Binding energy per nucleon solutions

From the binding-energy curve to reaction energy

Matching answers and examiner-style commentary

OCR A H556 | A-level physics | Module 6 | 38 marks

Specification points covered

6.4.4(e): binding energy per nucleon, its variation with nucleon number, and interpretation for fusion and fission.

Supporting 6.4.4(a), (b), (d): mass–energy, total binding energy and nuclear-energy calculations.

Prerequisites: nucleon/proton numbers and balanced nuclear equations. Numerical calculations use the supplied tables, not a scale read from the qualitative graph.

Write $b = E_B/A$ for binding energy per nucleon. Use 1 MeV = 1.602×10^{-13} J. A free proton or neutron has zero nuclear binding energy. Use the supplied rounded illustrative values throughout.

1 Compare like with like

A spacecraft-power research team compares nuclear reaction proposals. It must distinguish energy per nucleus, energy per nucleon and energy per complete reaction.

Figure 1 shows the broad trend. Fine structure and individual isotopes are not represented; use the data supplied for every numerical calculation.

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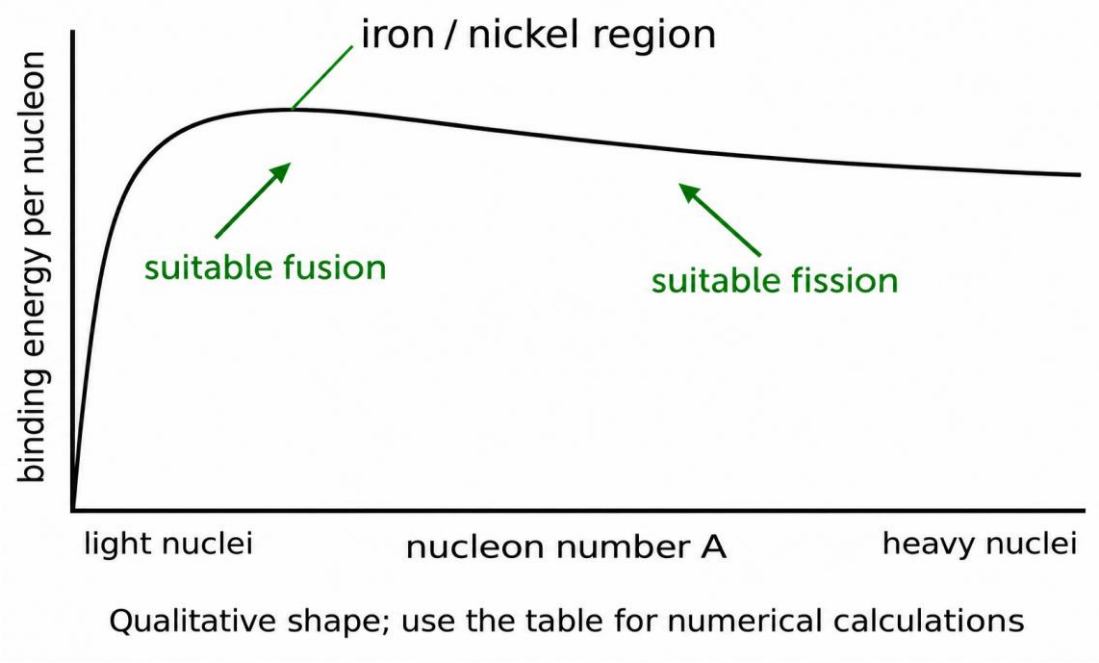


Figure 1 Suitable fusion or fission can move products towards greater binding per nucleon.

1(a) A model nucleus has total binding energy 522 MeV and A = 60. Calculate its binding energy per nucleon, giving an appropriate unit. **[1 mark]**

$b = E_B/A = 522/60 = 8.70$ MeV nucleon^{-1}.

The unit shows that a total energy has been divided by a nucleon count.

Marking guidance 1: 8.70 MeV nucleon^{-1}.

1(b) A second model nucleus has A = 200 and b = 7.90 MeV nucleon^{-1}. Calculate its total binding energy and decide which of the two nuclei has the greater average binding per nucleon. **[2 marks]**

For the second nucleus, $E_B = Ab = 200(7.90) = 1580$ MeV.

The A = 60 nucleus has the greater average binding per nucleon, because 8.70 > 7.90 MeV nucleon^{-1}, even though its total binding energy is smaller.

Marking guidance 1: 1580 MeV. 1: A = 60 nucleus correctly selected using b.

2 Read the physical meaning of the curve

2(a) Describe the broad change in binding energy per nucleon as A increases from very light nuclei through the iron/nickel region to very heavy nuclei. **[2 marks]**

Binding energy per nucleon rises steeply overall among very light nuclei, reaches a broad maximum in the iron/nickel region, then decreases gradually towards very heavy nuclei.

This is a qualitative trend, not a claim that every successive isotope lies above its neighbour or that iron-56 is uniquely the highest point.

Marking guidance 1: steep light-nucleus rise towards a maximum. 1: iron/nickel region followed by gradual heavy-nucleus decline.

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2(b) Explain why some fusion reactions between light nuclei release energy. Refer to total binding energy as well as the curve. **[2 marks]**

For a suitable reaction, the products have a greater total binding energy than the reactants. The products are then at lower total rest energy, and the difference is released.

The rising light-nucleus part of the curve helps identify promising reactions, but the calculation must weight each b value by its own A and account for every product.

Marking guidance 1: greater product total binding energy linked to lower rest energy/release. 1: curve trend interpreted with correct total-energy accounting.

2(c) Explain why splitting a very heavy nucleus into suitable medium-mass products can release energy. **[2 marks]**

The medium-mass products can have greater binding energy per nucleon than the very heavy parent. After multiplying by the nucleon numbers and including all products, their combined binding energy can be greater.

For the same overall protons and neutrons this means a decrease in rest energy, released as kinetic energy and radiation. Fission does not release energy simply because a large nucleus is broken.

Marking guidance 1: medium-mass products more tightly bound per nucleon. 1: increased total binding/lower rest energy gives release.

2(d) Why is it incorrect to read a height from this qualitative figure and quote it as a three-significant-figure reaction energy? **[1 mark]**

The graph has no numerical scale and shows a broad trend, not precise isotope data. A reaction energy also requires weighted total binding energies of every reactant and product. Use the supplied numerical table.

Marking guidance 1: unscaled qualitative shape cannot provide precise numerical values.

3 Calculate a complete fusion energy

Table 1 Rounded binding data for the fusion model

Nuclide	A	b / MeV nucleon^{-1}
Deuterium	2	1.112
Tritium	3	2.827
Helium-4	4	7.074
Free neutron	1	0

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3(a) For deuterium (A = 2), tritium (A = 3) and helium-4, calculate the total binding energies from Table 1. **[2 marks]**

Multiply each per-nucleon value by its own A.

Deuterium: 2(1.112) = 2.224 MeV. Tritium: 3(2.827) = 8.481 MeV. Helium-4: 4(7.074) = 28.296 MeV.

Marking guidance 1: both reactant totals. 1: helium total, each with A weighting.

3(b) The proposed reaction is deuterium plus tritium forming helium-4 and a neutron. Write the balanced nuclear equation. **[1 mark]**

$^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}.$

A totals 5 on each side and Z totals 2 on each side.

Marking guidance 1: balanced nuclides including the free neutron.

3(c) Calculate the rest-energy release Q per reaction from the binding energies, showing the separate reactant and product sums. **[3 marks]**

Step 1 Add reactant total binding energies: $E_{B,initial}$ = 2.224 + 8.481 = 10.705 MeV.

Step 2 Add product total binding energies: $E_{B,final}$ = 28.296 + 0 = 28.296 MeV. The free neutron has no nuclear binding energy.

Step 3 Q = $E_{B,final} - E_{B,initial}$ = 17.591 MeV ≈ 17.6 MeV, released.

Increased binding means lower rest energy; this sign is the opposite of final-minus-initial rest energy.

Marking guidance 1: correct initial total. 1: correct final total including zero free-neutron binding. 1: +17.6 MeV released.

3(d) Convert the fusion energy release to joules and calculate the energy release per reacting nucleon, counting all five initial nucleons. **[2 marks]**

Q = 17.591(1.602 × 10^{-13}) = 2.8180782 × 10^{-12} J ≈ 2.82 × 10^{-12} J per reaction.

Energy per reacting nucleon = 17.591/5 = 3.5182 MeV ≈ 3.52 MeV per reacting nucleon.

This reaction-energy normalisation is different from the final helium nucleus's binding energy per nucleon.

Marking guidance 1: 2.82 × 10^{-12} J. 1: 3.52 MeV per reacting nucleon.

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4 Apply the same method to a fission model

Table 2 Illustrative fission-model binding values

Species	A	b / MeV nucleon^{-1}
Compound uranium	236	7.60
Strontium product	94	8.60
Xenon product	140	8.40

4(a) A compound uranium-236 nucleus (Z = 92) splits into strontium-94 (Z = 38), xenon-140 (Z = 54) and free neutrons. Determine the number of free neutrons and write the balanced nuclear equation. **[2 marks]**

The two bound products contain 94 + 140 = 234 nucleons, so two free neutrons are needed to conserve A = 236.

$^{236}_{92}\mathrm{U} \rightarrow {}^{94}_{38}\mathrm{Sr} + {}^{140}_{54}\mathrm{Xe} + 2\ {}^{1}_{0}\mathrm{n}.$

The proton total is 38 + 54 = 92. The uranium-236 compound nucleus is already the starting system here; do not add an extra initial neutron.

Marking guidance 1: two neutrons. 1: correctly balanced complete equation.

4(b) Use Table 2 to calculate the initial total binding energy and the final combined total binding energy. **[2 marks]**

Initial: $E_{B,initial}$ = 236(7.60) = 1793.6 MeV.

Final: $E_{B,final}$ = 94(8.60) + 140(8.40) + 2(0) = 808.4 + 1176.0 = 1984.4 MeV.

A free neutron contributes rest mass to a mass balance but zero nuclear binding energy to this binding-energy sum.

Marking guidance 1: initial 1793.6 MeV. 1: final 1984.4 MeV with both products weighted.

4(c) Calculate Q per fission in MeV and the energy release per initial nucleon. Compare the latter with the fusion value from 3(d). **[2 marks]**

Q = 1984.4 − 1793.6 = 190.8 MeV per fission.

Per initial nucleon: 190.8/236 = 0.808475 MeV ≈ 0.808 MeV.

Fusion in 3(d) releases about 3.52 MeV per initial nucleon, so this fusion model gives more energy per reacting nucleon even though one fission releases more energy per event.

Marking guidance 1: Q = 190.8 MeV. 1: 0.808 MeV per initial nucleon and correct comparison.

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4(d) A colleague instead averages 8.60 and 8.40, subtracts 7.60, then multiplies by 236 to obtain 212.4 MeV. Identify two mistakes in that method. **[2 marks]**

The two bound products contain different numbers of nucleons (94 and 140), so an unweighted average of their b values is inappropriate.

Two final nucleons are free neutrons with zero binding energy. Assigning all 236 nucleons the bound-product average incorrectly gives those free neutrons binding energy. Sum Ab separately for every species instead.

Marking guidance 1: unequal product populations need A weighting. 1: two free neutrons must have zero binding contribution.

5 Check the limits of a quick conclusion

5(a) For a supplied model, two helium-4 nuclei form a beryllium-8 nucleus. Use b(He-4) = 7.074 and b(Be-8) = 7.062 MeV nucleon^{-1} to calculate Q. Does this fusion model release energy? **[2 marks]**

Both sides contain eight nucleons. Initial total binding energy = 2 × 4 × 7.074 = 56.592 MeV.

Final total binding energy = 8 × 7.062 = 56.496 MeV.

Q = 56.496 − 56.592 = −0.096 MeV. Energy is required in this model; not every fusion reaction releases energy.

Marking guidance 1: correctly weighted totals. 1: Q = −0.096 MeV, energy required.

5(b) Two candidate nuclei have A = 56, b = 8.790 and A = 62, b = 8.795 MeV nucleon^{-1}. Calculate each total binding energy, then explain which quantity answers “which has the greater mean binding per nucleon?”. **[2 marks]**

$E_B(56)$ = 56(8.790) = 492.240 MeV. $E_B(62)$ = 62(8.795) = 545.290 MeV.

The b values answer the mean-binding question: the A = 62 candidate has the slightly larger value, 8.795 MeV nucleon^{-1}. The larger total is not itself the reason.

Marking guidance 1: both total energies. 1: chooses the per-nucleon comparison and correct candidate.

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5(c) For the fusion and fission reactions above, the total numbers of protons and neutrons are separately unchanged. Explain why this lets us obtain Q from the increase in total binding energy. Why should that shortcut not be used uncritically for beta decay? **[2 marks]**

With the same total protons and neutrons, the separated-nucleon rest-energy reference is identical before and after. Greater binding reduces the system rest energy by the same amount, so Q equals the increase in total binding energy.

Beta decay changes a neutron into a proton or the reverse and creates leptons. Their rest-energy accounting changes, so a complete consistent mass/energy balance is needed; the binding-energy increase alone is not generally the whole Q.

Marking guidance 1: common separated-nucleon reference explains the shortcut. 1: beta changes particle rest-energy accounting.

5(d) The fission-model team quotes Q = 190.800000 MeV from Table 2. Explain why the displayed precision is misleading. **[1 mark]**

The input binding energies are rounded (for example to 0.01 MeV per nucleon), and weighting by large A amplifies their absolute rounding effects. The arithmetic does not create extra measurement precision. Quote an appropriate rounded model result, such as about 191 MeV.

Marking guidance 1: rounded input data do not justify eight/six decimal-place precision.

6 Give a defensible recommendation

6(a) A proposal claims “fission is always the better energy source because 190.8 is larger than 17.6”. Evaluate that numerical argument using the models here, then identify one further piece of information needed for an actual spacecraft design choice. **[2 marks]**

The quoted values compare energy per event. A fission event consumes many more nucleons: the models give about 0.808 MeV per initial nucleon for fission and 3.52 MeV for fusion. Therefore the event comparison alone does not establish which has the better fuel-mass energy yield.

A real design also needs supplied evidence about achievable reaction rate, conversion efficiency, fuel/system mass, control or heat removal. The worksheet does not establish that either is a practical spacecraft power system.

Marking guidance 1: meaningful normalised comparison. 1: relevant additional design evidence.

6(b) A researcher says the nucleus with the larger b must always have the longer radioactive half-life. Explain why that conclusion does not follow from this worksheet’s data. **[1 mark]**

Binding energy per nucleon describes an average binding relative to free nucleons. Half-life depends on the particular allowed decay process and its probability; the b comparison alone does not supply that probability or establish every possible decay route.

Marking guidance 1: b alone does not determine decay probability/half-life.

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6(c) Write a short checking sequence that would prevent a student from confusing per-nucleon binding values with a reaction energy. Include the sign of the final result. **[2 marks]**

First balance the complete reaction, including any free nucleons. Convert each b to a total Ab and sum totals separately for reactants and products, giving free nucleons zero binding energy.

For reactions with the same total protons and neutrons, Q = final total binding energy − initial total binding energy. Positive Q is released energy; negative Q is an energy requirement. Quote units per reaction and round only at the end.

Marking guidance 1: balanced, species-by-species A weighting and sums. 1: correct sign/order and interpretation.

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