physics UK
Smarter practice. Stronger understanding

SOLUTIONS

ATTEMPT CHECK CORRECT IMPROVE

OCR A A Level Gravitational Fields 2 solutions

Matching solutions and teaching prompts for the class review

Specification points covered

- 5.4.3(a) State Kepler's three laws of planetary motion.
- 5.4.3(b) Explain that the gravitational force provides the centripetal force for planetary motion.
- 5.4.3(c) Derive and use $T^2 = (4\pi^2/GM)r^3$ for circular orbits.
- 5.4.3(d) Apply Kepler's third law, $T^2 \propto r^3$, to systems beyond the Solar System.
- 5.4.3(e) Define geostationary orbit and describe uses of geostationary satellites.

Teaching focus

Make the model explicit: gravity provides the resultant centripetal force, while velocity is tangent to the orbit.
Use orbital radius from Earth's centre. Altitude is a different distance and must be calculated by subtracting Earth's radius.
A geostationary satellite needs all of its defining conditions, not only a period close to one day.

1. Building the circular-orbit model

1. State Kepler's three laws of planetary motion. **[3 marks]**

Answer / mark points: (1) Planets move in elliptical orbits with the Sun at one focus. (2) The line joining a planet to the Sun sweeps out equal areas in equal times. (3) For bodies orbiting the same central mass, T^2 is proportional to the cube of the semi-major axis; for a circular orbit, $T^2 \propto r^3$.

Teacher prompt / common error: For the third law, accept semi-major axis in the general case and orbital radius for a circular orbit. Avoid accepting a statement that T is directly proportional to r^3.

2. For a satellite of mass m in a circular orbit of radius r around Earth, show that $v = \sqrt{(GM/r)}$. Start by equating gravitational force to centripetal force. **[2 marks]**

Answer / mark points: For a circular orbit, gravitational force provides the centripetal force: $GMm/r^2 = mv^2/r$. Cancelling m and multiplying by r gives $GM/r = v^2$. Therefore $v = \sqrt{(GM/r)}$.

Teacher prompt / common error: The satellite mass cancels. The square root is essential; a common error is $v = GM/r$.

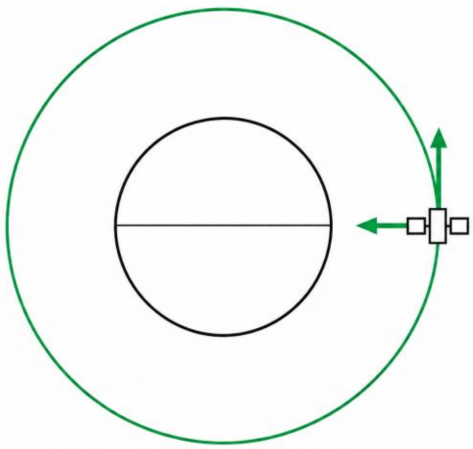

Figure 1 solution: upward is one possible tangential velocity direction; gravity points towards Earth's centre

3. Figure 1 is a section through Earth's equatorial plane. On the satellite, draw (i) one possible instantaneous velocity vector and (ii) the gravitational force vector. State what provides the centripetal force for the circular orbit. **[3 marks]**

Answer / mark points: The velocity vector is tangent to the orbit. In the figure, an upward tangent vector is one valid choice; a downward tangent vector would also be valid for the opposite sense of motion. The gravitational-force vector points radially inwards towards Earth's centre. The gravitational force provides the centripetal force.

Teacher prompt / common error: The velocity is tangent, not towards Earth. The gravitational force is the resultant centripetal force and points towards Earth's centre.

2. The low-orbit prototype

4. The low-orbit StormWatch prototype Aurora is 630 km above Earth's surface. Calculate (a) its orbital speed and (b) its orbital period in minutes. [4 marks]

Answer / mark points: $r = 6.37 \times 10^6 + 0.630 \times 10^6 = 7.00 \times 10^6$ m. $v = \sqrt{GM/r} = \sqrt{(6.67 \times 10^{-11} \times 5.97 \times 10^{24})/(7.00 \times 10^6)} = 7.54 \times 10^3$ m s^{-1}. $T = 2\pi r/v = 2\pi(7.00 \times 10^6)/(7.54 \times 10^3) = 5.83 \times 10^3$ s = 97.2 min.

Teacher prompt / common error: Require conversion of both altitude and radius to metres. The calculated period is a consequence of the speed, not a second independent formula.

5. Explain why Aurora is not suitable for providing an uninterrupted image sequence of the same tropical storm. [2 marks]

Answer / mark points: Aurora completes an orbit in about 97 min and moves relative to Earth's surface, so it passes over or near the storm only intermittently. It cannot remain above one longitude to provide a continuous image sequence.

Teacher prompt / common error: Use precise language: the satellite moves relative to Earth's surface and visits the location intermittently. Do not describe the storm as moving out from under a fixed low-orbit satellite.

3. Applying Kepler's third law

6. A second circular-orbit satellite has an orbital radius of 1.40×10^7 m. Aurora has orbital radius 7.00×10^6 m and a period of 97.2 min. Use Kepler's third law to calculate the period of the second satellite in minutes. [3 marks]

Answer / mark points: $T_2/T_1 = (r_2/r_1)^{3/2} = (1.40 \times 10^7 / 7.00 \times 10^6)^{3/2} = 2^{3/2} = 2.83$. $T_2 = 97.2 \times 2.83 = 275$ min.

Teacher prompt / common error: A doubling of radius increases period by $2^{3/2}$, not by two, four or eight. A ratio method is usually clearest.

7. Horizon has a circular orbit of radius 4.22×10^7 m. Use $T^2 = (4\pi^2/GM)r^3$ to calculate its orbital period in hours. Earth rotates once every 23.93 h. State whether Horizon has the required period for geostationary orbit. [3 marks]

Answer / mark points: $T = \sqrt{4\pi^2 r^3/GM} = \sqrt{(4\pi^2(4.22 \times 10^7)^3)/(6.67 \times 10^{-11} \times 5.97 \times 10^{24})} = 8.63 \times 10^4$ s = 23.98 h. This is approximately Earth's 23.93 h rotation period, so it has the required period.

Teacher prompt / common error: The required period is the sidereal day, close to 23.93 h; do not insist on an exact 24.0 h statement here.

4. Designing a geostationary weather satellite

8. State four conditions that a satellite must satisfy in order to be geostationary. [4 marks]

Answer / mark points: It must orbit Earth in a circular orbit, in the equatorial plane, in the same west-to-east direction as Earth's rotation, and with a period equal to Earth's rotation period. These conditions give the same angular velocity as Earth, so it remains above one longitude.	
Teacher prompt / common error: All four conditions are needed. 'High above Earth' or 'has a 24 hour period' alone is insufficient.	

9. Calculate Horizon's altitude above Earth's surface in kilometres. [2 marks]

Answer / mark points: $h = r - R_e = 4.22 \times 10^7 - 6.37 \times 10^6 = 3.583 \times 10^7$ m $= 3.583 \times 10^4$ km, or 35 800 km to three significant figures.	
Teacher prompt / common error: The orbital radius is measured from Earth's centre. Altitude requires subtracting Earth's radius.	

5. Beyond Earth's weather network

10. A data-analysis algorithm is tested using two planets orbiting the distant star Nysa. Planet P has a period of 48.0 Earth days. Planet Q has an orbital radius 2.25 times that of P. Calculate the period of Q in Earth days. State the assumption that permits this ratio calculation. [4 marks]

Answer / mark points: For the same central star, $T \propto r^{3/2}$. The period of Q $= 48.0 \times 2.25^{3/2} = 48.0 \times 3.375 = 162$ Earth days. The two planets must be orbiting the same central mass; the Kepler ratio also assumes the stated orbital radii are comparable semi-major axes.	
Teacher prompt / common error: The proportionality is valid for objects orbiting the same central mass. This is an explicit beyond-the-Solar-System application, not a new law.	

Table 1: candidate StormWatch satellite designs

Satellite	Orbit / plane	Direction	Period
Aurora	7.00×10^{6} m / polar	varies with polar path	97.2 min
Horizon	4.22×10^{7} m / equatorial	west to east	23.98 h
Drift	4.22×10^{7} m / equatorial	east to west	23.98 h

11. The StormWatch team needs continuous images of cyclones near 20° W longitude. Use Table 1 to select the only suitable satellite. Justify your choice using the orbital data and explain why each rejected option fails at least one geostationary condition. [4 marks]

Answer / mark points: Choose Horizon. Its 23.98 h period is approximately Earth's rotation period, it is equatorial and travels west to east, so it remains above a fixed longitude and can observe the target continuously. Aurora is polar and has a 97.2 min period, so it passes the target intermittently. Drift has the correct radius, plane and near-day period but travels east to west, opposite to Earth's rotation, so it is not stationary over one longitude.

Teacher prompt / common error: Horizon is the only valid recommendation. Drift's direction is deliberately wrong even though its period and plane look plausible.

12. State one use of a geostationary satellite other than weather observation. Explain why placing several geostationary weather satellites at different longitudes improves coverage, but does not give equally good coverage at the poles. [3 marks]

Answer / mark points: Accept communication, television broadcasting or relay as another use. Satellites at different longitudes observe different fixed regions, giving a more complete sequence of weather images. Geostationary satellites are above the equator, so their line of sight to high latitudes is low and the range is larger; polar coverage is consequently poorer.

Teacher prompt / common error: A global network improves longitudinal coverage but does not remove the poor geometry at the poles. Keep the distinction between longitude and latitude clear.

Language and model check

Say that gravitational field strength or orbital speed changes with radius. A velocity is not 'centripetal'; a force or acceleration is directed towards the centre.

Say 'a satellite remains above one longitude' for a geostationary orbit. It still moves at about 3.1 km s^{-1} relative to Earth's centre.