AC generators and flux linkage graphs solutions

Electromagnetism 5 Testing a research buoy generator

Matching answers and examiner-style commentary

OCR A H556 | A-level physics | Module 6 | 35 marks

Specification points covered

6.3.3(a–c), (d)(i): magnetic flux and flux linkage; induced e.m.f. and Faraday's and Lenz's laws.

6.3.3(e): a simple a.c. generator. Apply prior skills in period, frequency, gradients and energy transfers.

Use $\varepsilon = -\Delta(N\Phi)/\Delta t$ for an average e.m.f. Instantaneous e.m.f. is the negative gradient of the flux-linkage–time graph. Treat the stated field as uniform.

1 Power for a wave-monitoring buoy

A research buoy uses wave motion to drive a generator through a gearbox and flywheel. The tests below use a steady laboratory motor; do not assume the real waves produce constant rotation.

The rectangular coil has 180 turns, area 0.0120 m^2 per turn, and rotates in a uniform field of flux density 0.0900 T. Ignore resistance unless a later question states otherwise.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

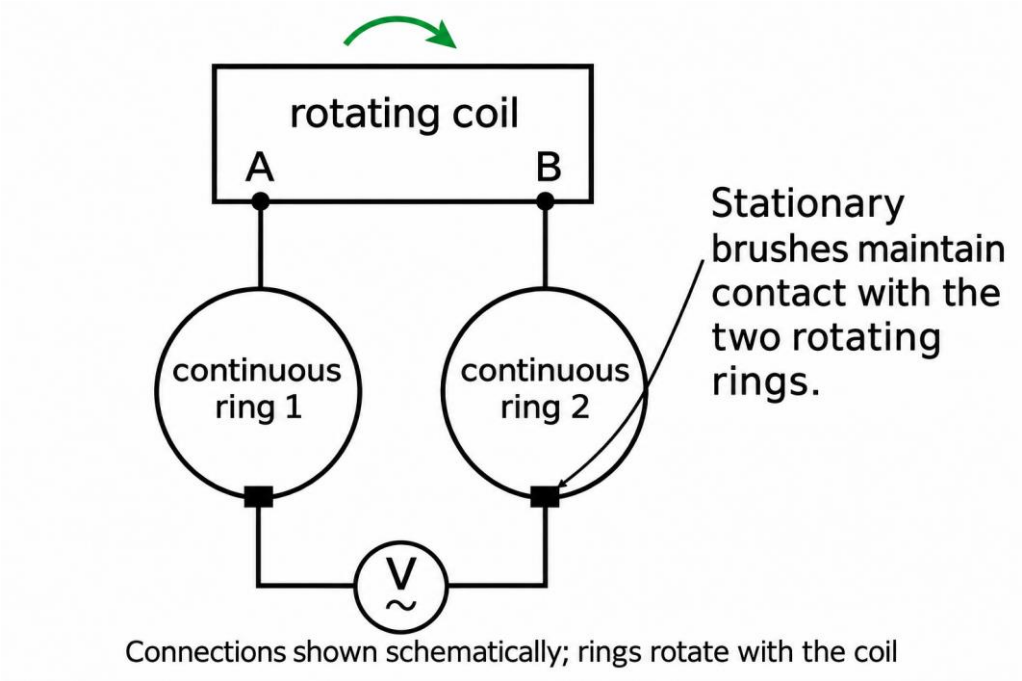


Figure 1 Stationary brushes contact separate continuous rings

1(a) A rotating coil is connected to two continuous slip rings and two brushes. Explain the roles of the rings and brushes. **[2 marks]**

Each end of the coil connects to its own continuous conducting ring. Stationary brushes maintain separate electrical connections to the rotating coil, so the external circuit is complete while the coil turns.

The rings do not swap the coil connections every half-turn; the induced alternating polarity is delivered to the external circuit.

Marking guidance One mark for maintained electrical contact during rotation; one for separate continuous connections to the two coil ends.

1(b) State the main energy transfer when the generator supplies the buoy's sensors. **[1 mark]**

Mechanical energy from the drive is transferred electrically to the sensors; some is dissipated as thermal energy in a real generator.

Marking guidance Mechanical to electrical energy transfer.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

2 Orientation and alternating output

2(a) At t = 0 the coil's area normal is parallel to B. Calculate the magnetic flux through one turn and the flux linkage. Explain why these are different quantities. **[2 marks]**

Step 1 Find the flux through one turn

$\Phi = BA\cos\theta = 0.0900 \times 0.0120 \times \cos 0° = 1.08 \times 10^{-3}$ Wb.

Step 2 Include all the turns

$N\Phi = 180 \times 1.08 \times 10^{-3} = 0.1944$ Wb. Flux linkage includes the 180 turns; flux is for one turn. Wb turn is also accepted for linkage.

Marking guidance One mark for $\Phi = 1.08 \times 10^{-3}$ Wb; one for linkage 0.1944 Wb with the one-turn/all-turn distinction.

2(b) The coil turns through 90° from its starting position. State the new flux linkage and explain the orientation used. **[1 mark]**

Zero. The area normal is now perpendicular to B, so $N\Phi = NBA\cos 90° = 0$. The plane of the coil is parallel to B.

Marking guidance Zero linkage with normal perpendicular to B, or plane parallel to B.

2(c) Explain why the magnitude of the e.m.f. is greatest when the flux linkage is zero, rather than when the flux linkage is greatest. **[2 marks]**

E.m.f. depends on the rate of change of flux linkage, not its value. For uniform rotation the linkage–time graph is steepest as the linkage passes through zero.

At a maximum or minimum of linkage the gradient is zero, so the instantaneous e.m.f. is zero.

Marking guidance Rate of change/gradient, not value; steepest at zero and zero slope at extrema.

2(d) Explain why the e.m.f. reverses during each full rotation. **[1 mark]**

The signed flux linkage first decreases and then increases. Its gradient changes sign, so Faraday's law gives an e.m.f. of the opposite polarity.

Marking guidance Reversal of the rate of change of signed linkage, or equivalent conductor-motion reasoning.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

3 Average e.m.f. is not the peak e.m.f.

3(a) The coil rotates at 12.0 Hz. Calculate its period and the time taken to turn through 60°. **[2 marks]**

Step 1 Use a complete turn

T = 1/f = 1/12.0 = 0.08333 s.

Step 2 Take the appropriate fraction of a turn

60° is one sixth of a full rotation, so Δt = T/6 = 0.01389 s.

Marking guidance Period 0.0833 s; time 0.0139 s.

3(b) Calculate the flux linkage after a 60° turn from t = 0. **[1 mark]**

NΦ = NBA cos 60° = 0.1944 × 0.500 = 0.0972 Wb.

Marking guidance 0.0972 Wb.

3(c) For this first 60° turn, calculate the signed average e.m.f. Use $\varepsilon_{av} = -\Delta(N\Phi)/\Delta t$ with the signed linkage just defined. **[2 marks]**

Step 1 Calculate the change, final minus initial

Δ(NΦ) = 0.0972 − 0.1944 = −0.0972 Wb.

Step 2 Divide by the elapsed time and include the minus sign

ε_{av} = −(−0.0972)/0.0138889 = +6.9984 V ≈ +7.00 V.

Marking guidance Correct change −0.0972 Wb; average +7.00 V including sign.

3(d) A pupil calls the answer to 3(c) “the peak e.m.f.” Explain why this conclusion is not justified. **[1 mark]**

The calculation used a finite linkage change divided by a finite time interval. It is the average e.m.f. over that interval. A peak value requires the greatest instantaneous gradient, not this chord gradient.

Marking guidance Average over interval distinguished from greatest instantaneous gradient.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

4 Construct and read the graphs

Keep f = 12.0 Hz. Use the unrounded model values where helpful. The graph data refer to the ideal induced e.m.f., with no load connected.

Table 1 One steady rotation; rounded values

Time / ms	NΦ / Wb	ε / V
0	+0.1944	0
20.83	0	+14.66
41.67	−0.1944	0
62.50	0	−14.66
83.33	+0.1944	0

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

4(a) Use Table 1 to sketch smooth flux-linkage–time and e.m.f.–time graphs for one turn. Label both axes, the extrema and the quarter-period times. The tabulated e.m.f. values follow the declared sign convention. **[3 marks]**

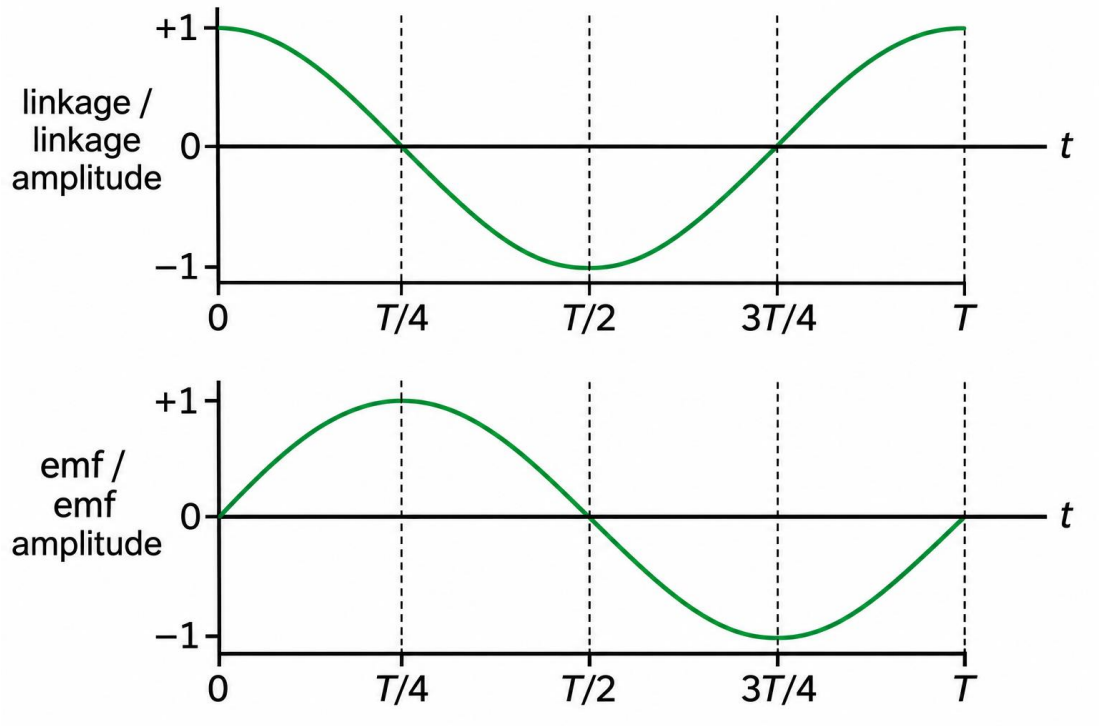
Step 1 Plot the linkage

At t = 0, T/4, T/2, 3T/4, T use +0.1944, 0, −0.1944, 0, +0.1944 Wb. Join with a smooth cosine-shaped curve.

Step 2 Plot the e.m.f.

At the same times use 0, +14.66, 0, −14.66, 0 V. Join with a smooth sine-shaped curve. E.m.f. is positive when linkage decreases and negative when linkage increases.

The two graphs use the same time scale. Their maxima occur a quarter-period apart; a maximum of linkage does not coincide with a maximum of e.m.f.



Model graph (normalised axes)

Marking guidance Correct linkage curve and labelled axes; correct e.m.f. curve with extrema; correct common time scale and phase.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

4(b) A tangent to the linkage graph at T/4 passes through the points (18.00 ms, +0.04153 Wb) and (23.00 ms, −0.03176 Wb). Use these tangent points to estimate the instantaneous e.m.f. at T/4. **[2 marks]**

Step 1 Find the tangent gradient, converting milliseconds

gradient = (−0.03176 − 0.04153)/[(23.00 − 18.00) × 10^{-3}] = −14.658 Wb s^{-1}.

Step 2 Apply Faraday's law

ε = −gradient = +14.658 V ≈ +14.7 V. The readings already describe flux linkage, so do not multiply by N again.

Marking guidance Correct tangent gradient using seconds; +14.7 V, with no extra N.

4(c) The steady rotation frequency is doubled. State the changes to the linkage amplitude, the period and the peak e.m.f. **[2 marks]**

Linkage amplitude stays at NBA = 0.1944 Wb. The period halves to 0.0417 s. The same linkage change occurs in half the time, so the peak e.m.f. doubles to about 29.3 V.

Marking guidance Unchanged linkage amplitude and halved period; doubled peak e.m.f. linked to faster change.

4(d) Calculate the signed area under the e.m.f.–time graph over the first half-turn. Explain why its area over a complete turn is zero although energy can be transferred to a load. **[2 marks]**

Step 1 Relate area to a change of linkage

Area = −Δ(NΦ) = −(−0.1944 − 0.1944) = +0.3888 V s over the first half-turn.

Step 2 Distinguish a net signed change from energy

Over a full turn the linkage returns to its starting value, so positive and negative areas cancel. This does not mean no electrical energy: with a resistive load, voltage and current reverse together and power remains positive.

Marking guidance First half-turn +0.3888 V s; full-cycle signed cancellation distinguished from energy transfer.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

5 Connect the load

5(a) When a resistive load is connected, the motor needs a greater driving torque to keep the same speed. Explain this using Lenz's law and energy conservation. **[2 marks]**

The induced current produces a magnetic effect that opposes the change causing it: the electromagnetic torque opposes the rotation.

The drive must do extra mechanical work to maintain speed. That energy is transferred electrically to the load and dissipated in resistances; it is not created by the generator.

Marking guidance Opposing electromagnetic torque; extra mechanical input linked to electrical output/losses.

5(b) At one instant the terminal p.d. is +8.00 V and the outward load current is +0.450 A. Calculate the instantaneous power transferred to the load. **[1 mark]**

$P = VI = 8.00 \times 0.450 = 3.60$ W. These are simultaneous actual values, so the product is instantaneous power.

Marking guidance 3.60 W.

5(c) Over many cycles the mean electrical output is 8.00 W and mean mechanical input is 13.0 W. Calculate the efficiency. Explain why peak e.m.f. × peak current is not generally the mean power. **[2 marks]**

Efficiency = mean useful output/mean input = 8.00/13.0 = 0.615 = 61.5%.

Mean power is an average over the varying simultaneous voltage and current. The product of peak values is a peak power for an ideal resistive load, not the cycle average. No peak-value average formula is required here.

Marking guidance 61.5%; peak product distinguished from the mean over the cycle.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

6 Choose a viable design

6(a) For the same field and coil area, the peak induced e.m.f. is $\varepsilon_0 = 2\pi fNBA$. Calculate it for each proposed design: P: N = 180, f = 10.0 Hz; Q: N = 120, f = 14.0 Hz; R: N = 180, f = 14.0 Hz. **[2 marks]**

Step 1 Keep field and area fixed

BA = 0.0900 × 0.0120 = 0.00108 Wb per turn.

Step 2 Apply the supplied expression to each design

P: 2π × 10.0 × 180 × 0.00108 = 12.2145 V.

Q: 2π × 14.0 × 120 × 0.00108 = 11.4002 V.

R: 2π × 14.0 × 180 × 0.00108 = 17.1003 V.

Marking guidance Correct use of 2πfNBA for at least two designs; all three values about 12.2, 11.4 and 17.1 V.

6(b) The buoy requires a no-load peak e.m.f. from 12.0 V to 16.0 V and a frequency from 10.0 Hz to 14.0 Hz, inclusive. Select a design and justify it against both requirements. **[1 mark]**

Choose P: its peak e.m.f. is 12.2 V and its frequency is 10.0 Hz, both within the limits. Q has too little peak e.m.f.; R has too much. All three frequencies meet the frequency condition.

Marking guidance P justified using both voltage and frequency limits.

6(c) In a separate loaded test, the instantaneous induced e.m.f. is +10.0 V, the coil resistance is 2.00 Ω, and the outward current is +0.400 A. Calculate the terminal p.d. Assume $\varepsilon = V + Ir$ at this instant. **[1 mark]**

$V = \varepsilon - Ir = 10.0 - 0.400 \times 2.00 = 9.20$ V. Part of the induced e.m.f. drives current through the coil's own resistance.

Marking guidance 9.20 V.

6(d) Correct both claims: "Maximum flux means maximum e.m.f." and "A faster generator changes only the frequency, not the amplitude." **[2 marks]**

Maximum linkage gives zero instantaneous gradient and therefore zero e.m.f. in this steady rotating-coil model.

Faster rotation increases frequency and the rate of change of linkage, so it increases the e.m.f. amplitude as well. The linkage amplitude itself stays fixed if B, N and A are unchanged.

Marking guidance Correct value/rate distinction; faster rotation increases e.m.f. amplitude as well as frequency.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk