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SOLUTIONS
ATTEMPT CHECK CORRECT IMPROVE

Capacitors 1 solutions

Charge combinations and stored energy

Matching answers and examiner-style commentary

OCR A H556 and AQA 7408 | A-level physics

Specification and route

OCR A H556: 6.1.1(a)–(e), 6.1.2(a)–(c).

AQA 7408: 3.7.4.1 and 3.7.4.3. Combination work in Questions 4–7 is a supported extension for AQA; the relationships are supplied.

A-level only. Parallel-plate geometry and dielectric action are outside this sheet.

23 core parts | 43 marks

A seabed sampling robot must close its sample cartridge and save one final record if its main supply fails. Engineers are choosing a capacitor bank for these two jobs. Use ideal capacitors with constant capacitance and negligible leakage unless told otherwise.

Original PhysicsUK worked solutions. Use the explanations to check the method, then use the separate marking guidance to award credit.

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1 Understand the charge

1(a) A component is labelled 2200 μF. State what its capacitance means in terms of charge and potential difference. **[1 mark]**

It stores a charge magnitude of 2200 μC on either plate for each 1 V of p.d. across it: C = Q/V. This is 0.002200 C per volt. Capacitance is a charge-to-p.d. ratio, not the charge at every voltage.

Marking 1: correct charge per volt interpretation, with units.

1(b) Explain how the robot's supply charges a capacitor, and why an ideal charged capacitor can have zero net charge. **[2 marks]**

The supply removes electrons from one plate and delivers the same number to the other through the external circuit. Electrons do not pass through the insulating gap.

The plates acquire equal and opposite charges, +Q and −Q. Their sum is zero, although charge has been separated and energy is stored. In Q = CV, Q is the magnitude of the charge on either plate.

Marking 1: electron transfer through the external circuit; 1: equal/opposite plate charges give zero net charge.

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2 Test a single component

For Questions 2–3 use an ideal 2200 μF capacitor. It is initially uncharged.

2(a) Calculate the charge magnitude on either plate when the p.d. is 18.0 V. Give your answer in mC. **[1 mark]**

Charge is proportional to p.d. for this fixed capacitor, so use Q = CV. Convert μF to F first.

C = 2200 × 10^{-6} F = 0.002200 F.

Q = 0.002200 × 18.0 = 0.0396 C.

Since 1 C = 1000 mC, Q = 39.6 mC on either plate.

Marking 1: 39.6 mC, or a correctly rounded equivalent.

2(b) The capacitor later holds 18.0 mC on either plate. Calculate its p.d. **[1 mark]**

Use Q = CV and divide both sides by C to make the p.d. the subject: V = Q/C.

Q = 18.0 mC = 0.0180 C; C = 0.002200 F.

V = 0.0180/0.002200 = 8.18 V.

The capacitance stays the same even though the stored charge and p.d. have changed.

Marking 1: 8.18 V or equivalent.

2(c) In a separate test a constant current of 4.40 mA charges an initially uncharged component for 9.00 s. Its final p.d. is 18.0 V. Use these measurements to determine its capacitance. **[2 marks]**

Step 1 Find the charge transferred

The current is constant, so Q = It gives the total charge moved during the charging interval. Initially the capacitor is uncharged.

I = 4.40 mA = 0.00440 A.

Q = 0.00440 × 9.00 = 0.0396 C.

Step 2 Use charge per volt to find capacitance

Rearrange Q = CV by dividing by V: C = Q/V.

C = 0.0396/18.0 = 0.00220 F = 2200 μF.

The final 18.0 V is the p.d. corresponding to that charge. One instantaneous current from an ordinary resistor-charging circuit could not be multiplied by the whole interval, because that current changes.

Marking 1: charge from It with correct conversion; 1: capacitance 2.20 mF.

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3 Connect charge and energy

For a constant-capacitance component, a graph of p.d. V vertically against plate-charge magnitude Q horizontally is a straight line through the origin.

3(a) Sketch this V–Q graph up to 39.6 mC and 18.0 V. Use its area to show that W = ½QV, then calculate the stored energy at 18.0 V. **[3 marks]**

Step 1 Connect the graph to energy

Draw a straight line from the origin to (39.6 mC, 18.0 V), with Q horizontal and V vertical. As each small extra amount of charge moves, the energy transferred is the charge multiplied by the p.d. at that stage. The total energy is therefore the area under V against Q.

Step 2 Find the triangular area

The line passes through the origin, so the area is a triangle with base Q and height V.

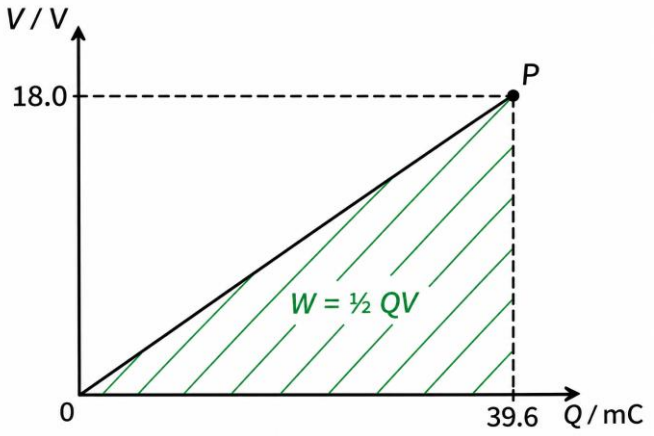
W = ½ × base × height = ½QV.

Step 3 Calculate using coulombs

Q = 39.6 mC = 0.0396 C.

W = ½ × 0.0396 × 18.0 = 0.3564 J.

The energy stored at 18.0 V is 0.356 J to three significant figures.



Model graph for 3(a). Convert mC to C when calculating the triangular energy area.

Marking 1: labelled straight-line graph with correct endpoint; 1: triangle gives ½QV; 1: energy 0.356 J.

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3 Derive and explain the energy equations

3(b) Use Q = CV to obtain both $W = \frac{1}{2}CV^2$ and $W = Q^2/(2C)$ from $W = \frac{1}{2}QV$. **[2 marks]**

Step 1 Obtain the form using C and V

Start with $W = \frac{1}{2}QV$. Since Q = CV, replace Q by CV.

$W = \frac{1}{2}(CV)V$.

The two factors of V give $W = \frac{1}{2}CV^2$. Use this form when capacitance and p.d. are known.

Step 2 Obtain the form using Q and C

Rearrange Q = CV to V = Q/C. This time replace V in $W = \frac{1}{2}QV$.

$W = \frac{1}{2}Q(Q/C)$.

The two factors of Q give $W = Q^2/(2C)$. Use this form when charge and capacitance are known.

Marking 1: each valid substitution leading to its stated energy form.

3(c) An engineer uses W = QV because “all the charge ends up at 18 V”. Explain why this doubles the energy for charging from zero. **[2 marks]**

The p.d. rises from zero as the capacitor charges. Earlier charge was moved through a lower p.d.; it was not all moved through the final 18 V.

For constant capacitance, p.d. increases linearly with charge from zero; the mean p.d. for the transferred charge is V/2. The rectangle QV is twice the triangular area $\frac{1}{2}QV$.

Marking 1: p.d. increases during charging; 1: average is V/2 / triangle rather than rectangle.

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4 Compare capacitor banks

Each bank in Figure 1 is initially uncharged, then connected between X and Y across an ideal 18.0 V supply. All quoted capacitances and ratings are component values. The two components in B are matched.

OCR required / supported AQA extension. For two capacitors: series $1/C = 1/C_1 + 1/C_2$; parallel $C = C_1 + C_2$. In series the initially neutral isolated junction gives equal charge magnitudes; in parallel each branch has the same p.d.

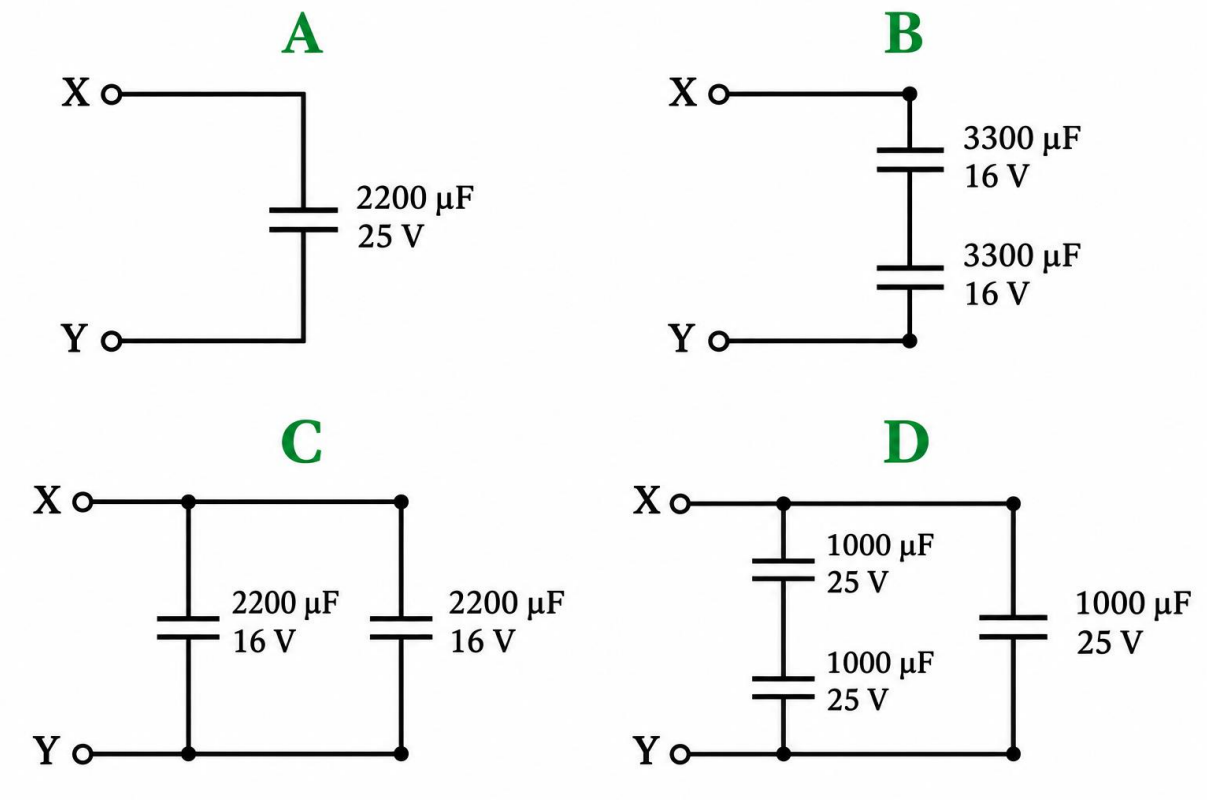


Figure 1 Four independent banks. The second label on each component is its maximum permitted p.d.

4(a) Calculate the equivalent capacitance of bank B. **[1 mark]**

The two 3300 µF capacitors are in series. Add their reciprocals, then take the reciprocal of that result.

$1/C_{eq} = 1/3300 + 1/3300 = 2/3300$ in μF^{-1}.

$C_{eq} = 3300/2 = 1650\ \mu F = 1.650\ mF$.

The equivalent is smaller than either component, as expected for series capacitors.

Marking 1: 1650 µF.

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4 Series charge and potential difference

4(b) Calculate the charge magnitude on each capacitor in B at a total p.d. of 18.0 V. **[2 marks]**

Step 1 Find the charge supplied to the series bank

Use the bank capacitance from 4(a) with the total 18.0 V across the bank. Convert 1650 μF to 0.001650 F.

$Q = C_{eq}V = 0.001650 \times 18.0 = 0.0297$ C.

Step 2 Identify the charge on each component

The initially neutral isolated junction between the series capacitors cannot gain net charge. Equal and opposite charges form on its two connected plates, so the capacitors have equal charge magnitudes.

Charge on each capacitor = 0.0297 C = 29.7 mC.

The 29.7 mC is not divided between the two capacitors.

Marking 1: correct bank charge; 1: same 29.7 mC on each component.

Series and parallel reasoning

Refer to Figure 1 on the preceding page. The supply p.d. remains 18.0 V.

4(c) Calculate the p.d. across each capacitor in B and decide whether its 16 V rating is exceeded. **[2 marks]**

Step 1 Use the charge on one capacitor

Each component carries 0.0297 C from 4(b). Use the capacitance of one component, not the equivalent bank capacitance, to find its p.d.

C = 3300 μF = 0.003300 F.

V = Q/C = 0.0297/0.003300 = 9.0 V.

Step 2 Compare with the component rating

The components are matched and carry equal charge magnitudes, so each has 9.0 V across it. These p.d.s add to the supplied 18.0 V.

9.0 V < 16 V, so neither nominal component rating is exceeded. Equal voltage sharing would not apply to unequal capacitances.

Marking 1: 9.0 V each; 1: correct rating comparison.

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4 Voltage ratios and parallel charge

4(d) In a separate 18.0 V series test, 1000 μF and 2000 μF capacitors are used. Without calculating the charge, explain which has the larger p.d. and give the two p.d.s. **[2 marks]**

Step 1 Find the voltage ratio without calculating charge

The capacitors in series have equal charge magnitudes. Since V = Q/C, the smaller capacitance has the larger p.d.

$V_{1000}/V_{2000} = C_{2000}/C_{1000} = 2000/1000 = 2$.

Step 2 Share the total 18.0 V in that ratio

The p.d.s are in the ratio 2:1, so the total is three equal shares.

One share = 18.0/3 = 6.0 V.

The 1000 μF capacitor has 2 × 6.0 = 12.0 V; the 2000 μF capacitor has 6.0 V. Their sum is 18.0 V.

Marking 1: equal Q and inverse-C reasoning; 1: 12.0 V and 6.0 V.

5(a) Calculate the equivalent capacitance of bank C. **[1 mark]**

Parallel branches have the same p.d. and their supplied charges add. Their capacitances therefore add.

$C_{eq} = C_1 + C_2 = 2200 + 2200 = 4400$ μF.

Marking 1: 4400 μF.

5(b) Calculate the charge on each capacitor in C and the total charge supplied to the bank at 18.0 V. **[2 marks]**

Step 1 Calculate the charge on one branch

Each parallel capacitor has the full 18.0 V across it. Convert its capacitance to farads and use Q = CV.

$Q_{each} = 0.002200 \times 18.0 = 0.0396$ C = 39.6 mC.

Step 2 Add the charges supplied to the branches

The two identical branches each require 39.6 mC, so the supply transfers their sum.

$Q_{total} = 39.6 + 39.6 = 79.2$ mC.

This is charge transferred by the supply, not the net charge after adding all positive and negative capacitor plates together.

Marking 1: 39.6 mC each; 1: 79.2 mC supplied.

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5 Check the energy and the ratings

Use Figure 1. Bank C is assessed as an ideal arrangement before its component ratings are checked; it must not be used above a rating.

5(c) Calculate the energy in each component of C at 18.0 V and show that their sum equals the energy calculated using the equivalent capacitance. **[2 marks]**

Step 1 Calculate one component's ideal stored energy

Each parallel component has C = 0.002200 F and V = 18.0 V. Use $W = \frac{1}{2}CV^2$.

$W_{each} = \frac{1}{2} \times 0.002200 \times 18.0^2 = 0.3564$ J.

Step 2 Add the two component energies

$W_{total} = 0.3564 + 0.3564 = 0.7128$ J.

Step 3 Check using the equivalent capacitance

For the whole bank, $C_{eq} = 4400\ \mu F = 0.004400$ F. The bank still has 18.0 V across it.

$W_{bank} = \frac{1}{2} \times 0.004400 \times 18.0^2 = 0.7128$ J.

The results agree: adding the branch energies gives the same ideal energy as using the equivalent capacitor.

Marking 1: each/summed component energy; 1: valid equivalent-bank check.

5(d) Bank C stores more energy than B at the same total p.d. Explain why C is nevertheless unsuitable at 18.0 V. **[1 mark]**

In parallel each capacitor has the full 18.0 V across it. This exceeds each 16 V component rating, so the bank is unsuitable even though its ideal stored energy is larger.

Marking 1: full 18 V across each branch exceeds 16 V.

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6 Reduce the mixed bank and find released energy

6(a) Find the equivalent capacitance of bank D by reducing its series pair first. **[2 marks]**

Step 1 Replace the series pair by one equivalent capacitor

The left branch contains two 1000 μF capacitors in series.

$1/C_{pair} = 1/1000 + 1/1000 = 2/1000$ in μF^{-1}.

$C_{pair} = 1000/2 = 500$ μF.

Step 2 Add the parallel branch

The 500 μF equivalent branch is in parallel with the single 1000 μF capacitor, so these capacitances add.

$C_{eq} = 500 + 1000 = 1500$ μF.

Marking 1: 500 μF series pair; 1: 1500 μF bank.

6(b) The robot's electronics stop using the bank when its p.d. reaches 9.0 V. Calculate the energy released by bank A between 18.0 V and 9.0 V. **[2 marks]**

Step 1 Calculate the energy before discharge

For bank A, C = 2200 μF = 0.002200 F. Use $W = \frac{1}{2}CV^2$ at the initial p.d. of 18.0 V.

$W_{initial} = \frac{1}{2} \times 0.002200 \times 18.0^2 = 0.3564$ J.

Step 2 Calculate the energy left at the cut-off

The electronics stop at 9.0 V, so the capacitor still stores energy that cannot be used for the specified jobs.

$W_{final} = \frac{1}{2} \times 0.002200 \times 9.0^2 = 0.0891$ J.

Step 3 Find the energy released

Subtract the energy still stored from the starting energy.

$\Delta W = W_{initial} - W_{final} = 0.3564 - 0.0891 = 0.2673$ J.

Using the full 0.3564 J would include energy remaining below the operating cut-off.

Marking 1: difference of stored energies; 1: 0.2673 J.

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6 Decide what the robot can use

From 18.0 V down to 9.0 V, 70% of the energy released by the bank reaches the robot's useful jobs. The cartridge closure needs 0.100 J and the final record needs 0.0400 J. Both jobs must be completed.

For the same voltage interval, the energy released is 0.200475 J for B, 0.5346 J for C and 0.18225 J for D. Use your answer for A from 6(b).

6(c) Calculate the useful energy delivered by A and decide whether it meets the combined demand. **[2 marks]**

Step 1 Apply efficiency to the energy released

Only 70% of the released 0.2673 J reaches the useful jobs. Multiply by the efficiency once.

$W_{useful} = 0.70 \times 0.2673 = 0.18711$ J.

Step 2 Compare with the combined useful demand

Both jobs must be completed, so add their energy requirements.

$W_{required} = 0.100 + 0.0400 = 0.1400$ J.

0.18711 J > 0.1400 J. Bank A meets the nominal energy demand, supplying about 0.0471 J more useful energy than required.

Marking 1: 0.187 J useful; 1: comparison with combined 0.140 J.

6(d) Determine the minimum bank capacitance needed to deliver 0.140 J usefully over this voltage interval at 70% efficiency. **[2 marks]**

Step 1 Work back from useful energy to released energy

Useful energy = efficiency × released energy. The capacitor must release more energy than the useful output because the efficiency is below 100%. Divide by 0.70.

$\Delta W_{required} = 0.140/0.70 = 0.200$ J.

Step 2 Use the energy difference over the permitted voltage interval

$\Delta W = \frac{1}{2}C(18.0^2 - 9.0^2)$.

At the minimum capacitance the released energy just equals the required 0.200 J:

$0.200 = \frac{1}{2}C(324 - 81) = \frac{1}{2}C \times 243$.

Step 3 Make C the subject

Multiply both sides by 2, then divide by 243.

$C = (2 \times 0.200)/243 = 0.00164609$ F.

Since 1 F = 1000 mF, C = 1.64609 mF ≈ 1.65 mF.

This is the minimum actual bank capacitance for the stated voltage interval and efficiency. It does not yet allow for a lower-than-nominal capacitance.

Marking 1: correct energy/efficiency equation or rearrangement; 1: 1.65 mF.

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7 Compare all four designs

7(a) Compare A–D at their nominal values. State which satisfy both the useful-energy demand and their component voltage ratings. Support your decisions numerically. **[3 marks]**

Step 1 Compare useful energies with the same 0.140 J demand

The supplied figures are released energies. Multiply each by 0.70 to compare useful output with useful demand.

A: 0.70 × 0.2673 = 0.18711 J, which exceeds 0.140 J.

B: 0.70 × 0.200475 = 0.1403325 J, which just exceeds 0.140 J.

C: 0.70 × 0.5346 = 0.37422 J, which exceeds 0.140 J.

D: 0.70 × 0.18225 = 0.127575 J, which is below 0.140 J.

Step 2 Check the p.d. across each actual component

A has 18.0 V across its one 25 V component: its rating is satisfied.

B has 9.0 V across each matched series component, below 16 V: both ratings are satisfied.

C has the full 18.0 V across each parallel component, above 16 V: both ratings are exceeded.

D has 9.0 V across each matched component of its series pair and 18.0 V across its single parallel component. These are all below 25 V, so its ratings are satisfied.

Step 3 Require both conditions to pass

Only A and B satisfy both conditions at nominal values. C fails the component ratings; D fails the useful-energy requirement. B has very little nominal energy margin.

Marking 1: correct useful-energy comparison; 1: voltage-rating decisions; 1: A and B only pass both.

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7 Test the margin and the measurement

For this tolerance test only, each bank's actual capacitance is 10% below its nominal equivalent value. Matched series components remain matched, so nominal voltage-sharing ratios are unchanged.

7(b) Decide whether A and B still meet the useful-energy demand at the lower capacitance. Recommend one bank. **[2 marks]**

Step 1 Apply the lower-capacitance factor

For fixed voltage limits and efficiency, useful energy is proportional to C because $\Delta W = \frac{1}{2}C(V_{high}^2 - V_{low}^2)$. A 10% reduction in C leaves 90% of the nominal useful energy.

A: $W_{useful} = 0.90 \times 0.18711 = 0.168399$ J.

B: $W_{useful} = 0.90 \times 0.1403325 = 0.12629925$ J.

Step 2 Compare with 0.140 J and recommend a bank

A still exceeds 0.140 J; B is now below 0.140 J. Under the stated matched-component assumption, the voltage-sharing ratios and rating decisions are unchanged.

Recommend A. It retains enough useful energy in this lower-capacitance case; B's small nominal margin is insufficient.

Marking 1: lower-capacitance energy values; 1: A recommended with correct comparison.

7(c) Describe how a controlled constant-current charging test, using an ammeter, voltmeter and timer, could check the equivalent capacitance of the chosen bank. Include one useful measurement precaution. **[3 marks]**

Start with the bank discharged. Connect it to a controlled constant-current charging source, with an ammeter in series and a high-resistance voltmeter across the bank. Record the current I, charging interval Δt and change in p.d. ΔV.

The transferred charge is $\Delta Q = I\Delta t$, so $C = I\Delta t/\Delta V$. Check that current stays constant, rather than using one instantaneous current in an ordinary resistor-charging circuit.

For example, repeat the test over a substantial measurable voltage interval below the ratings, keeping the current constant; or use sufficiently sensitive meters/timing to reduce relative uncertainty. Discharge the bank before each repeated test. Accept another specific justified precaution.

Marking 1: workable constant-current measurement arrangement; 1: $C = I\Delta t/\Delta V$ analysis; 1: relevant justified precaution.

Use consistent signs and retain enough digits during working. Small graph-reading differences and physically equivalent explanations are acceptable where the method is sound.

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