Motion of charged particles solutions

Electric fields 4 Steering an ion beam

Matching answers and examiner-style commentary

OCR A H556 | A-level physics | Module 6 | 40 marks

Specification points covered

6.2.3(a,c), 6.2.1(d) Uniform fields, electric force and charged-particle motion.

Prior learning applied: linear/projectile motion (3.1), forces (3.2), energy (3.3) and W = VQ (4.2.2).

Use e = 1.60 × 10^{-19} C and g = 9.81 m s^{-2}.

1 Read the beam test rig

An ion-engine test rig accelerates singly charged positive xenon ions and steers them towards an offset detector. Each ion has q = +1.60 × 10^{-19} C and m = 2.18 × 10^{-25} kg.

For steering calculations use u_x = 2.00 × 10^{4} m s^{-1}, L = 20.0 mm, d = 10.0 mm and D = 30.0 mm. The beam enters at mid-gap. Take upwards as positive.

Model a dilute beam in a vacuum: negligible drag and ion–ion interactions. The field is uniform between the plates and zero outside; neglect edge effects. The detector centre is h = 8.10 mm above the entry line.

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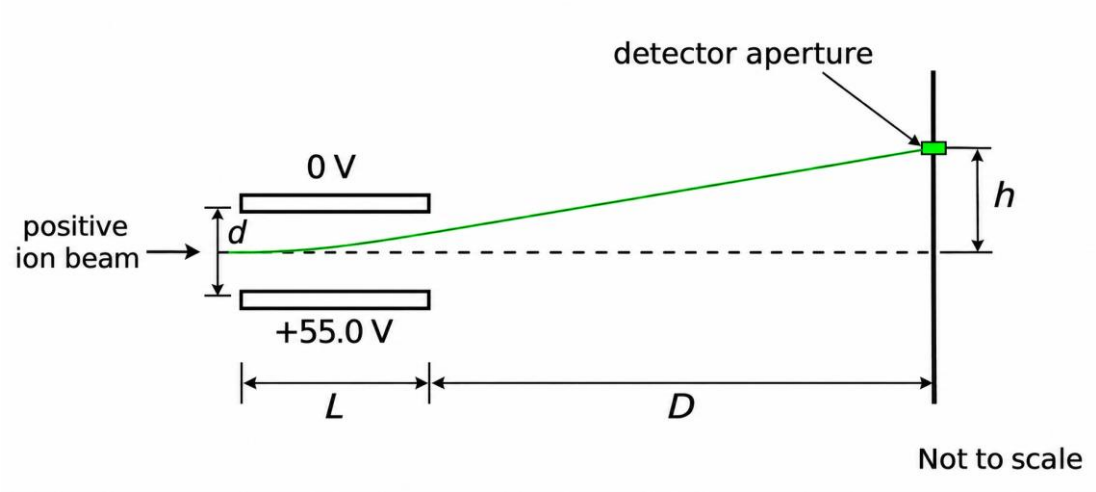


Figure 1 Steering plates and detector

1(a) State the directions of the electric field and the electric force on a positive ion between the steering plates in Figure 1. **[1 mark]**

The field is upwards, from the +55.0 V lower plate towards the 0 V upper plate. A positive ion feels an upward force, in the field direction.

Marking guidance 1: both directions upwards.

1(b) The beam enters horizontally at speed 2.00×10^4 m s^{-1}. State its initial horizontal and vertical velocity components. **[1 mark]**

$u_x = 2.00 \times 10^4$ m s^{-1}; $u_y = 0$. A horizontal launch does not give an initial upward velocity.

Marking guidance 1: both components correct.

2 Accelerate the ions upstream

The accelerating p.d. U_a is separate from the steering p.d. U_s. For Question 2 only, calculate the speed specified in each part; return to the stated 2.00×10^4 m s^{-1} for Question 3 onwards.

2(a) Before the steering region, an ion starts from rest and moves through a 300 V accelerating p.d. Calculate its kinetic energy gain in joules. **[1 mark]**

Electrical work becomes kinetic energy: $\Delta K = |q|U_a$.

$\Delta K = (1.60 \times 10^{-19})(300) = 4.80 \times 10^{-17}$ J.

Marking guidance 1: 4.80×10^{-17} J.

2(b) Calculate the speed produced by that 300 V accelerating p.d. **[1 mark]**

Use $\frac{1}{2}mv^2 = |q|U_a$, because the ion starts from rest.

$v = \sqrt{(2|q|U_a/m)} = \sqrt{[2(1.60 \times 10^{-19})(300)/(2.18 \times 10^{-25})]} = 2.10 \times 10^4$ m s^{-1}.

Marking guidance 1: about 2.10×10^4 m s^{-1}.

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2(c) Find the accelerating p.d. that produces the 2.00×10^4 m s^{-1} entry speed used elsewhere in the worksheet. Explain whether a positive ion moves to a higher or lower electric potential while gaining this energy. **[2 marks]**

Step 1 Rearrange the energy equation for the accelerating-p.d. magnitude.

$U_a = mv^2/(2q) = (2.18 \times 10^{-25})(2.00 \times 10^4)^2/[2(1.60 \times 10^{-19})] = 272.5\ \text{V} \approx 273\ \text{V}$.

Step 2 Interpret the sign physically.

For $q > 0$, a decrease in potential gives $\Delta U = q\Delta V < 0$. The electric field does positive work and K increases. The positive ion therefore accelerates towards lower potential.

Marking guidance 1: accelerating p.d. about 273 V. 1: lower potential, with correct energy reasoning.

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3 Establish the transverse acceleration

3(a) At U_s = 55.0 V, calculate the electric field magnitude between the steering plates. **[1 mark]**

d = 10.0 mm = 0.0100 m.

$E = U_s/d = 55.0/0.0100 = 5.50 \times 10^3$ V m^{-1}.

Marking guidance 1: 5.50×10^3 V m^{-1}.

3(b) Calculate the electric force magnitude on one ion. **[1 mark]**

$F = qE = (1.60 \times 10^{-19})(5.50 \times 10^3) = 8.80 \times 10^{-16}$ N, upwards.

Marking guidance 1: 8.80×10^{-16} N.

3(c) Calculate the vertical acceleration and explain why the horizontal acceleration is zero in the given model. **[2 marks]**

$a_y = F/m = (8.80 \times 10^{-16})/(2.18 \times 10^{-25}) = 4.0367 \times 10^9$ m s^{-2}, upwards.

There is no horizontal component of the electric force and other horizontal forces are neglected. Therefore $a_x = 0$ and horizontal velocity stays constant.

Marking guidance 1: vertical acceleration about 4.04×10^9 m s^{-2}. 1: horizontal-force explanation.

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3 Establish the transverse acceleration continued

3(d) Calculate the ratio of electric force to weight and use it to justify neglecting gravity. Does this calculation also justify neglecting drag? **[2 marks]**

Weight = mg = $(2.18 \times 10^{-25})(9.81) = 2.1386 \times 10^{-24}$ N.

$F_{electric}/mg = (8.80 \times 10^{-16})/(2.1386 \times 10^{-24}) = 4.12 \times 10^{8}$.

Weight is negligible compared with the electric force. This ratio says nothing about drag; negligible drag is a separate supplied vacuum-model assumption.

Marking guidance 1: ratio and gravity conclusion. 1: distinguishes drag assumption from the weight comparison.

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4 Follow the motion inside the plates

4(a) Calculate the time an ion spends between the steering plates. **[1 mark]**

Use horizontal motion because L is a horizontal distance and u_x is constant.

$t = L/u_x = (20.0 \times 10^{-3})/(2.00 \times 10^{4}) = 1.00 \times 10^{-6}$ s.

Marking guidance 1: 1.00 μs from horizontal motion.

4(b) Calculate the vertical displacement at the plate exit. **[2 marks]**

Step 1 Use the vertical initial condition $u_y = 0$.

$y = u_y t + \frac{1}{2}a_y t^2 = \frac{1}{2}a_y t^2$.

Step 2 Substitute the field-region time and acceleration.

$y = \frac{1}{2}(4.0367 \times 10^{9})(1.00 \times 10^{-6})^2 = 2.01835 \times 10^{-3}$ m ≈ 2.02 mm upwards.

Marking guidance 1: correct vertical-motion equation/initial condition. 1: about 2.02 mm.

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4 Follow the motion inside the plates continued

4(c) Determine the exit vertical velocity, then the exit speed and angle above the horizontal. **[3 marks]**

Step 1 Find the vertical component gained in the field.

$v_y = u_y + a_y t = 0 + (4.0367 \times 10^9)(1.00 \times 10^{-6}) = 4.0367 \times 10^3$ m s^{-1}.

Step 2 Combine perpendicular components, rather than adding their magnitudes.

$v = \sqrt{(v_x^2 + v_y^2)} = \sqrt{[(2.00 \times 10^4)^2 + (4.0367 \times 10^3)^2]} = 2.0403 \times 10^4$ m s^{-1}.

$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(4036.7/20000) = 11.4°$ above the horizontal.

Marking guidance 1: correct vertical component. 1: correct speed. 1: correct angle.

4(d) Treat each ion as a point. Check whether the original beam clears the upper plate. **[1 mark]**

The beam starts at mid-gap, so the available vertical clearance is $d/2 = 5.00$ mm.

The displacement rises throughout the field and is largest at the exit, 2.02 mm. Since $2.02 < 5.00$ mm, the beam clears the plate.

Marking guidance 1: comparison with half the gap, with a correct conclusion.

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5 Continue to the detector

5(a) Calculate the time taken to travel the drift distance D after leaving the plates. **[1 mark]**

Horizontal speed is still 2.00×10^4 m s^{-1}.

$t_{drift} = D/u_x = (30.0 \times 10^{-3})/(2.00 \times 10^4) = 1.50 \times 10^{-6}$ s.

Marking guidance 1: 1.50 μs.

5(b) Calculate the additional vertical displacement in the drift region and the total displacement at the detector. Explain why the path in this region is straight. **[3 marks]**

Step 1 There is no field outside the plates, so v_y stays at its exit value and $a_y = 0$.

$\Delta y_{drift} = v_y t_{drift} = (4.0367 \times 10^3)(1.50 \times 10^{-6}) = 6.05505 \times 10^{-3}$ m.

Step 2 Add the displacement already gained inside the field.

$Y = y_{exit} + \Delta y_{drift} = 2.01835 + 6.05505 = 8.07340$ mm ≈ 8.07 mm.

Both velocity components are constant during drift, so the beam travels on a straight line at a fixed angle; a non-zero vertical velocity does not itself require a vertical force.

Marking guidance 1: drift displacement. 1: total about 8.07 mm. 1: straight-path explanation from constant velocity.

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5 Continue to the detector continued

5(c) Sketch v_x against time and v_y against time from entry at t = 0 to arrival at the detector at t = 2.50 μs. Mark the plate-exit time and the main velocity values. **[2 marks]**

v_x stays at 2.00×10^4 m s^{-1} throughout.

v_y starts at zero and increases linearly to 4.04×10^3 m s^{-1} at 1.00 μs, because acceleration in the field is constant. It then stays at that value until 2.50 μs.

Plot these coordinates. Join the horizontal-component points with a horizontal line. Join the vertical-component points with a straight rising line to 1.00 μs, then a horizontal line to 2.50 μs. There is no instantaneous fall to zero at arrival.

t / μs	v_x / m s^{-1}	v_y / m s^{-1}
0	20000	0
1.00	20000	4037
2.50	20000	4037

Marking guidance 1: correct horizontal-component graph. 1: correct vertical-component graph including the change at 1.00 μs.

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6 Align the detector

6(a) Starting from the field force and the two stages of motion, show that the detector displacement is $Y = qU_sL(D + L/2)/(md\,u_x^2)$. Define the role of the two terms D and L/2. **[3 marks]**

Step 1 Link steering voltage to transverse acceleration.

$a_y = qE/m = qU_s/(md)$, and time in the field is L/u_x.

Step 2 Find the displacement and vertical velocity at the plate exit.

$y_{exit} = \tfrac{1}{2}a_y(L/u_x)^2$; $v_y = a_yL/u_x$.

Step 3 Add drift displacement v_yD/u_x.

$Y = a_yL^2/(2u_x^2) + a_yLD/u_x^2 = a_yL(D + L/2)/u_x^2$.

Substitute $a_y = qU_s/(md)$ to obtain the stated expression. The L/2 term comes from accelerated displacement inside the plates; D comes from constant-velocity drift.

Marking guidance 1: acceleration/field-region relations. 1: adds the correct drift term. 1: correct combined expression with terms interpreted.

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6 Align the detector continued

6(b) Find the steering p.d. needed to reach the detector centre at Y = 8.10 mm. Check both an 80.0 V supply limit and the 5.00 mm exit-clearance limit. Use the original ion and geometry data. **[3 marks]**

Step 1 For fixed beam and geometry, Y is proportional to U_s.

$U_{s,needed} = 55.0 \times (8.10/8.0734) = 55.181\ V \approx 55.2\ V$.

Alternatively rearrange the expression in 6(a): $U_s = Ymd\,u_x^2/[qL(D + L/2)]$.

Step 2 Check the exit displacement at the selected p.d.

$y_{exit}/Y = L/(L + 2D) = 20.0/(20.0 + 60.0) = 1/4$.

$y_{exit} = 8.10/4 = 2.025$ mm.

The required 55.2 V is below 80.0 V and 2.025 mm is below 5.00 mm, so both conditions are satisfied.

Marking guidance 1: steering p.d. about 55.2 V. 1: exit displacement about 2.03 mm. 1: explicitly checks both limits.

6(c) A pupil calculates vertical deflection using $y = (2.00 \times 10^4)t + \frac{1}{2}a_y t^2$. Explain the error and give the correct initial vertical velocity. **[2 marks]**

The 2.00×10^4 m s^{-1} value is horizontal speed, so it cannot be used as u_y. The beam enters horizontally and $u_y = 0$. Vertical displacement inside the plates is therefore $y = \frac{1}{2}a_y t^2$.

Marking guidance 1: identifies component confusion. 1: $u_y = 0$ and a correct vertical relation.

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7 Transfer the method to a different setting

7(a) At the same entry speed, geometry, steering p.d. and charge, an ion's mass is doubled. State and explain the change in detector displacement. **[2 marks]**

The electric force qE is unchanged, but a = qE/m halves. The field and drift times remain unchanged because horizontal speed and distances are unchanged. Both displacement terms therefore halve, so Y halves.

Marking guidance 1: half displacement. 1: reasoning includes fixed entry speed/times and half acceleration.

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7 Transfer the method to a different setting continued

7(b) Instead, both ion types start from rest and are accelerated through the same U_a before entering the same steering arrangement. Substitute $u_x^2 = 2qU_a/m$ into 6(a). Explain why the mass conclusion from 7(a) no longer applies. **[3 marks]**

Step 1 Under the same accelerating-p.d. condition, entry speed changes with mass.

$u_x^2 = 2qU_a/m$.

Step 2 Substitute into the detector-displacement expression.

$Y = qU_sL(D + L/2)/[md(2qU_a/m)] = U_sL(D + L/2)/(2dU_a)$.

Mass and charge cancel in this ideal model. A heavier ion accelerates less in the steering field but enters more slowly and spends longer in and beyond the plates. The fixed-entry-speed comparison in 7(a) is a different condition.

Marking guidance 1: correct energy/speed substitution. 1: correct simplified expression. 1: explains the changed operating condition and cancellation.

7(c) Return to the original positive ions and the original entry speed. Change only the lower plate from +55.0 V to −55.0 V. Describe the trajectory and state whether the original upper detector aperture will collect the beam. **[2 marks]**

The field and force reverse downwards with the same magnitudes. The beam curves downwards between the plates and then travels along a straight downward-sloping line. Its detector displacement is −8.07 mm in the ideal model, so it misses the aperture above the baseline.

Marking guidance 1: correct reversed curved-then-straight trajectory. 1: misses upper aperture, with sign/direction clear.

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