OCR A A Level Circular Motion solutions

Matching solutions and teaching prompts for the class review

Specification points covered

- 5.2.1(a), 5.2.1(b) and 5.2.1(c) Use radians, period, frequency and angular velocity, with $\omega = 2\pi/T = 2\pi f$.
- 5.2.2(a) Explain that a constant net force perpendicular to velocity causes circular motion.
- 5.2.2(b) and 5.2.2(c) Use $v = r\omega$ and centripetal acceleration $a = v^2/r = r\omega^2$.
- 5.2.2(d)(i) and 5.2.2(d)(ii) Use $F = mv^2/r = mr\omega^2$ and investigate circular motion using a whirling bung.

Teaching focus

Insist on vector language: speed may stay constant while velocity changes because its direction changes.
Centripetal force is the inward resultant of real forces, never an additional force drawn beside them.

1. Circular-motion kinematics

1. State what is meant by an angle of one radian. [1 mark]

Answer / mark points: One radian is the angle subtended at the centre of a circle by an arc whose length is equal to the radius.

Teacher prompt / common error: Require the arc length to equal the radius and the angle to be at the centre. ‘About 57 degrees’ is not a definition.

2. A calibration arm of radius 0.280 m turns through 135°. Convert the angle to radians and calculate the arc length travelled by its tip. [2 marks]

Answer / mark points: $\theta = 135 \times \pi/180 = 2.36$ rad. Arc length $s = r\theta = 0.280 \times 2.356 = 0.660$ m.

Teacher prompt / common error: Convert degrees to radians before using $s = r\theta$. The radian is dimensionless, while arc length is in metres.

3. An ocean-current sensor rotor completes 18 revolutions in 7.50 s. Calculate its frequency, period and angular velocity. [3 marks]

Answer / mark points: $f = 18/7.50 = 2.40$ Hz. $T = 1/f = 0.417$ s. $\omega = 2\pi f = 15.1$ rad s^{-1}.

Teacher prompt / common error: Keep frequency and period reciprocal. Angular velocity is in rad s^{-1}, not revolutions per second.

4. A blade tip on the rotor is 0.420 m from the axis. Calculate its linear speed when the rotor turns at 2.40 Hz. [1 mark]

Answer / mark points: $v = r\omega = 2\pi rf = 2\pi \times 0.420 \times 2.40 = 6.33$ m s^{-1}.

Teacher prompt / common error: Either $v = r\omega$ after finding ω or $v = 2\pi rf$ is valid. Do not use $v = rf$ without 2π.

2. Direction, acceleration and resultant force

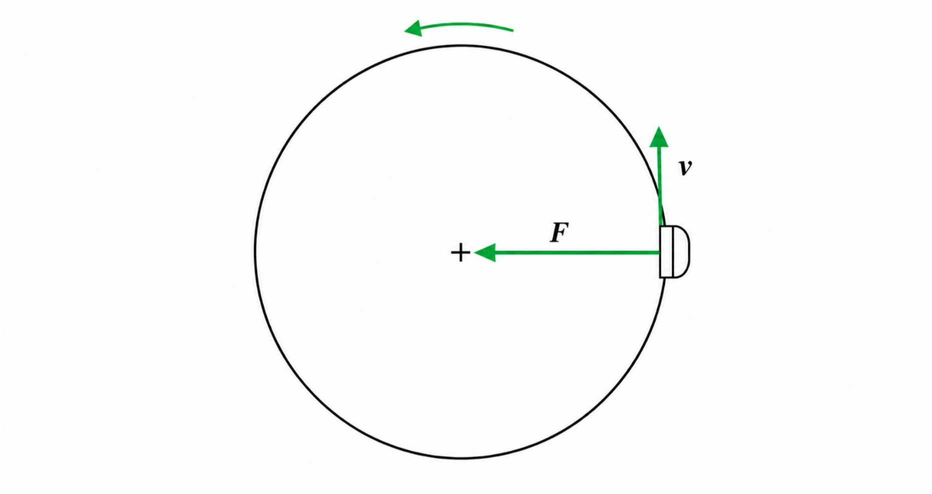

Figure 1 solution: tangential velocity and inward resultant force

5. Figure 1 shows a sample capsule moving anticlockwise at constant speed. On Figure 1, draw and label an arrow for its velocity v and an arrow for the resultant force F. Explain why the capsule accelerates even though its speed is constant. [4 marks]

Answer / mark points: At the right-hand point, v is tangential and vertically upward; F is radial and points left towards the centre. The speed stays constant, but the direction of velocity changes continuously. A changing velocity means acceleration, directed towards the centre.
Teacher prompt / common error: Velocity is tangent; acceleration and resultant force are inward. Reject any outward ‘centrifugal force’ on the capsule in an inertial-frame force diagram.

6. An asteroid-sample centrifuge moves a 0.180 kg capsule at 14.0 m s^{-1} in a circle of radius 0.650 m. Calculate its centripetal acceleration and the resultant force. State the factor by which the force changes if the speed increases by 20%. [3 marks]

Answer / mark points: $a = v^2/r = 14.0^2/0.650 = 302$ m s^{-2}. $F = ma = 0.180 \times 301.5 = 54.3$ N. Since $F \propto v^2$, a 20% speed increase multiplies the force by $1.20^2 = 1.44$.
Teacher prompt / common error: The 20% change applies before squaring: $1.20^2 = 1.44$, not 1.20 or 1.40.

3. Forces that provide the inward resultant

7. A 5.00 kg inspection drone flies in a horizontal circle of radius 32.0 m at 16.0 m s^{-1} while maintaining constant height. The vertical component of lift balances its weight and the horizontal component supplies the centripetal force. Calculate the horizontal component of lift, the total lift and the angle of the lift force to the vertical. Take g = 9.81 N kg^{-1}. **[3 marks]**

Answer / mark points: Horizontal lift component = $mv^2/r = 5.00 \times 16.0^2/32.0 = 40.0$ N. Vertical component = mg = 49.1 N. Total lift = $\sqrt{(40.0^2 + 49.1^2)} = 63.3$ N. Angle to the vertical = $\tan^{-1}(40.0/49.1) = 39.2°$.

Teacher prompt / common error: The horizontal lift component is the inward resultant. Weight is balanced vertically, so do not add weight to the centripetal force as scalars.

8. In a proton test ring, the speed increases from 4.20 × 10^6 m s^{-1} to 10.5 × 10^6 m s^{-1} as the frequency increases from 0.700 × 10^5 Hz to 1.75 × 10^5 Hz. Calculate the gradient of a v-against-f graph and hence determine the ring radius. **[2 marks]**

Answer / mark points: Gradient = $\Delta v/\Delta f = (10.5 - 4.20) \times 10^6 / [(1.75 - 0.700) \times 10^5] = 60.0$ m. Since $v = 2\pi rf$, gradient = $2\pi r$, so $r = 60.0/(2\pi) = 9.55$ m.

Teacher prompt / common error: For v against f, the gradient is $2\pi r$. The 10^6 and 10^5 scale factors must be included.

9. A rotating space-habitat ring has radius 95.0 m. Determine the angular velocity, period and rim speed required for the centripetal acceleration at the rim to be 9.81 m s^{-2}. **[3 marks]**

Answer / mark points: $a = r\omega^2$, so $\omega = \sqrt{(9.81/95.0)} = 0.321$ rad s^{-1}. $T = 2\pi/\omega = 19.6$ s. Rim speed $v = r\omega = 30.5$ m s^{-1}.

Teacher prompt / common error: The simulated gravity is the centripetal acceleration supplied by the ring. Use the radius from the axis, not the diameter.

4. Vertical circles and proportional reasoning

10. A 120 kg carrier travels at 12.0 m s^{-1} around a vertical circular guide of radius 8.00 m. At both the top and bottom, state the direction of the resultant force. Calculate the normal contact force from the guide at the top and at the bottom. Take g = 9.81 N kg^{-1}. [4 marks]

Answer / mark points: The resultant force is towards the centre: downward at the top and upward at the bottom. Required resultant $F = mv^2/r = 120 \times 12.0^2/8.00 = 2160$ N. At the top, N + mg = 2160, so N = 2160 - 1177 = 983 N. At the bottom, N - mg = 2160, so N = 2160 + 1177 = 3.34×10^3 N.

Teacher prompt / common error: Centripetal force is the resultant. At the top, weight helps provide it; at the bottom, the contact force must exceed weight.

11. A technician says, ‘The capsule moves at constant speed, so it has no acceleration. The centripetal force is an extra force acting in addition to the real forces.’ Explain both errors using precise physics language. [3 marks]

Answer / mark points: Constant speed does not mean constant velocity: the velocity direction changes, so the object has centripetal acceleration towards the centre. ‘Centripetal force’ names the inward resultant of the real forces; it is not a separate additional force.

Teacher prompt / common error: Train the chain: direction of velocity changes → velocity changes → acceleration exists → inward resultant force is required.

12. A tethered sensor rotates with constant angular velocity. Its radius is reduced from 12.0 m to 8.00 m. State the ratios new/old for its linear speed, centripetal acceleration and centripetal force. [2 marks]

Answer / mark points: At constant ω, $v = r\omega$, $a = r\omega^2$ and $F = mr\omega^2$. Each ratio new/old is 8.00/12.0 = 2/3.

Teacher prompt / common error: At constant angular velocity, v, a and F are each proportional to r. This differs from changing r at constant linear speed.

5. Practical: validating F = mv^2/r

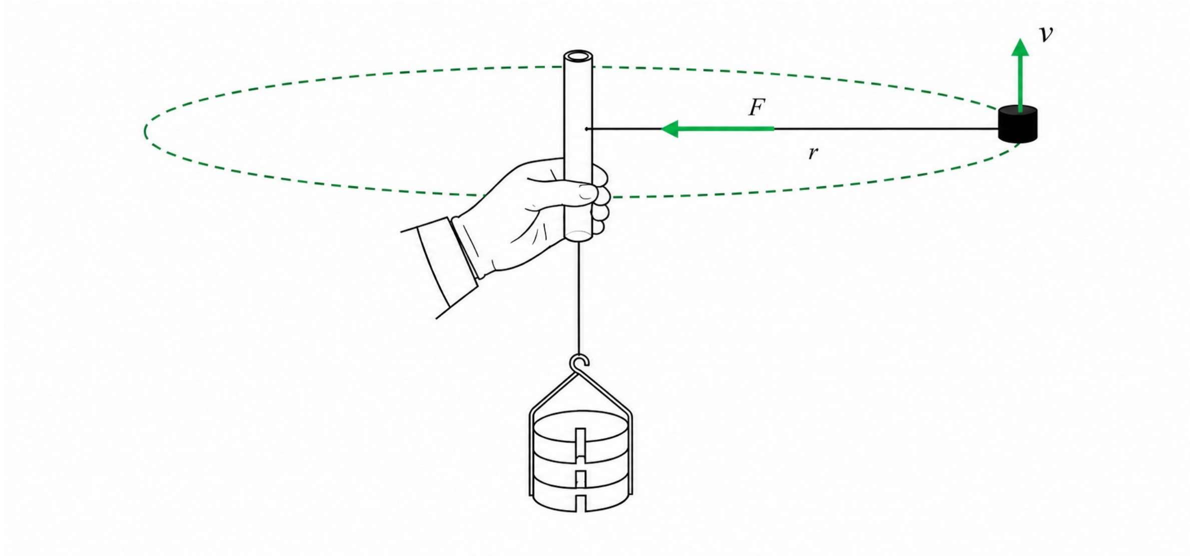

Figure 2 solution: radius, tangential speed and inward tension

13. Scaffolded six-mark practical. Figure 2 shows a whirling bung of mass m connected through a tube to a stationary hanging mass M. Describe how to validate $F = mv^2/r$. Organise your answer around: how F, r and v are measured; how M is varied while m and r are controlled; reliability and safety; the graph to plot; and how its gradient and intercept test the equation. [6 marks]

Answer / mark points: Measure the bung mass m with a balance and the hanging mass M including its hanger; when M is stationary, the string tension is $F = Mg$. Measure r from the tube axis to the centre of the bung. For each M, rotate the bung so the hanging mass remains at a fixed height. Time N revolutions, repeat, use the mean time t and calculate $v = 2\pi rN/t$. Keep m and r fixed; use several M values. Use a clear area, eye protection, a secure bung/string and controlled rotation. Calculate v^2 and plot F on the y-axis against v^2 on the x-axis. A straight line through the origin supports $F \propto v^2$. The predicted gradient is m/r; compare the measured gradient with m/r, including uncertainty, to validate $F = mv^2/r$.
Teacher prompt / common error: For full credit, pupils need measured quantities, controlled variables, repeats, a safe method, F against v^2, and the gradient m/r. A single numerical agreement does not validate a relationship.

6. Unfamiliar application

14. In microgravity, a 0.800 kg debris-capture sensor moves on a 4.50 m tether at angular velocity 1.80 rad s^{-1}. Calculate its linear speed, centripetal acceleration and tether tension. The tether is rated to 15.0 N. Calculate the maximum safe angular velocity. **[4 marks]**

Answer / mark points: $v = r\omega = 4.50 \times 1.80 = 8.10$ m s^{-1}. $a = r\omega^2 = 4.50 \times 1.80^2 = 14.6$ m s^{-2}. $F = mr\omega^2 = 0.800 \times 14.58 = 11.7$ N. From $F_{max} = mr\omega_{max}^2$, $\omega_{max} = \sqrt{[15.0/(0.800 \times 4.50)]} = 2.04$ rad s^{-1}.	
Teacher prompt / common error: In microgravity the tether tension is the inward resultant. The limiting calculation must use the rated force, not the working tension already calculated.	

Language and method check

Say ‘the resultant force is towards the centre’; do not treat centripetal force as another interaction.
A validation uses a range of values, repeats and a predicted graph gradient, not one matching calculation.