Charged particles in magnetic fields solutions

Electromagnetism 2 Guiding an electron beam

Matching answers and examiner-style commentary

OCR A H556 | A-level physics | Module 6 | 39 marks

Specification points covered

6.3.2(a–b) Magnetic force on moving charge and circular orbits in a uniform field.

Prior learning applied: 5.2 circular motion; kinetic energy and electrical work.

Entry velocity is perpendicular to B. Crossed-field selection belongs to Worksheet 3.

Use $m_e = 9.11 \times 10^{-31}$ kg, $|e| = 1.60 \times 10^{-19}$ C and $g = 9.81$ m s^{-2}.

1 Read the beam guide

An evacuated materials-testing instrument bends an electron beam through a quarter-circle towards an adjustable target. Figure 1 is a model of the bend, not a scale drawing.

Unless a question changes a value, use entry speed $v = 1.20 \times 10^{7}$ m s^{-1} and B = 3.00 mT. The field is uniform and perpendicular to the beam plane. Neglect gravity, collisions, fringing and interactions between electrons.

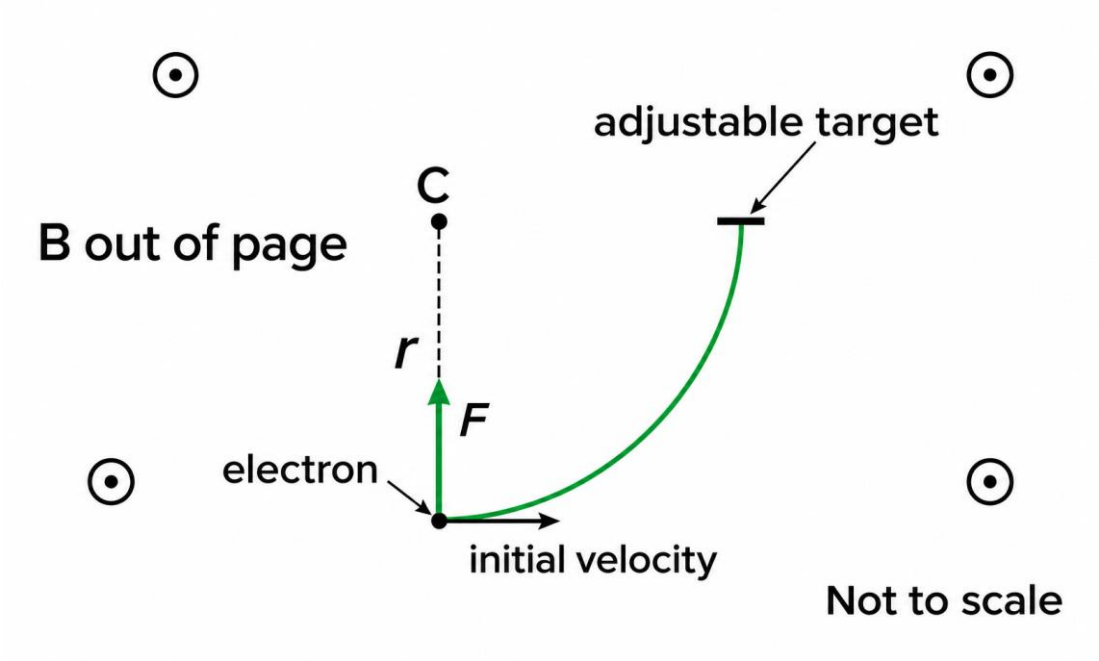


Figure 1 Electron beam guide

1(a) An electron enters to the right and B points out of the page. State its initial magnetic force direction. **[1 mark]**

For a positive charge travelling right, an outward B would give a downward force. The electron has negative charge, so its force is upwards.

Marking guidance 1: upwards.

1(b) In the magnetic region, which can change: the electron's speed, its velocity direction, or both? Explain briefly. **[2 marks]**

The velocity direction changes. Its speed stays constant because the magnetic force is perpendicular to the instantaneous velocity and does no work.

Marking guidance 1: changing direction with constant speed. 1: perpendicular-force/no-work explanation.

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2 Connect force and motion

2(a) Calculate the magnetic force magnitude on one electron at the baseline setting. **[1 mark]**

Convert B = 3.00 mT = 3.00 × 10^{-3} T. The entry is perpendicular, so F = B|q|v.

F = (3.00 × 10^{-3})(1.60 × 10^{-19})(1.20 × 10^{7}) = 5.76 × 10^{-15} N.

Marking guidance 1: 5.76 × 10^{-15} N.

2(b) Calculate the ratio of magnetic force to electron weight and use it to assess the gravity approximation. **[2 marks]**

Weight is $m_e g$ = (9.11 × 10^{-31})(9.81) = 8.937 × 10^{-30} N.

$F/(m_e g)$ = (5.76 × 10^{-15})/(8.937 × 10^{-30}) = 6.45 × 10^{14}.

Magnetic force is vastly greater than weight, so neglecting gravity is justified to this model's precision. The ratio does not itself test the separate collision assumption.

Marking guidance 1: ratio about 6.4 × 10^{14}. 1: relevant gravity conclusion.

2(c) Explain why the magnetic field can produce acceleration without increasing the electron's kinetic energy. **[2 marks]**

Acceleration is change of velocity, so it can arise from a changing direction at constant speed.

The force is perpendicular to displacement at every instant, so magnetic work is zero. Therefore kinetic energy and speed remain constant.

Marking guidance 1: direction change gives acceleration. 1: perpendicular force/no work linked to unchanged kinetic energy.

2(d) A pupil draws the electron's curved orbit as a magnetic field line. Explain why these are different. **[2 marks]**

The field lines here point out of the page and represent the direction of B. The electron's orbit lies in the page.

Its force is perpendicular to both its velocity and B, so the path followed by the moving electron is not a field line.

Marking guidance 1: correct distinct field and orbit directions. 1: role of perpendicular force/trajectory.

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3 Derive the bend radius

3(a) Starting with the magnetic force and the force needed for circular motion, derive $r = mv/(B|q|)$. **[2 marks]**

The magnetic force provides the centripetal resultant; a second “centripetal force” is not added.

$B|q|v = mv^2/r$.

Multiply by r and divide by $B|q|v$: $r = mv/(B|q|)$.

Marking guidance 1: equates magnetic and centripetal-force expressions. 1: valid cancellation/rearrangement.

3(b) Calculate the baseline electron orbit radius in millimetres. **[1 mark]**

$r = mv/(B|q|) = [(9.11 \times 10^{-31})(1.20 \times 10^{7})]/[(3.00 \times 10^{-3})(1.60 \times 10^{-19})] = 0.022775$ m = 22.8 mm.

Marking guidance 1: about 22.8 mm.

3(c) A particle has the same mass and speed as the electron but charge +e. In blank space, sketch its path from the same entry point and compare its radius with the electron’s. **[2 marks]**

It curves downwards, opposite to the electron, because its charge sign is opposite.

The radius magnitude is the same: m, v, B and |q| are unchanged. Draw a downward quarter-circle of equal radius for the same bend stage.

Marking guidance 1: opposite curvature. 1: equal radius represented and explained.

3(d) The speed becomes 1.25 times its baseline value and B is halved. Find the factor by which the radius changes. **[1 mark]**

At fixed m and |q|, r is proportional to v/B.

$r_{new}/r_{old} = 1.25/0.50 = 2.50$.

Marking guidance 1: factor 2.50.

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4 Link the upstream accelerator

The accelerator and magnetic bend are separate regions. The accelerating p.d. is changed only when the question says so.

4(a) Before the bend, electrons accelerate from rest through a p.d. of magnitude U. Find U for the baseline entry speed. **[2 marks]**

Electrical work becomes kinetic energy before the magnetic region: $|e|U = \tfrac{1}{2}m_e v^2$.

$U = m_e v^2/(2|e|) = [(9.11 \times 10^{-31})(1.20 \times 10^{7})^2]/[2(1.60 \times 10^{-19})] = 409.95\ \text{V} \approx 410\ \text{V}$.

Marking guidance 1: correct energy relationship. 1: about 410 V.

4(b) At B = 3.00 mT, a new test requires r = 25.0 mm. Calculate the required accelerating p.d. from rest. **[3 marks]**

Step 1 First find the speed required by the orbit geometry.

$v = B|e|r/m_e = [(3.00 \times 10^{-3})(1.60 \times 10^{-19})(0.0250)]/(9.11 \times 10^{-31}) = 1.3172 \times 10^{7}\ \text{m s}^{-1}$.

Step 2 The upstream electrical work must provide that kinetic energy.

$U = m_e v^2/(2|e|) = 493.96\ \text{V} \approx 494\ \text{V}$.

The magnetic bend itself supplies no kinetic energy.

Marking guidance 1: radius-to-speed relationship. 1: required speed or equivalent combined algebra. 1: about 494 V.

4(c) At fixed B, the accelerating p.d. is doubled. By what factor does the orbit radius increase? Assume non-relativistic motion from rest. **[1 mark]**

Since $\tfrac{1}{2}mv^2 = |q|U$, v is proportional to $\sqrt{U}$. At fixed B, r is proportional to v, so the radius increases by $\sqrt{2} \approx 1.41$.

Marking guidance 1: $\sqrt{2}$ or about 1.41.

4(d) A positive ion's mass is increased by a factor of four with |q| unchanged. Compare the radius factors at (i) fixed speed and (ii) fixed accelerating p.d. from rest, with B unchanged. **[2 marks]**

At fixed speed, $r = mv/(B|q|)$, so the radius increases by 4.

At fixed accelerating p.d., $r = \sqrt{(2mU/|q|)}/B$, so the radius increases by $\sqrt{4} = 2$. The speed falls when mass rises under this second condition.

Marking guidance 1: factor 4 for fixed speed. 1: factor 2 for fixed p.d. with valid reasoning.

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5 Use time and calibration data

Table 1

B / mT	r / mm
2.00	34.2
2.50	27.3
3.00	22.8
4.00	17.1

5(a) Calculate the time for a baseline electron to complete the quarter-circle. Use the arc length, not the chord. **[2 marks]**

The arc length is one quarter of $2\pi r$: $s = \pi r/2 = \pi(0.022775)/2 = 0.035775$ m.

$t = s/v = 0.035775/(1.20 \times 10^{7}) = 2.981 \times 10^{-9}$ s ≈ 2.98 ns.

Marking guidance 1: correct quarter-arc length. 1: about 2.98 ns.

5(b) Derive the full-orbit period in terms of m, B and |q|. State what the result predicts if only speed is increased. **[2 marks]**

$T = 2\pi r/v = (2\pi/v)[mv/(B|q|)] = 2\pi m/(B|q|)$.

The period is unchanged when only speed increases. The orbit becomes larger in the same proportion as speed, so its greater circumference compensates.

Marking guidance 1: correct substitution and period expression. 1: unchanged period with a relevant explanation.

5(c) The entry speed is held fixed while B is varied. Choose graph axes using Table 1 that should give a straight line through the origin, and identify its gradient. **[1 mark]**

Plot r vertically against 1/B horizontally, using SI units. Since $r = (mv/|q|)(1/B)$, the gradient is $mv/|q|$.

Marking guidance 1: axes and gradient both correct.

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5(d) Use the readings at 2.00 mT and 4.00 mT to find the gradient in T m and infer m/|q|. The speed is 1.20×10^{7} m s^{-1}. **[2 marks]**

Step 1 Convert the coordinates before taking their differences.

At 2.00 mT, 1/B = 500 T^{-1} and r = 0.0342 m. At 4.00 mT, 1/B = 250 T^{-1} and r = 0.0171 m.

gradient = $(0.0342 - 0.0171)/(500 - 250) = 6.84 \times 10^{-5}$ T m.

Step 2 Since gradient = mv/|q|, divide by the fixed speed.

$m/|q| = (6.84 \times 10^{-5})/(1.20 \times 10^{7}) = 5.70 \times 10^{-12}$ kg C^{-1}.

Marking guidance 1: 6.84×10^{-5} T m. 1: about 5.70×10^{-12} kg C^{-1}.

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6 Check the channel limit

6(a) The maximum permitted orbit radius is 26.0 mm. At the baseline speed, find the minimum permissible B. **[1 mark]**

The largest allowed radius sets the smallest field: $B_{min} = mv/(|q|r_{max})$.

$B_{min} = [(9.11 \times 10^{-31})(1.20 \times 10^{7})]/[(1.60 \times 10^{-19})(0.0260)] = 2.628 \times 10^{-3}$ T ≈ 2.63 mT.

Marking guidance 1: about 2.63 mT.

6(b) Available nominal settings are 2.60 mT and 2.80 mT. Calculate the orbit radius at each and select a setting that meets the 26.0 mm limit. **[2 marks]**

At 2.60 mT, r = 0.026279 m = 26.3 mm, which exceeds the limit.

At 2.80 mT, r = 0.024402 m = 24.4 mm. Choose 2.80 mT because 24.4 mm is below 26.0 mm.

Marking guidance 1: both radii. 1: correct justified choice.

6(c) For the 2.80 mT setting, B may be 5% below nominal and speed may be 2% above baseline. Does this setting guarantee the radius limit? Use the combined worst case. **[3 marks]**

Step 1 A larger speed and a smaller field both increase radius.

$v_{max} = 1.02(1.20 \times 10^{7}) = 1.224 \times 10^{7}$ m s^{-1}; $B_{min} = 0.95(2.80$ mT$) = 2.66$ mT.

Step 2 Use both limits together.

$r_{max} = [(9.11 \times 10^{-31})(1.224 \times 10^{7})]/[(1.60 \times 10^{-19})(2.66 \times 10^{-3})] = 0.02620$ m = 26.2 mm.

This exceeds 26.0 mm, so the nominally suitable setting does not guarantee the limit over the stated tolerance range.

Marking guidance 1: correct worst-case choices. 1: about 26.2 mm. 1: justified no-guarantee conclusion.

6(d) Correct this response: “A stronger magnetic field gives a larger force, so it speeds up the electrons and increases their kinetic energy.” **[2 marks]**

For a given entry speed, a stronger B gives a larger perpendicular force and a smaller orbit radius.

The magnetic field still does no work, so speed and kinetic energy stay unchanged in the bend. A change in direction, not an increase in speed, accounts for the acceleration.

Marking guidance 1: greater force linked to tighter curvature. 1: no work and unchanged speed/kinetic energy.

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