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SOLUTIONS

ATTEMPT | CHECK | CORRECT | IMPROVE

OCR A A Level Astrophysics 3 cosmology and the expanding Universe solutions

Matching solutions and teaching prompts for the class review

Specification points covered

- 5.5.3(a) Define the astronomical unit, light-year and parsec.
- 5.5.3(b) Describe stellar parallax and use $d = 1 / p$, where p is in arcseconds and d is in parsecs.
- 5.5.3(c) State the Cosmological Principle: on a large scale the Universe is homogeneous and isotropic, and the laws of physics are universal.
- 5.5.3(d) Describe the Doppler effect and the shift of electromagnetic radiation from moving astronomical sources.
- 5.5.3(e) Use $\Delta\lambda / \lambda = v / c$ for sources moving at speeds much less than the speed of light.
- 5.5.3(f) State and use Hubble's law $v = H_{0d}$.
- 5.5.3(g) Explain how cosmological redshift provides evidence for an expanding Universe.
- 5.5.3(h) Use the units km s^{-1} Mpc^{-1} and s^{-1} for the Hubble constant.
- 5.5.3(i) Describe the Big Bang model and supporting observations, including cosmic microwave background radiation at about 2.7 K.
- 5.5.3(j) Describe the expansion of the Universe as expansion of spacetime itself.
- 5.5.3(k) Estimate the age of the Universe using $t \approx 1 / H_0$.
- 5.5.3(l) Describe the evolution of the Universe from the Big Bang to the present.

Teaching focus

Treat the distance ladder as a chain of models: parallax for nearby stars, redshift for recession speed, and Hubble's law for large-scale distance.
One redshift gives one fractional wavelength change. The absolute change in wavelength is therefore greater for a longer-wavelength line.
Separate local gravitational motion from the large-scale expansion of spacetime, and separate the estimate $1 / H_0$ from an exact measured age.

1. Cosmic distance and stellar parallax

1. The survey uses the astronomical unit (AU), light-year (ly) and parsec (pc). State what each unit represents. **[3 marks]**

Answer / mark points: One AU is the mean distance between Earth and the Sun. One light-year is the distance travelled by light in vacuum in one year. One parsec is the distance at which an object has a parallax angle of one arcsecond.

Teacher prompt / common error: Keep the units conceptually distinct: AU is a Solar-System scale, light-year is a distance travelled by light, and parsec is defined through parallax.

2. A nearby calibration star is photographed six months apart. Its total angular displacement against distant stars is 0.000120°. Explain why the parallax angle p is half this displacement. Convert p to arcseconds, then calculate the distance to the star in parsecs and light-years. Use 1° = 3600 arcseconds and 1 pc = 3.26 ly. **[4 marks]**

Answer / mark points: The two observations show the two opposite extremes of the apparent motion, so the full displacement is $2p$. Total displacement $= 0.000120 \times 3600 = 0.432$ arcseconds, hence $p = 0.216$ arcseconds. $d = 1 / p = 1 / 0.216 = 4.63$ pc. In light-years, $d = 4.63 \times 3.26 = 15.1$ ly.

Teacher prompt / common error: The measured six-month displacement is twice the parallax angle. Require degree-to-arcsecond conversion before using $d = 1 / p$.

3. Explain why stellar parallax becomes unsuitable for very distant stars. State one practical improvement that allows a smaller parallax angle to be measured more reliably. **[2 marks]**

Answer / mark points: As distance increases, the apparent angular displacement becomes too small compared with the angular resolution or measurement uncertainty. A telescope with better angular resolution, repeated measurements and averaging, or a space telescope that avoids atmospheric distortion can improve the measurement.

Teacher prompt / common error: Do not accept ‘the star is too faint’ as the fundamental parallax limitation. The issue is that the angle becomes too small to resolve reliably.

2. Reading a cosmological redshift

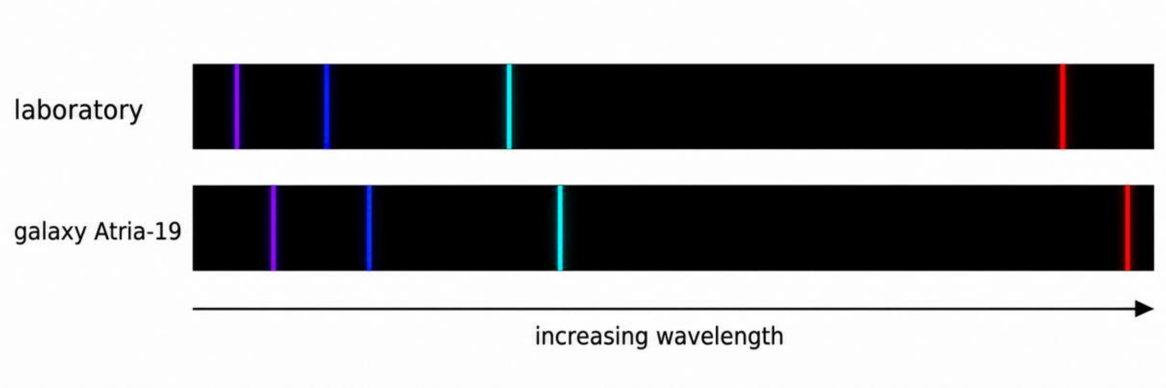

Figure 1 solution: the four lines share one fractional redshift

4. Figure 1 compares laboratory hydrogen lines with light from galaxy Atria-19. State whether the galaxy spectrum is redshifted or blueshifted and hence state whether Atria-19 is approaching or receding from Earth. Use the Doppler effect to explain the change in observed wavelength. [3 marks]

Answer / mark points: Every identified line is displaced towards longer wavelength, so the spectrum is redshifted and Atria-19 is receding from Earth. Successive wavefronts from a receding source reach the observer with a greater separation, so the observed wavelength is longer and the observed frequency is lower.

Teacher prompt / common error: Use ‘observed wavelength is longer’ and ‘the galaxy is receding’. A wavelength does not move, and the galaxy is not ‘redder because it is hotter’. Accept a wavefront-spacing or lower-observed-frequency explanation of the Doppler effect.

5. The laboratory Hα line has wavelength 656.3 nm. In Atria-19 it is observed at 679.9 nm. Calculate the wavelength change Δλ and the fractional change $z = \Delta\lambda / \lambda_{rest}$. [2 marks]

Answer / mark points: $\Delta\lambda = 679.9 - 656.3 = 23.6$ nm. $z = \Delta\lambda / \lambda_{rest} = 23.6 / 656.3 = 0.03596 \approx 0.0360$.

Teacher prompt / common error: Use the laboratory wavelength in the denominator. Redshift z is dimensionless.

6. For the same galaxy, the laboratory Hδ line at 410.2 nm is observed at 425.0 nm. Show that its fractional change is consistent with your answer to Question 5. Explain why the two lines do not have the same change in wavelength measured in nanometres. [2 marks]

Answer / mark points: For Hδ, $\Delta\lambda = 425.0 - 410.2 = 14.8$ nm and $\Delta\lambda / \lambda_{rest} = 14.8 / 410.2 = 0.03608$, consistent with 0.03596. One redshift multiplies all rest wavelengths by the same factor $(1 + z)$, so equal fractional changes produce larger absolute changes for longer wavelengths.

Teacher prompt / common error: One source has one fractional wavelength change. Longer rest wavelengths therefore have larger changes in nanometres; the shifts are not equal absolute additions.

3. From redshift to distance

7. Use the Hα data and $\Delta\lambda / \lambda_{rest} = v / c$ to calculate the recession speed of Atria-19 in km s^{-1}. Use $c = 3.00 \times 10^5$ km s^{-1}. **[2 marks]**

Answer / mark points: $v = c\Delta\lambda / \lambda_{rest} = 3.00 \times 10^5 \times (23.6 / 656.3) = 1.08 \times 10^4$ km s^{-1}.
Teacher prompt / common error: This non-relativistic relation is valid here because the speed is much less than c. Keep c and the answer in km s^{-1}.

8. The best-fit line on the survey's recession-speed–distance graph passes through the origin and the point d = 240 Mpc, v = 16 800 km s^{-1}. Calculate the Hubble constant H_0 and give its usual astronomical unit. **[2 marks]**

Answer / mark points: H_0 is the gradient of the graph: $H_0 = v / d = 16\,800 / 240 = 70.0$ km s^{-1} Mpc^{-1}.
Teacher prompt / common error: H_0 is the gradient of recession speed against distance. Include km s^{-1} Mpc^{-1}, not km s^{-1} alone.

9. Use your answers to Questions 7 and 8 to estimate the distance to Atria-19 in megaparsecs. **[2 marks]**

Answer / mark points: $d = v / H_0 = (1.08 \times 10^4) / 70.0 = 154$ Mpc.
Teacher prompt / common error: Hubble distance is an estimate based on the large-scale relationship. Retain enough significant figures from the recession speed.

4. The Hubble constant, age and uncertainty

10. Convert H_0 = 70.0 km s^{-1} Mpc^{-1} to s^{-1}. Hence estimate the age of the Universe in seconds and in years using t ≈ 1 / H_0. Use 1 Mpc = 3.09 × 10^{22} m and 1 year = 3.16 × 10^7 s. [4 marks]

Answer / mark points: H_0 = 70.0 × 10^3 / (3.09 × 10^{22}) = 2.27 × 10^{-18} s^{-1}. Therefore t ≈ 1 / H_0 = 4.41 × 10^{17} s. In years, t = (4.41 × 10^{17}) / (3.16 × 10^7) = 1.40 × 10^{10} years, about 14.0 billion years.

Teacher prompt / common error: Convert kilometres to metres before dividing by one megaparsec. Once H_0 is in s^{-1}, its reciprocal has units of seconds.

11. The value 1 / H_0 is an estimate rather than an exact age. State the main assumption made about the expansion and explain why the assumption may not be valid throughout the history of the Universe. [2 marks]

Answer / mark points: The calculation assumes that the present expansion rate has remained constant. The expansion rate has changed because the contents of the Universe and effects such as gravity and dark energy influence its expansion, so 1 / H_0 is only an estimate.

Teacher prompt / common error: The common oversimplification is that 1 / H_0 must equal the exact age. The expansion history has not been uniform.

12. The observed Hα wavelength is 679.9 ± 0.8 nm; uncertainty in the laboratory wavelength is negligible. Calculate the percentage uncertainty in the recession speed and hence the absolute uncertainty in the distance found in Question 9. [3 marks]

Answer / mark points: The wavelength change is 23.6 nm and its absolute uncertainty is 0.8 nm. Percentage uncertainty = (0.8 / 23.6) × 100 = 3.39%, approximately 3.4%. Since v and d are proportional to Δλ here, the distance has the same percentage uncertainty: absolute uncertainty = 0.0339 × 154 = 5.2 Mpc, so d ≈ 154 ± 5 Mpc.

Teacher prompt / common error: The laboratory wavelength uncertainty is stated negligible, so the uncertainty in Δλ is 0.8 nm. The same fractional uncertainty transfers to v and d.

5. Interpreting the expanding Universe

13. A nearby galaxy in the Lumen Group has a small blueshift even though most distant galaxies are redshifted. Explain why this observation does not contradict Hubble's law. [2 marks]

Answer / mark points: A nearby galaxy can have a local or peculiar velocity caused by gravitational attraction within its galaxy group. At small distance this local motion can be larger than the Hubble recession speed. Hubble's law describes the large-scale trend, not every nearby galaxy individually.

Teacher prompt / common error: Use ‘peculiar velocity’ or explain local gravitational motion. A nearby exception does not remove the large-scale correlation.

14. On scales greater than about 500 Mpc, the survey finds approximately the same mean galaxy density in different regions and approximately the same appearance in every direction. Define homogeneous and isotropic. Explain why the existence of local clusters and voids does not disprove the Cosmological Principle. [3 marks]

Answer / mark points: Homogeneous means the mean matter distribution or density is the same from place to place on a sufficiently large scale. Isotropic means the Universe looks the same in every direction on that scale. Clusters and voids are local structure; averaging over a much larger volume can still give homogeneity and isotropy. The Cosmological Principle also assumes that the laws of physics are the same everywhere.

Teacher prompt / common error: Homogeneous concerns place; isotropic concerns direction. Both claims apply only after averaging over a sufficiently large scale.

15. A student says: ‘Every distant galaxy is moving through space away from Earth, so Earth must be at the centre of the Big Bang.’ Correct this description using the expansion of spacetime. [3 marks]

Answer / mark points: The model describes space itself expanding, increasing the separation between widely separated, non-bound galaxies. An observer in any galaxy sees other distant galaxies receding, so the observation does not select Earth as a centre. The Big Bang was not an explosion from one point into pre-existing space.

Teacher prompt / common error: Correct the central-explosion picture. Distances grow because spacetime expands, and every comoving observer sees the same large-scale pattern.

6. Evidence and the early Universe

16. Explain how cosmological redshift and cosmic microwave background radiation support the Big Bang model. Include the significance of the present CMB temperature of about 2.7 K. **[3 marks]**

Answer / mark points: The systematic redshift of distant galaxies, together with the Hubble relationship, shows that the Universe is expanding; reversing the expansion implies a hotter, denser earlier state. The CMB is relic electromagnetic radiation released when atoms formed and the Universe became transparent. Expansion has stretched and cooled this radiation to a near-uniform microwave background at about 2.7 K, as predicted by the Big Bang model.

Teacher prompt / common error: Link each observation to the model: redshift to expansion, and the near-uniform 2.7 K CMB to cooled relic radiation from a hot early Universe.

17. Write a coherent account of the evolution of the Universe from the Big Bang to the present. Your answer should link expansion to falling temperature and should place these stages in a physically sensible order: quarks and leptons; hadrons; nuclei; atoms and released background radiation; stars and galaxies. **[6 marks]**

Answer / mark points: The Universe began in an extremely hot, dense state and expanded. As it expanded it cooled, so the typical particle energy fell. Quarks and leptons existed first; quarks then combined to form hadrons such as protons and neutrons. Light nuclei formed when it was cool enough for them to remain stable. Later, electrons combined with nuclei to form neutral atoms. Photons could then travel freely, producing the radiation now observed as the redshifted 2.7 K CMB. Gravity gathered matter into stars and galaxies. Expansion has continued to the present Universe.

Teacher prompt / common error: Award for a linked chronological account, not a memorised list. Look for hot and dense; expansion and cooling; quarks/leptons; hadrons; nuclei; atoms and photon decoupling/CMB; stars and galaxies; continued expansion.

Language and model check

A spectral line has a longer or shorter wavelength; it does not move faster along the spectrum.
A galaxy approaches or recedes. A velocity is not ‘redshifted’ and a wavelength does not ‘accelerate’.
Homogeneous means the same from place to place; isotropic means the same in every direction. Both apply after averaging on a large scale.
The Universe expands because spacetime expands. The Big Bang was not an explosion away from Earth through pre-existing empty space.