Electric potential and energy solutions

Electric fields 5 A charged spacecraft probe and dust

Matching answers and examiner-style commentary

OCR A H556 | A-level physics | Module 6 | 42 marks

Specification points covered

6.2.4(a–e) Electric potential, energy, force–distance work and isolated-sphere capacitance.

Prior learning applied: radial fields, kinetic energy, conservation of energy and C = Q/V.

Use $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1} and $k = 8.99 \times 10^{9}$ N m^2 C^{-2}. Take V = 0 at infinity.

1 Choose the reference

A spherical spacecraft probe of radius R = 0.500 m carries a fixed uniformly distributed charge Q = +20.0 nC. Model it as isolated, with its source charge fixed.

Study a tiny dust grain relative to the freely falling probe. Neglect external gravitational gradients, drag, magnetic effects and changes to the source caused by the grain.

Position A is r_A = 2.00 m from O. Position B is at the surface. Distances in radial-field equations are measured from O, not from the surface.

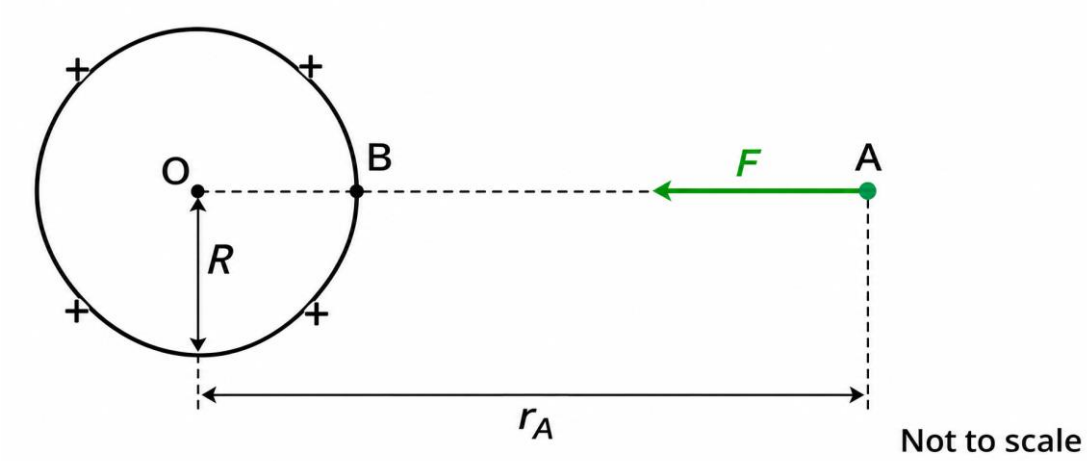


Figure 1 Ideal isolated probe and radial positions Negative-grain force shown

1(a) Define electric potential at a point. Be clear about the charge and the reference position used. **[2 marks]**

Electric potential is the work done per unit positive charge in bringing a small positive charge from infinity to the point, with no change in kinetic energy. This is external work against the electric force; the reference is V = 0 at infinity.

Marking guidance 1: work per unit positive charge. 1: from infinity to the point.

1(b) State whether electric potential V, electric field strength E and electric potential energy U are scalar or vector quantities. **[1 mark]**

V and U are scalars. E is a vector: it has direction as well as magnitude.

Marking guidance 1: all three classifications correct.

2 Calculate the probe potential

2(a) Calculate the potential at the probe surface. **[1 mark]**

The surface is at r = R = 0.500 m.

$V_B = kQ/R = (8.99 \times 10^9)(20.0 \times 10^{-9})/0.500 = +359.6$ V ≈ +360 V.

Marking guidance 1: surface potential about +360 V.

2(b) A sensor 1.50 m from the probe surface reads +90.0 V. Use this reading and the 0.500 m radius to estimate Q independently. **[2 marks]**

Step 1 Convert the surface gap to the centre distance.

r = 1.50 + 0.500 = 2.00 m.

Step 2 Rearrange V = kQ/r.

$Q = Vr/k = (90.0)(2.00)/(8.99 \times 10^9) = 2.002 \times 10^{-8}$ C ≈ +20.0 nC.

Marking guidance 1: correct centre distance. 1: charge estimate about +20.0 nC.

2(c) Using the stated Q = +20.0 nC, calculate the change in potential from A to B. Explain why this potential change does not depend on the dust grain's charge. **[2 marks]**

$V_A = kQ/2.00 = +89.9$ V; $V_B = +359.6$ V.

$\Delta V = V_B - V_A = 359.6 - 89.9 = +269.7$ V.

The potential is determined by the prescribed source Q and position, not by the small test grain. The grain's charge will affect its energy change $q\Delta V$.

Marking guidance 1: +269.7 V, with final minus initial. 1: source/test-charge distinction.

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3 Compare radial relationships

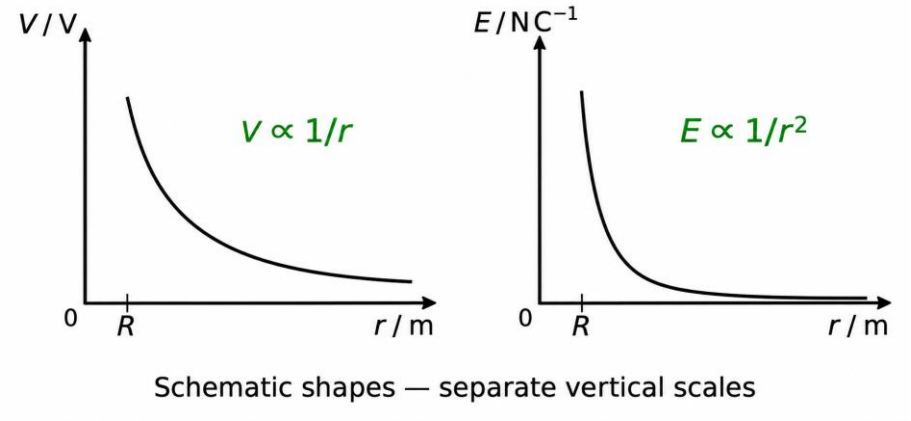
3(a) The centre distance is doubled from any point outside the probe. State the factors by which the potential magnitude and field magnitude change. **[1 mark]**

$V \propto 1/r$, so $|V|$ halves. $E \propto 1/r^2$, so $|E|$ becomes one quarter.

Marking guidance 1: both factors correct.

3(b) Sketch V against r and, on separate axes, E against r for the positive probe outside its surface. Label the axes and show how each approaches zero. The sketches need not be to scale. **[2 marks]**

Both graphs are positive and decrease non-linearly towards zero as r increases. Draw only $r \geq R$. The potential follows $1/r$ and the field follows $1/r^2$; neither reaches zero at a finite r. Use separate vertical axes because V and E have different units.



Schematic shapes — separate vertical scales

Model sketch Numerical calculations use the stated values and data tables

Marking guidance 1: appropriate positive inverse-r potential sketch with units. 1: appropriate inverse-square field sketch with units.

3(c) If the probe carried −20.0 nC instead, describe how the V–r graph and the field direction would change. **[2 marks]**

The potential graph is reflected below the horizontal axis: V is negative and approaches zero from below as r increases. The field points radially inward rather than outward.

Marking guidance 1: negative potential tending to zero from below. 1: inward field.

3(d) A pupil uses the electric force at the surface multiplied by the entire distance from A to B to calculate work. Explain why that method is invalid. **[1 mark]**

The force changes with distance as $1/r^2$. Its surface value is not constant over the path, so multiplying that single force by the full distance does not give the work. Use potential-energy differences or the force–distance area.

Marking guidance 1: identifies the changing force and why the constant-force calculation fails.

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4 Track the dust grain’s energy

4(a) A grain has $q = -5.00 \times 10^{-15}$ C and mass 2.00×10^{-12} kg. Calculate its potential energy at A and B, the change from A to B and the work done by the electric field. **[3 marks]**

Step 1 Use U = qV and retain the grain’s negative charge.

$U_A = (-5.00 \times 10^{-15})(89.9) = -4.495 \times 10^{-13}$ J.

$U_B = (-5.00 \times 10^{-15})(359.6) = -1.798 \times 10^{-12}$ J.

Step 2 Form final minus initial.

$\Delta U = U_B - U_A = -1.3485 \times 10^{-12}$ J.

Step 3 Work by the electric field is the decrease in potential energy.

$W_{field} = -\Delta U = +1.3485 \times 10^{-12}$ J. The attractive force acts in the same direction as the inward motion.

Marking guidance 1: both potential energies. 1: correct signed change. 1: positive field work with correct magnitude.

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4 Track the dust grain's energy continued

4(b) If the negative grain starts from rest at A, calculate its speed immediately before reaching B.
[2 marks]

Step 1 With only electric work changing kinetic energy, $K_B - K_A = -\Delta U$. Here $K_A = 0$.

$\frac{1}{2}mv_B^2 = 1.3485 \times 10^{-12}$ J.

Step 2 Rearrange for speed and substitute.

$v_B = \sqrt{[2(1.3485 \times 10^{-12})/(2.00 \times 10^{-12})]} = 1.161$ m s^{-1} ≈ 1.16 m s^{-1}.

Marking guidance 1: correct energy equation. 1: final speed about 1.16 m s^{-1}.

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4 Initial speed and closest approach

4(c) Instead, the same negative grain is already moving inward at 0.800 m s^{-1} at A. Find its speed at B. **[2 marks]**

The initial kinetic energy must be included; the electrical gain is unchanged.

$\frac{1}{2}mv_B^2 = \frac{1}{2}mu^2 + 1.3485 \times 10^{-12}$.

$v_B^2 = (0.800)^2 + 2(1.3485 \times 10^{-12})/(2.00 \times 10^{-12}) = 1.9885\ m^2\ s^{-2}$.

$v_B = 1.410\ m\ s^{-1} \approx 1.41\ m\ s^{-1}$. Do not add the separate speed gains.

Marking guidance 1: includes initial kinetic energy correctly. 1: about 1.41 m s^{-1}.

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4 Initial speed and closest approach continued

4(d) A positive grain has the same mass and |q| and starts at A moving inward at 0.800 m s^{-1}. Determine its closest centre distance and decide whether it reaches the surface. **[3 marks]**

Step 1 A positive grain is repelled. Its kinetic energy falls to zero at the turning point.

$K_A = \frac{1}{2}(2.00 \times 10^{-12})(0.800)^2 = 6.40 \times 10^{-13}$ J.

$U_A = kQq/r_A = +4.495 \times 10^{-13}$ J.

Step 2 Conserve total energy between A and the turning point.

$kQq/r_{min} = K_A + U_A = 1.0895 \times 10^{-12}$ J.

$r_{min} = kQq/(K_A + U_A) = 8.99 \times 10^{-13}/(1.0895 \times 10^{-12}) = 0.825$ m.

Step 3 Compare with the probe radius.

0.825 m > 0.500 m, so it turns before reaching the surface.

Marking guidance 1: turning-point/conservation equation. 1: closest distance about 0.825 m. 1: comparison with R and correct conclusion.

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5 Connect force and work

Use $q = -5.00 \times 10^{-15}$ C and the original source. The table values are rounded. Numerical work must use the table, not a measurement from a sketch.

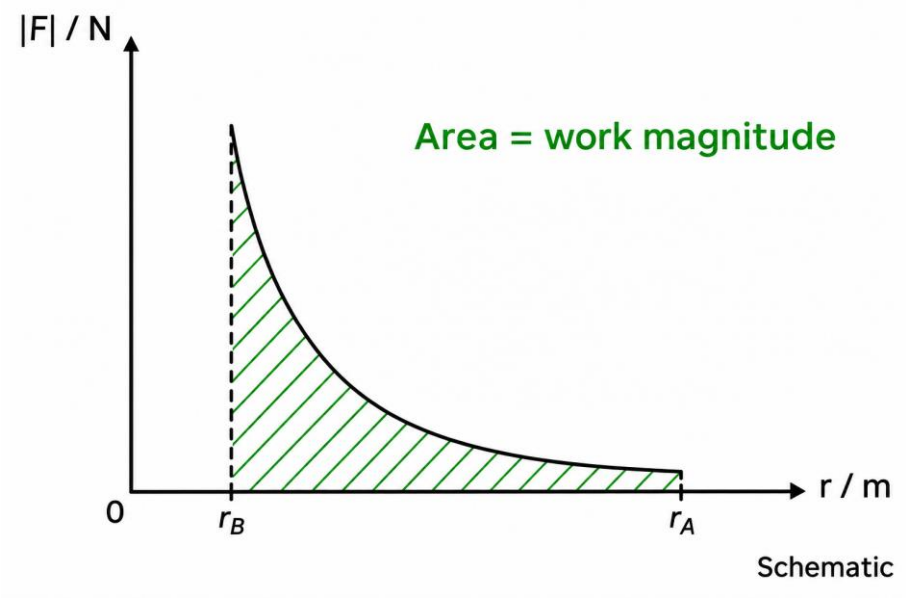
Table 1

r / m	$\lvert F \rvert / 10^{-12}$ N
0.500	3.60
0.750	1.60
1.00	0.899
1.50	0.400
2.00	0.225

5(a) Table 1 gives the force magnitude on the negative grain. Sketch |F| against r, include units and shade the area between r = 0.500 m and r = 2.00 m. State what the area represents. **[2 marks]**

Draw a positive decreasing inverse-square curve over the stated radial interval. Shade the region between the curve and the r axis from 0.500 m to 2.00 m.

Because the graph uses force magnitude, the area gives the magnitude of the electric work for this radial path, not its sign by itself.



Model sketch Numerical calculations use the stated values and data tables

Marking guidance 1: suitable labelled curve and interval shaded. 1: work magnitude, rather than an automatic sign.

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5 Estimate and interpret the work

Use the rounded force readings in Table 1 in Question 5.

5(b) Estimate that area using a trapezium for each adjacent pair of readings in Table 1. Show your calculation in joules. **[2 marks]**

For each interval use $\frac{1}{2}(F_{left} + F_{right})\Delta r$, remembering the 10^{-12} N scale.

Area = $[\frac{1}{2}(3.60 + 1.60)(0.250) + \frac{1}{2}(1.60 + 0.899)(0.250) + \frac{1}{2}(0.899 + 0.400)(0.500) + \frac{1}{2}(0.400 + 0.225)(0.500)] \times 10^{-12}$.

Area = $(0.650 + 0.312375 + 0.324750 + 0.156250) \times 10^{-12} = 1.4434 \times 10^{-12}$ J $\approx 1.44 \times 10^{-12}$ J.

Marking guidance 1: correct sum of trapezia including the force scale. 1: about 1.44×10^{-12} J.

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5 Estimate and interpret the work continued

5(c) For the negative grain travelling from A to B, state the sign of work by the field. Compare your area estimate with the exact potential-energy result and explain a likely source of the difference.
[2 marks]

The attractive force and inward displacement have the same direction, so field work is positive.

The trapezium estimate, 1.44×10^{-12} J, is about 7% above the potential-energy result 1.3485×10^{-12} J. Straight chords lie above this curved decreasing $1/r^2$ graph, so the coarse trapezium sum overestimates the area. Rounding the table contributes a much smaller difference.

Marking guidance 1: positive work linked to force/displacement. 1: meaningful comparison and numerical-approximation explanation.

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6 Derive isolated-sphere capacitance

6(a) Starting with the surface-potential equation and C = Q/V, derive $C = 4\pi\varepsilon_0 R$ for an isolated conducting sphere. **[2 marks]**

Step 1 The sphere's surface is an equipotential at $V = Q/(4\pi\varepsilon_0 R)$.

Step 2 Substitute that potential into the capacitance definition.

$C = Q/V = Q/[Q/(4\pi\varepsilon_0 R)] = 4\pi\varepsilon_0 R$.

Q cancels, so the capacitance of this isolated geometry does not depend on how much charge is already stored.

Marking guidance 1: uses both appropriate starting equations. 1: clear substitution/cancellation to the required result.

6(b) Calculate the capacitance of the original 0.500 m radius probe. **[1 mark]**

$C = 4\pi\varepsilon_0 R = 4\pi(8.85 \times 10^{-12})(0.500) = 5.5606 \times 10^{-11}$ F = 55.6 pF.

Marking guidance 1: about 55.6 pF.

6(c) At the same radius, the probe's charge is doubled. State and explain the changes in its capacitance and surface potential. **[2 marks]**

Capacitance stays the same because $C = 4\pi\varepsilon_0 R$ depends on radius and the surrounding medium/model. The surface potential doubles because V = Q/C and C is unchanged.

Marking guidance 1: unchanged C with correct reason. 1: doubled surface V with correct reason.

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7 Advise the probe designer

7(a) A design must hold +24.0 nC while keeping surface potential at or below 450 V. Compare isolated spheres of radius 0.250 m and 0.500 m. Choose a suitable design using calculated potentials. **[3 marks]**

Step 1 Apply V = kQ/R to both candidate radii.

For 0.250 m: $V = (8.99 \times 10^{9})(24.0 \times 10^{-9})/0.250 = 863$ V.

For 0.500 m: $V = (8.99 \times 10^{9})(24.0 \times 10^{-9})/0.500 = 432$ V.

Step 2 Compare with the same limit.

Choose the 0.500 m sphere: about 432 V is below 450 V, whereas the smaller sphere exceeds the limit.

Marking guidance 1: potential for each candidate (two marks). 1: justified choice against 450 V.

7(b) Evaluate this response: 'The negative grain has negative potential energy, so its kinetic energy must also be negative and a real speed cannot be calculated.' **[2 marks]**

Potential energy can be negative because its zero was chosen at infinity. This does not require kinetic energy to be negative.

For the approaching negative grain, ΔU is negative, so $\Delta K = -\Delta U$ is positive. Starting from rest, it gains the positive kinetic energy 1.3485×10^{-12} J and has a real speed.

Marking guidance 1: explains the reference-dependent sign of U. 1: correctly links negative ΔU to positive ΔK.

7(c) In the actual spacecraft, a large conducting body is placed close to the probe. Explain why the isolated-sphere capacitance calculation may then be inaccurate. **[2 marks]**

The nearby conductor changes the electric field and the charge distribution/potential of the probe for a given total charge. The probe is no longer an isolated spherical source, so the relationship between Q and surface V need not be $C = 4\pi\varepsilon_0 R$.

Marking guidance 1: identifies changed field/charge distribution or potential. 1: links this to failure of the isolated-sphere model.

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