravitational force is directed inwards' and 'the actuator force is directed outwards'. Avoid saying that a potential or a velocity is a force.

Escape velocity is a minimum initial speed, not a speed that an object keeps throughout its journey.

Continue your physics journey

Videos, practice and support for your next step

www.physicsuk.co.uk