Capacitors 3

In graphs charging and equation manipulation

35–40 minutes independent work + 15–20 minutes review

OCR A H556 and AQA 7408 | A-level physics

Specification and route

OCR A H556: 6.1.1(e)(i), 6.1.2(b), 6.1.3(a)–(c) and (e). Optional modelling: 6.1.3(d).

AQA 7408: 3.7.4.3–3.7.4.4; energy, logarithmic graph skills and RP9 preparation. The separately labelled numerical-model extension is not assumed AQA recall.

A-level only. This is capacitor application; the separate ln and log skills worksheet provides extra preparatory practice.

21 core parts | 41 marks | Optional extension: 3 marks

A remote volcano-monitoring station has become unreliable between measurement cycles. Engineers bench-test its sealed capacitor module, infer its capacitance from logged discharge data and set an appropriate recharge interval. Use the switching circuit in Figure 1; the capacitor value is initially unknown.

Show your working, include units and justify decisions using the information given.

The monitoring-station test

The switch is held at A, then moved to B when the logger clock reads 15.0 s. The measured discharge resistance is R = (150 ± 2) kΩ. The logger has input resistance at least 100 MΩ; neglect its loading and capacitor leakage for the initial analysis.

Use $V = V_0 \exp(-t/RC)$ for discharge and $V_C = V_S[1 - \exp(-t/RC)]$ for charging from zero. Here t is elapsed time from switching. In logarithms, divide a voltage by a stated voltage reference so the argument is dimensionless.

1 Choose and transform the equation

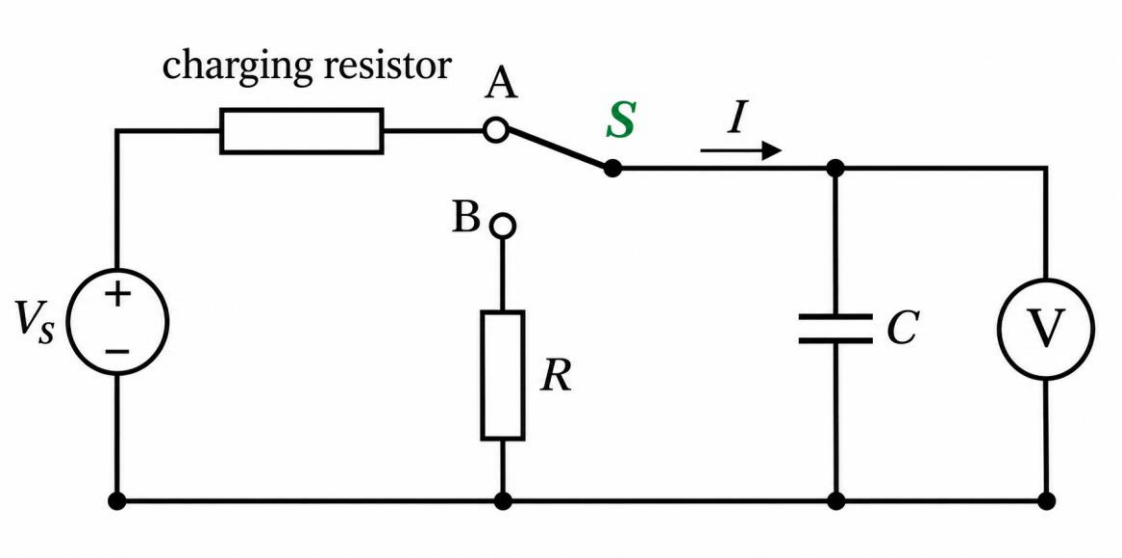


Figure 1 The voltage logger is connected across C. At B the supply is isolated and R determines the nominal discharge path.

1(a) For the readings taken after switching to B, identify the appropriate exponential model and explain your choice. **[1 mark]**

1(b) Starting from $V/V_0 = \exp(-t/RC)$, take ln of both sides to obtain a linear relationship. **[1 mark]**

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2 Linearise the discharge equation

2(a) Show that $y = \ln(V / 1\ \text{V})$ plotted against elapsed t gives a straight line. State its gradient and intercept. **[2 marks]**

2(b) If $\ln(V/V_0)$ is plotted instead, what is its intercept when t is measured from the start of discharge? Explain. **[1 mark]**

2(c) A fit using t in seconds has gradient $-0.01710\ \text{s}^{-1}$. Give the numerical gradient if t is plotted in milliseconds. Also state how the intercept changes if $\ln(V / 1\ \text{mV})$ replaces $\ln(V / 1\ \text{V})$. **[2 marks]**

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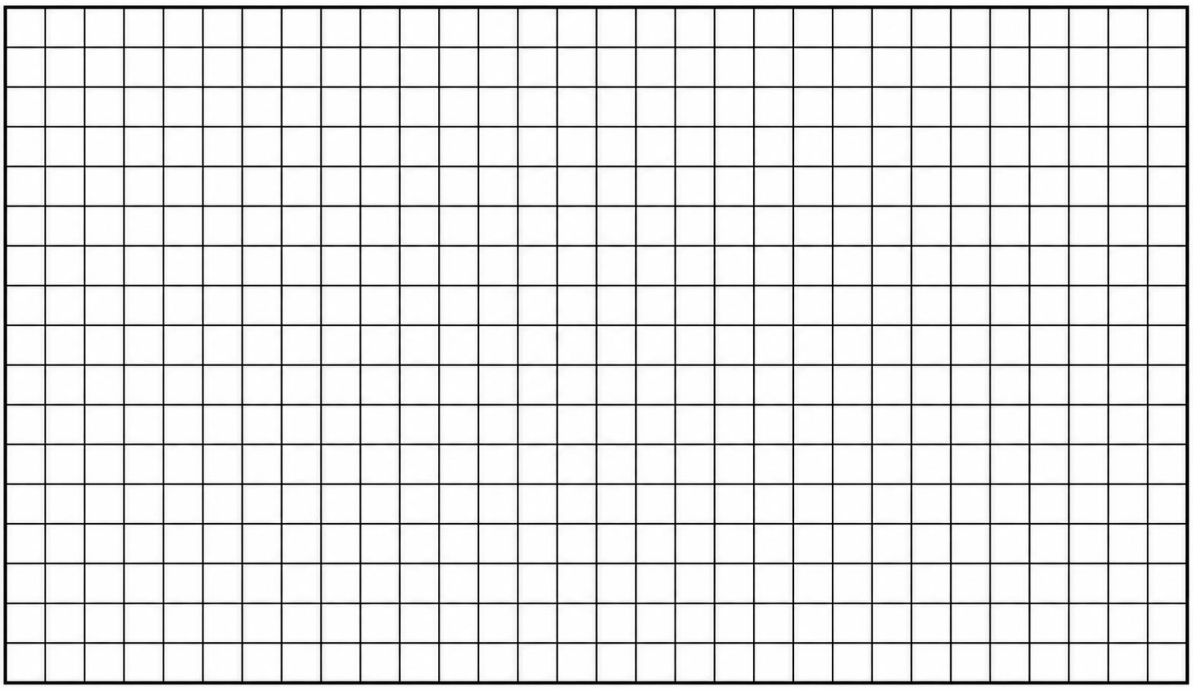
3 Construct the ln graph

The table contains measured-style data. Each p.d. reading has uncertainty ±0.02 V. Complete the elapsed-time and ln columns; use t = logger clock − 15.0 s. Give ln values to three decimal places.

Clock / s	V / V	Elapsed t / s	ln(V / 1 V)
15	6.60		
27	5.37		
39	4.39		
51	3.56		
63	2.91		
75	2.36		
87	1.93		

3(a) Complete the two missing columns in the table. **[2 marks]**

3(b) Plot ln(V / 1 V) against elapsed t and draw a straight line of best fit. Use labelled linear axes and a scale that makes good use of the grid. **[2 marks]**



Choose and label suitable linear scales. Plot from the exact data table.

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Extract the model parameters

Use your fitted line, not a gradient between two individual data points.

3(c) Use a large triangle on your fitted line to find its gradient and hence determine RC. **[2 marks]**

3(d) Use R = 150 kΩ to find C. Use the graph intercept to recover the initial p.d. V_0. **[2 marks]**

4(a) The label says 470 μF ±10%. For this evaluation use |m| = 0.01710 ± 0.00040 s^{-1} and R = 150 ± 2 kΩ. Add percentage uncertainties to estimate the uncertainty in C, then judge whether the inferred value agrees with the label. **[3 marks]**

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4 Evaluate what the test establishes

4(b) Explain why the late, low-voltage readings have a larger relative uncertainty than the early readings, even though every p.d. reading has the same ±0.02 V uncertainty. **[1 mark]**

4(c) Could the discharge trace alone prove that the capacitor is the faulty component? Explain how an unintended parallel path could affect the inference. **[2 marks]**

4(d) A logger later displays 0.00 V. Explain why this reading cannot simply be included in a ln(V / 1 V) plot. **[1 mark]**

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5 Transform the charging curve

For the following charging tests, use a stated design capacitance of 390 μF and a charging resistor of 100 kΩ. The ideal supply has V_S = 6.60 V. This charging resistor replaces the unspecified charging resistor in Figure 1; the discharge resistor R is isolated at A.

For an initially uncharged capacitor, $V_C = V_S[1 - \exp(-t/RC)]$.

5(a) Define the remaining voltage gap $U = V_S - V_C$. Show that ln(U / 1 V) plotted against t is a straight line, and identify its gradient and intercept. **[2 marks]**

5(b) Explain why $\ln(V_C / 1\ V)$ itself is not the correct straight-line transformation for charging from zero. **[1 mark]**

5(c) A straight-line fit for ln(U / 1 V) against t in seconds has gradient $-0.0256\ s^{-1}$. Determine the charging time constant. **[1 mark]**

5(d) When V_C = 4.00 V, calculate the charging current. Explain why using V_C/R would give the wrong answer. **[2 marks]**

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6 Solve the timing conditions

Use RC = 58.5 s for the discharge calculations in 6(a) and 6(c). For charging in 6(b), use RC = 39.0 s and V_S = 6.60 V.

6(a) Starting at 6.60 V, determine the discharge time needed to reach 1.00 V. Show the logarithmic rearrangement. **[2 marks]**

6(b) Find the charging time from zero to 80% of V_S. In a second test the starting p.d. is V_i = 1.20 V: find the time to reach 5.90 V using $V_C = V_S - (V_S - V_i)\exp(-t/RC)$. **[3 marks]**

6(c) Show that the energy remaining during discharge is proportional to $\exp(-2t/RC)$. Find its half-time and compare it with the voltage half-time. **[2 marks]**

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7 Plan an independent investigation

An unknown capacitor is to be tested with a low-voltage dc supply, a suitable two-position switch, a fixed resistor, a resistance meter and a high-input-resistance voltage logger. Use your earlier graph work to organise the method; do not assume that listing apparatus is sufficient.

7 Describe how to determine the capacitance from a discharge and a ln graph. Include a labelled circuit sketch, the measurements and switching procedure, how the graph leads to C, and one justified point about measurement quality. **[6 marks]**

Labelled circuit sketch

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Optional extension Step modelling

OCR H556 6.1.3(d) / supplied extension for AQA. Allow about 5 extra minutes; this is outside the common timed route.

In a separate ideal model, RC = 60.0 s, Q_0 = 3.00 mC and Δt = 10.0 s. Each spreadsheet row uses $Q_{next} = Q_{now} - Q_{now}\,\Delta t/(RC)$.

E(a) Calculate Q after the first and second model steps. **[2 marks]**

E(b) Explain why using a smaller time step improves the approximation to exponential discharge. **[1 mark]**

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