physicsUK
Smarter practice. Stronger understanding

SOLUTIONS
ATTEMPT CHECK CORRECT IMPROVE

Capacitors 2 solutions

Discharge time constants and changing current

Matching answers and examiner-style commentary

OCR A H556 and AQA 7408 | A-level physics

Specification and route

OCR A H556: 6.1.1(a)–(b), 6.1.1(e)(i), 6.1.2(b), 6.1.3(a)–(c) and (e), selected discharge content.

AQA 7408: 3.7.4.1, 3.7.4.3 and 3.7.4.4, selected discharge content. Practical preparation supports RP9 / OCR PAG9.

A-level only. Charging algebra and ln graphs are developed in Capacitors 3.

22 core parts | 41 marks

A moving lunar-lander museum exhibit uses a capacitor timer to delay its reset after a visitor's command. A high-input-resistance controller resets the exhibit when the capacitor p.d. falls to a threshold. A separate supply powers the controller and the mechanism.

Original PhysicsUK worked solutions. Use the explanations to check the method, then use the separate marking guidance to award credit.

The exhibit timer

In Figure 1 the switch is first held at A until C reaches 9.0 V. It is then moved to B. The charging supply is ideal. During discharge, C = 470 μF and R = 68.0 kΩ. Treat meter/controller loading and leakage as negligible unless told otherwise.

The arrow I defines positive current into the upper capacitor plate during charging. In calculation questions asking for magnitude, give a positive value. In signed-current sketches, use this printed reference direction.

For discharge use $V = V_0 \exp(-t/RC)$, $Q = CV$ and $|I| = V/R$. Here t is elapsed time since moving to B. You may use the half-time $t_{1/2} = 0.693RC$.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

1 Explain and establish the starting values

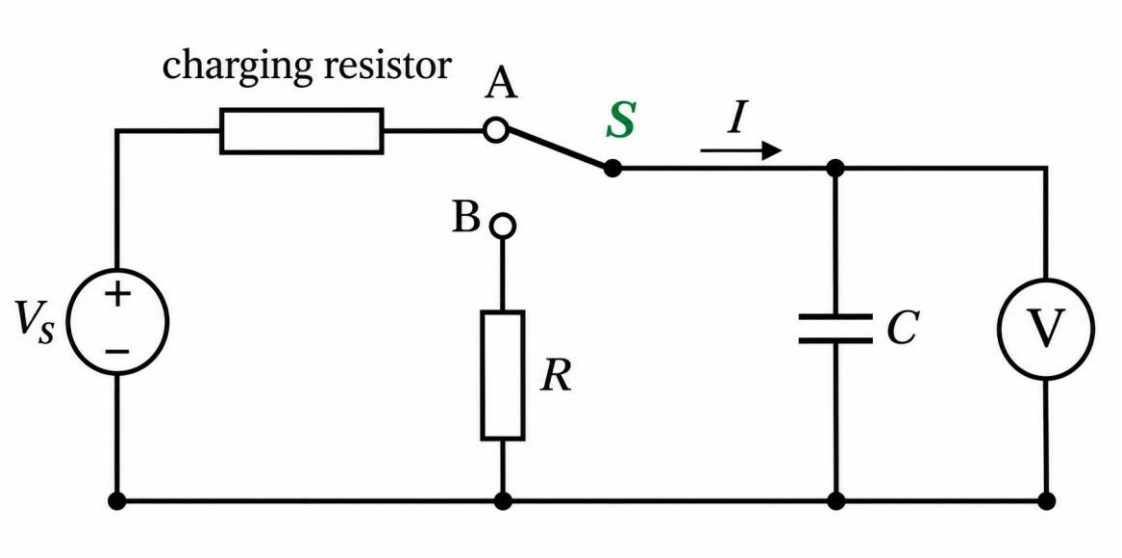


Figure 1 The switch is shown at A. At B the capacitor discharges through R. V represents a high-resistance voltmeter or voltage logger.

1(a) When the switch moves to B, identify the complete discharge path and explain why the charging supply does not drive this current. **[2 marks]**

The discharge loop runs from the upper capacitor plate through the switch at B, through R, along the bottom return and back to the lower plate.

Contact A is open, so the supply/charging-resistor branch is disconnected from the upper plate. The capacitor's stored energy drives the discharge, not the battery.

Marking 1: correct complete discharge loop; 1: A open / supply isolated from that loop.

1(b) Explain why the discharge current magnitude is largest initially and then decreases non-linearly, although R stays constant. **[2 marks]**

The initial capacitor p.d. is greatest, so $|I| = V/R$ is greatest then. As charge leaves the positive plate, Q decreases and $V = Q/C$ decreases.

With a smaller driving p.d., the current magnitude and rate of charge loss become smaller. The rate of decrease therefore changes: it is an exponential, non-linear discharge rather than a constant-rate fall.

Marking 1: decreasing Q gives decreasing V; 1: V/R and the changing discharge rate explain the shape.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

2 Establish the discharge values

2(a) Calculate the initial charge magnitude on either plate. **[1 mark]**

Use $Q_0 = CV_0$. Convert the capacitance to farads so the charge is obtained in coulombs.

$C = 470\ \mu F = 470 \times 10^{-6}\ F = 0.000470\ F$.

$Q_0 = 0.000470 \times 9.0 = 0.00423\ C = 4.23\ mC$.

This is the charge magnitude on either plate, not the net charge of the two plates together.

Marking 1: 4.23 mC.

2(b) Calculate the initial discharge current magnitude. **[1 mark]**

At the start of the isolated discharge, the resistor is directly across the capacitor, so its p.d. is the capacitor's initial 9.0 V. Use $|I_0| = V_0/R$.

$R = 68.0\ k\Omega = 68\ 000\ \Omega$.

$|I_0| = 9.0/68\ 000 = 0.00013235\ A$.

Multiply amperes by 10^6 to express this in microamperes: $|I_0| = 132\ \mu A$.

The question asks for a magnitude, so give a positive value.

Marking 1: 132 μA.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

2 Calculate and interpret the time constant

2(c) Calculate RC and state the physical meaning of this time constant during discharge. **[2 marks]**

Step 1 Convert R and C to SI units

R = 68 000 Ω; C = 0.000470 F.

Step 2 Multiply to find the time constant

RC = 68 000 × 0.000470 = 31.96 s ≈ 32.0 s.

Step 3 Give its physical meaning

At t = RC, the exponential factor is exp(−t/RC) = exp(−1) = 1/e ≈ 0.368.

The time constant is therefore the time for the p.d., plate-charge magnitude or current magnitude to fall to 1/e, about 37%, of its initial value. It is not the time until the capacitor is empty.

Marking 1: 32.0 s; 1: physical 1/e definition.

2(d) Determine the p.d. and charge magnitude after one time constant. **[2 marks]**

Step 1 Find the p.d. after one time constant

Putting t = RC into $V = V_0 \exp(-t/RC)$ makes the exponent −1.

V = 9.0 × exp(−1) = 9.0 × 0.367879 = 3.31091 V ≈ 3.31 V.

Step 2 Find the remaining plate charge

Use Q = CV at this new p.d.; the capacitance is still 0.000470 F.

Q = 0.000470 × 3.31091 = 0.00155613 C = 1.56 mC.

Both Q and V have fallen to the same fraction of their initial values because C is constant.

Marking 1: 3.31 V; 1: 1.56 mC.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

3 Calculate and check the measured data

The switch moved to B when the logger clock read 18.0 s. The table gives rounded p.d. readings during that discharge. Use these exact table values for calculations.

Logger clock / s	V / V
18	9
26	7.01
34	5.46
42	4.25
50	3.31
58	2.57
66	2
82	1.21
98	0.737

3(a) Use the exponential law to calculate the capacitor p.d. 16.0 s after switching to B, as a check on the table value. **[1 mark]**

Use the elapsed 16.0 s since switching to B. The time constant is 31.96 s from 2(c).

$V = V_0 \exp(-t/RC) = 9.0 \times \exp(-16.0/31.96)$.

The exponent is $-16.0/31.96 = -0.500626$, giving an exponential factor of 0.606151.

$V = 9.0 \times 0.606151 = 5.45536\ V \approx 5.46\ V$.

This agrees with the table value at logger clock 34 s, since 34 − 18 = 16 s of discharge.

Marking 1: 5.46 V.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

3 Check charge current and the decay pattern

3(b) At that same instant, calculate the charge magnitude and discharge current magnitude. **[2 marks]**

Step 1 Use the p.d. at the same physical instant

At 16.0 s the unrounded p.d. from 3(a) is about 5.45536 V. Keep extra digits until the final answers.

Step 2 Calculate charge from Q = CV

Q = 0.000470 × 5.45536 = 0.00256402 C.

Q = 2.56 mC.

Step 3 Calculate current magnitude using the resistor p.d.

The discharge resistor has the same 5.45536 V across it as the capacitor, so |I| = V/R.

|I| = 5.45536/68 000 = 0.0000802259 A = 80.2 μA.

The signed current is negative relative to Figure 1, but this question requests its positive magnitude. Allow sensible follow-through from 3(a).

Marking 1: 2.56 mC; 1: 80.2 μA; allow follow-through from 3(a).

3(c) Use successive readings separated by 16 s to test whether the data are consistent with exponential decay. Show two ratios. **[2 marks]**

Step 1 Choose equal time intervals

Use logger clocks 18 s, 34 s and 50 s. Each interval is 16 s, with p.d.s 9.00 V, 5.46 V and 3.31 V.

Step 2 Divide each later p.d. by the preceding p.d.

First remaining fraction = 5.46/9.00 = 0.606667 ≈ 0.607.

Second remaining fraction = 3.31/5.46 = 0.606227 ≈ 0.606.

Step 3 Interpret the ratios

The ratios are approximately equal within the precision of the rounded data. Nearly the same fraction remains in equal time intervals, supporting exponential decay. The decreases in volts need not be equal.

Marking 1: two correct equal-interval ratios; 1: constant-ratio conclusion.

4(a) On the grid, plot V against logger clock time and draw a smooth curve. Choose labelled linear scales and use at least six readings. **[2 marks]**

Plot the tabulated pairs, including (18, 9.00), (34, 5.46), (50, 3.31), (66, 2.00), (82, 1.21) and (98, 0.737). Join the trend with a smooth decreasing curve whose gradient becomes less negative.

The curve starts at clock time 18 s. Do not force it through V = 0 at a later reading: the ideal discharge approaches zero asymptotically.

Marking 1: labelled linear axes/scales and correct plots; 1: smooth appropriate curve.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

4 Extract time and current from your graph

Use your graph from 4(a). Show constructions on it where appropriate.

4(b) Determine RC from the graph using the time to fall from 9.00 V to 9.00/e V. Show how the logger readings give an elapsed time. **[2 marks]**

Step 1 Calculate the target p.d.

One time constant is the elapsed time for V to fall to V_0/e.

$V_0/e = 9.00/e = 3.3109\ V \approx 3.31\ V$.

Step 2 Read the two clock coordinates from the graph

The discharge begins at clock 18 s. The curve reaches about 3.31 V at clock 50 s.

Step 3 Subtract to obtain the duration

$RC \approx 50 - 18 = 32\ s$.

The clock coordinate 50 s is not the time constant: it includes 18 s before discharge began. Accept about 30–34 s from a sensible plotted graph.

Marking 1: target level/read-off evidence; 1: elapsed interval about 32 s. Do not award the graph-method mark for simply repeating calculated RC.

4(c) Draw a tangent at clock time 50 s. Estimate its gradient and hence the signed current I at that instant, using I = C × gradient of V against time. **[2 marks]**

Step 1 Use a tangent to obtain the instantaneous rate

Draw a tangent to the curve at clock time 50 s, where V is about 3.31 V. Choose two well-separated points on that straight tangent. They need not be measured data points.

For example, a suitable tangent passes approximately through (30 s, 5.39 V) and (70 s, 1.23 V).

Step 2 Calculate its gradient

Change in p.d. = 1.23 − 5.39 = −4.16 V.

Change in time = 70 − 30 = 40 s.

Gradient = −4.16/40 = −0.104 V s^{-1}.

Step 3 Convert the voltage rate into signed current

For fixed C, Q = CV gives I = C × gradient of V against time. Use $C = 470 \times 10^{-6}$ F.

$I = (470 \times 10^{-6}) \times (-0.104) = -0.00004888$ A.

I = −48.9 μA ≈ −49 μA.

The negative sign means current is opposite to the charging-current arrow in Figure 1. Use a tangent, not a chord through the curved data. Accept approximately −42 to −56 μA for a defensible tangent.

Marking 1: sensible tangent/negative gradient; 1: C × gradient with correct units/sign.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

4 Current signs and the reset time

4(d) Sketch the signed current I against elapsed discharge time. Label its initial value and the value after one RC. Show how it approaches zero. **[2 marks]**

Step 1 Place the initial signed current

The arrow defines positive charging current, so discharge current is negative. The initial magnitude from 2(b) is 132 μA: plot I = −132 μA at elapsed t = 0.

Step 2 Find the current after one RC

Current magnitude falls by the same exponential factor as p.d. At t = RC, the factor is 1/e.

I = −132.35/e ≈ −48.7 μA at t ≈ 32.0 s.

Step 3 Sketch the approach to zero

Draw an exponential curve rising from −132 μA towards zero from below, becoming less steep. It does not cross zero in the ideal model.

The signed current increases towards zero while its magnitude decreases. These statements describe different quantities.

Marking 1: correct negative starting/current-after-RC values; 1: correct curve approaching zero from below.

5 Set the reset delay

The controller resets at a capacitor p.d. of 2.25 V. The required delay is 42–47 s after switching to B. The controller draws negligible current from C.

5(a) Use the half-time relation to determine when the threshold is reached, and judge whether the delay is suitable. **[2 marks]**

Step 1 Count the halving intervals

The threshold fraction is 2.25/9.00 = 0.25. Falling to one quarter requires two halvings: 9.00 V to 4.50 V, then 4.50 V to 2.25 V.

Step 2 Calculate one half-time

Use the supplied relation $t_{1/2} = 0.693RC$.

$t_{1/2} = 0.693 \times 31.96 = 22.1483$ s.

Step 3 Calculate and assess the total delay

Delay = 2 × 22.1483 = 44.2966 s ≈ 44.3 s.

This is within the required 42–47 s window, so the nominal delay is suitable.

The corresponding logger clock reading would be 18.0 + 44.3 = 62.3 s. The requested delay is the elapsed 44.3 s.

Marking 1: two half-times / 44.3 s; 1: justified suitability.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

5 Select the resistor and check the model

5(b) Engineers can choose R = 56.0 kΩ, 68.0 kΩ or 82.0 kΩ with the same capacitor. Determine the threshold delay for each and select R. **[2 marks]**

Step 1 Use the same threshold rule for every resistor

The starting p.d. and quarter-voltage threshold are unchanged, so each delay is two half-times: delay = 2 × 0.693RC. Keep C = 0.000470 F and express R in ohms.

Step 2 Calculate each delay

56.0 kΩ: RC = 56 000 × 0.000470 = 26.32 s.

Delay = 2 × 0.693 × 26.32 = 36.5 s.

68.0 kΩ: RC = 68 000 × 0.000470 = 31.96 s.

Delay = 2 × 0.693 × 31.96 = 44.3 s.

82.0 kΩ: RC = 82 000 × 0.000470 = 38.54 s.

Delay = 2 × 0.693 × 38.54 = 53.4 s.

Step 3 Select against the stated requirement

Choose 68.0 kΩ: only its 44.3 s delay lies within 42–47 s. The smaller resistor resets too early; the larger one resets too late.

Marking 1: valid delay comparisons; 1: 68.0 kΩ alone satisfies the window.

5(c) Explain why this exponential model could not simply be applied to a real lamp or the exhibit motor in place of the fixed resistor. **[1 mark]**

The exponential model assumes a constant discharge resistance. A lamp's resistance changes with temperature, and a motor's electrical behaviour also depends on its motion. Such loads cannot be assumed to have the same constant R without evidence.

Marking 1: identifies the fixed-resistance assumption and a relevant changing-load effect.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

6 Change the settings

6(a) Compare doubling R with doubling C, starting each test at 9.0 V. State what happens to RC and to the initial current magnitude in each case. **[2 marks]**

Doubling R doubles RC, so the discharge takes twice as long to reach the same fraction. Since $|I_0| = V_0/R$, it halves the initial current.

Doubling C also doubles RC, but $|I_0| = V_0/R$ is unchanged because V_0 and R are unchanged. The larger C initially holds twice as much charge.

Marking 1: both RC changes; 1: correct contrasting initial-current changes.

6(b) Instead double the starting p.d., leaving R and C unchanged. The threshold is set to one quarter of this new starting p.d. State the effects on the initial current and threshold time. **[2 marks]**

The initial current magnitude doubles because $|I_0| = V_0/R$.

RC is unchanged, and reaching one quarter still requires two half-times, so the threshold time is unchanged. This conclusion uses a fractional threshold; a fixed absolute threshold would be a different condition.

Marking 1: initial current doubles; 1: unchanged fractional-threshold time with reasoning.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

6 Energy remaining and residual charge

6(c) For the original circuit, calculate the energy remaining after 16.0 s. Explain why the fraction of energy remaining is smaller than the fraction of p.d. remaining. **[2 marks]**

Step 1 Find the energy at the p.d. after 16.0 s

Use the unrounded V ≈ 5.45536 V from 3(a) and C = 0.000470 F. $W = \frac{1}{2}CV^2$ gives energy still stored, not energy already released.

$W = \frac{1}{2} \times 0.000470 \times 5.45536^2 = 0.00699383$ J.

W ≈ 6.99 mJ.

Step 2 Relate the energy fraction to the voltage fraction

At constant C, divide $W = \frac{1}{2}CV^2$ by $W_0 = \frac{1}{2}CV_0^2$. The common factor ½C cancels.

$W/W_0 = (V/V_0)^2$.

Voltage fraction = 5.45536/9.00 ≈ 0.606.

Energy fraction ≈ $0.606151^2 = 0.367$.

Squaring a fraction between zero and one makes it smaller. About 36.7% of the original energy remains, although about 60.6% of the original p.d. remains.

Marking 1: 6.99 mJ (allow follow-through); 1: squared-fraction reasoning.

7 Evaluate the timing test

7(a) An engineer says the capacitor is “empty after five RC”. Calculate the percentage of initial charge still present at that time and comment on this description. **[2 marks]**

At t = 5RC, the discharge equation gives the remaining fraction of charge:

$Q/Q_0 = \exp[-(5RC)/(RC)] = \exp(-5) = 0.00673795$.

Convert the fraction to a percentage by multiplying by 100.

Percentage remaining = 0.00673795 × 100 = 0.674%.

The charge may be negligible for a stated application, but it is not exactly zero in the exponential model. One RC leaves about 37%; five RC leaves about 0.674%.

Marking 1: 0.674%; 1: practical approximation distinguished from exact zero.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

7 Improve the method and test an extra path

7(b) A proposed investigation leaves the battery connected to the upper capacitor plate, uses a low-resistance voltmeter across C and takes only one late reading. Give three justified changes that would produce useful discharge data. **[3 marks]**

Move the switch to B so the supply is isolated from the upper plate. Otherwise the intended free-discharge model does not apply.

Use a high-input-resistance voltmeter/logger so it does not provide a significant extra parallel discharge path.

Record a series of readings from the switching instant at intervals short compared with RC, over a useful voltage range. One late reading cannot establish the decay curve or time constant. Accept a specific justified measurement improvement addressing the stated faults.

Marking 1 each: isolation with reason; reduced loading with reason; suitable time series with reason.

7(c) An unintended 68.0 kΩ resistor is connected in parallel with the original discharge resistor. Find the new RC and explain the effect on the reset delay. **[2 marks]**

Step 1 Calculate the resistance of the parallel paths

The two 68.0 kΩ resistors provide two discharge paths. Their reciprocals add.

$1/R_{effective} = 1/68\,000 + 1/68\,000 = 2/68\,000$.

$R_{effective} = 34\,000\ \Omega = 34.0\ k\Omega$.

Step 2 Find the new time constant

The capacitance stays at 0.000470 F. Use the effective resistance, since both paths remove charge.

$RC = 34\,000 \times 0.000470 = 15.98\ s \approx 16.0\ s$.

Step 3 Calculate the new reset delay

The threshold is still one quarter of the initial p.d., so the delay is two half-times.

$\text{Delay} = 2 \times 0.693 \times 15.98 = 22.1483\ s \approx 22.1\ s$.

This is half the original delay and is below the required 42–47 s. The exhibit resets too early because the extra path allows charge to leave faster.

Marking 1: RC ≈ 16.0 s; 1: half-delay / too-early reset with explanation.

Use consistent signs and retain enough digits during working. Small graph-reading differences and physically equivalent explanations are acceptable where the method is sound.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk