

Synoptic mechanics and electricity — solutions

Matching worked solutions and teaching prompts | 39 marks

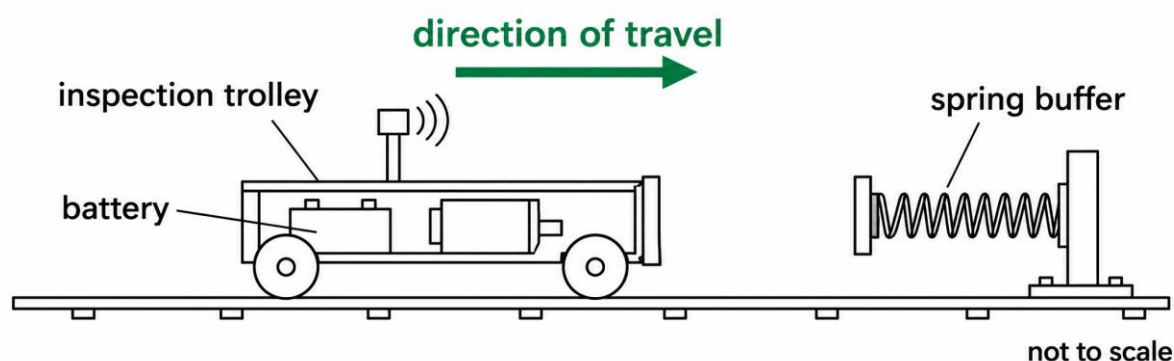
AQA AS Physics 7407 • First-year A Level 7408

Specification references

AQA AS 7407 / first-year A Level 7408: 3.4.1.3 — motion graphs; 3.4.1.5 — Newton's laws; 3.4.1.7 — work, power and efficiency; 3.4.1.8 — conservation of energy; 3.4.2.1 — Hooke's law and elastic strain energy. Electricity: 3.5.1.1 — current, charge and potential difference; 3.5.1.4 — electrical energy and power; 3.5.1.6 — emf, terminal potential difference and internal resistance.

Inspecting a flood-defence tunnel

A battery-powered inspection trolley carries acoustic sensors along a rail inside a flood-defence tunnel. Engineers test its motion, energy use and emergency stopping system before deciding how many inspections one battery can support.



The loaded trolley has mass 12.0 kg. On the level rail a constant 4.00 N resistive force opposes motion. Neglect rotational kinetic energy. The motor and controller are one energy converter; no motor-effect theory is needed. Show working in the blank spaces.

1. Check the battery [3 marks]

The battery has emf 12.0 V. During steady travel it supplies 2.00 A and its terminal potential difference is 11.2 V.

(a) Explain what an emf of 12.0 V means in terms of energy and charge. [1 mark]

The battery transfers 12.0 J of chemical energy to electrical energy for each 1 C of charge passing through it. [1] This is not necessarily the energy per coulomb delivered to the external circuit when current flows.

(b) Determine the internal resistance of the battery. [2 marks]

Lost potential difference = $\epsilon - V = 12.0 - 11.2 = 0.800$ V. [1] Using $\epsilon = V + Ir$, $r = (\epsilon - V)/I = 0.800/2.00 = 0.400$ Ω . [1]

Teaching prompt: An emf gives chemical energy transferred per coulomb. Terminal potential difference gives electrical energy delivered per coulomb to the external circuit.

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2. Model the inspection cycle [6 marks]

The table gives the velocity at the boundaries of each stage of one level-rail cycle. Acceleration is constant within each stage. The trolley always moves forwards or is instantaneously at rest. Use these exact data for your calculations.

Boundary	Time / s	Velocity / m s ⁻¹
Start	0	0
End of acceleration	4.00	2.00
Start of braking	14.0	2.00
End of cycle	16.0	0

(a) Sketch a velocity–time graph for the complete cycle in the blank space. Label both axes with quantities and units, and label the time and velocity at each corner. [2 marks]

Use t / s horizontally and $v / \text{m s}^{-1}$ vertically. Join (0, 0), (4.00, 2.00), (14.0, 2.00) and (16.0, 0) with three straight segments: rising, horizontal, falling. [1] Both axes correctly labelled with units and all four corner coordinates correctly labelled, with sensible relative stage durations and no negative-velocity branch. [1] The flat stage lasts 10.0 s and the final stage 2.00 s; the sketch need not be on graph paper.

(b) Calculate the acceleration during the first 4.00 s. [1 mark]

$$a = \Delta v / \Delta t = (2.00 - 0) / (4.00 - 0) = +0.500 \text{ m s}^{-2} \text{ in the direction of travel. [1]}$$

(c) Calculate the total distance travelled during the cycle. [2 marks]

Use the area under the velocity–time graph: $(\frac{1}{2} \times 4.00 \times 2.00) + (10.0 \times 2.00) + (\frac{1}{2} \times 2.00 \times 2.00)$. [1] Distance = 4.00 + 20.0 + 2.00 = 26.0 m. [1] Velocity is never negative, so this area is both displacement and distance.

(d) Taking the direction of travel as positive, calculate the acceleration from 14.0 s to 16.0 s. [1 mark]

$$a = (0 - 2.00) / (16.0 - 14.0) = -1.00 \text{ m s}^{-2}. [1] \text{ The velocity decreases linearly; the acceleration is constant and negative, not increasing in magnitude.}$$

Teaching prompt: A gradient gives acceleration; an area gives displacement. State which quantity changes and whether that change is linear. Constant velocity means zero resultant force, not zero driving force.

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3. Account for the driving force [6 marks]

Use mass 12.0 kg and the constant 4.00 N resistive force. During 0–4.00 s the acceleration is uniform and the driving force is constant.

(a) Calculate the resultant force and hence the motor's driving force during the first 4.00 s. [2 marks]

Resultant force = $ma = 12.0 \times 0.500 = 6.00$ N forwards. [1] Driving force – 4.00 = 6.00, so driving force = 10.0 N forwards. [1] The driving force is not equal to ma because resistance also acts.

(b) Determine the driving force during steady travel from 4.00 s to 14.0 s. [1 mark]

Driving force = 4.00 N forwards, balancing the 4.00 N resistance. [1] The resultant force is zero, but both individual forces are nonzero.

(c) In the first 4.00 s the trolley travels 4.00 m. Calculate the mechanical work done by the driving force, the increase in kinetic energy and the energy transferred against resistance. Check that these energies balance. [3 marks]

Driving work = $Fs = 10.0 \times 4.00 = 40.0$ J. [1] Increase in kinetic energy = $\frac{1}{2}mv^2 = \frac{1}{2} \times 12.0 \times (2.00)^2 = 24.0$ J. [1] Work against resistance = $4.00 \times 4.00 = 16.0$ J, and $40.0 = 24.0 + 16.0$ J. [1] This balance concerns mechanical energy at the wheels, not the battery's chemical input.

Teaching prompt: Work by the resultant force equals the change in kinetic energy. Work by the driving force also supplies energy dissipated against resistance.

4. Follow the power transfers [5 marks]

During steady travel, the trolley speed is 2.00 m s⁻¹. Use battery current 2.00 A, emf 12.0 V and terminal potential difference 11.2 V. The motor/controller is the only external electrical load in this idealised test.

(a) Calculate the rate of chemical energy transfer in the battery and the electrical power delivered to the motor/controller. [2 marks]

Chemical power = $\epsilon I = 12.0 \times 2.00 = 24.0$ W. [1] Terminal electrical power = $VI = 11.2 \times 2.00 = 22.4$ W. [1] Their difference, 1.60 W, is dissipated inside the battery: $I^2r = (2.00)^2 \times 0.400$.

(b) Calculate the mechanical power delivered by the driving force. Hence determine the terminal-electrical-to-mechanical efficiency of the motor/controller. [2 marks]

Mechanical power = $F_{\text{drive}} v = 4.00 \times 2.00 = 8.00$ W. [1] Efficiency = $8.00/22.4 \times 100 = 35.7\%$. [1] Use terminal input for this efficiency; $8.00/24.0 = 33.3\%$ would describe the whole battery-to-wheel chain instead.

(c) A learner uses the zero resultant force in $P = Fv$ and concludes that the motor supplies no mechanical power. Explain the mistake. [1 mark]

For the motor's output use its driving force, not the resultant force: it supplies 8.00 W to balance dissipation against resistance. [1] Zero resultant power means no change in kinetic energy; it does not mean no energy transfer.

Teaching prompt: Always name the input and output boundaries of an efficiency. In $P = Fv$, use the force whose power is being calculated; the resultant force gives the rate of change of kinetic energy.



5. Separate braking from regeneration [5 marks]

From 14.0 s the motor is electrically disconnected. A mechanical brake stops the trolley while the usual 4.00 N resistance still acts. The total acceleration is -1.00 m s^{-2} . No regenerative charging occurs.

(a) Determine the additional retarding force exerted by the mechanical brake. [2 marks]

Total retarding force has magnitude $m|a| = 12.0 \times 1.00 = 12.0 \text{ N}$. [1] Mechanical-brake force = $12.0 - 4.00 = 8.00 \text{ N}$ backwards. [1] In signed form, $-F_{\text{brake}} - 4.00 = 12.0 \times (-1.00)$.

(b) Use the cycle data to calculate the braking distance and hence the energy transferred by the mechanical brake and by the usual resistance during this stop. [2 marks]

Braking distance = $\frac{1}{2} \times (16.0 - 14.0) \times 2.00 = 2.00 \text{ m}$. Brake transfer = $8.00 \times 2.00 = 16.0 \text{ J}$. [1]
Resistance transfer = $4.00 \times 2.00 = 8.00 \text{ J}$. [1] Total 24.0 J equals the initial kinetic energy; do not add another 24.0 J to these transfers.

(c) The cycle starts and finishes at rest. Explain why this does not mean that the battery's initial chemical energy has been restored. [1 mark]

The kinetic energy is transferred to thermal energy in the brake, rail/wheels and surroundings; it is not returned to the electrically disconnected battery. [1] Equal initial and final kinetic energy does not imply zero chemical energy use.

Teaching prompt: A negative change in kinetic energy is not a negative battery-energy use. Identify where the energy goes and whether a real charging path exists.

6. Estimate the inspection capacity [6 marks]

The table gives the measured traction current in each stage of the same cycle. For this ideal estimate omit sensor/controller standby currents, return journeys and reserve charge. The battery can supply $1.80 \times 10^4 \text{ C}$; take its emf as constant at 12.0 V.

Stage	Time interval / s	Battery current / A
Acceleration	0–4.00	4.00
Steady travel	4.00–14.0	2.00
Mechanical braking	14.0–16.0	0

(a) Calculate the total charge supplied by the battery during one complete cycle. [2 marks]

Charge during acceleration = $4.00 \times 4.00 = 16.0 \text{ C}$; during cruise = $2.00 \times 10.0 = 20.0 \text{ C}$. [1] Braking charge = 0, so total $Q = 16.0 + 20.0 = 36.0 \text{ C}$. [1] Do not use 2.00 A for all 16.0 s or average stage currents without weighting by duration.

(b) Calculate the chemical energy transferred from the battery during one cycle. [1 mark]

$E_{\text{chemical}} = \epsilon Q = 12.0 \times 36.0 = 432 \text{ J}$. [1] Use emf for chemical input, not the cruising terminal voltage; terminal voltage also changes with current.

(c) Calculate the ideal number of complete cycles one battery can support. Give two modelling limitations that could make the achieved number smaller, explaining the effect of each. [3 marks]

Ideal full cycles = $\text{floor}[(1.80 \times 10^4)/36.0] = 500$. [1] Operating sensors/standby electronics draw additional charge between and during cycles, reducing the charge available for traction. [1] A return journey, safety reserve, greater resistance or extra starts requires charge omitted from the model, reducing completed inspection cycles. [1] Accept two distinct justified limitations; do not award twice for the same additional electrical load.

Teaching prompt: Charge is current multiplied by duration in each stage, then summed. This is an ideal capacity estimate, not a guarantee of 500 real inspections.

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7. Select an emergency buffer [8 marks]

In a separate failed-brake test, the unpowered trolley first contacts an initially uncompressed buffer at 2.00 m s^{-1} . The track remains level and the 4.00 N resistance still acts throughout compression. Springs are massless and obey Hooke's law throughout the stated ranges. The maximum allowed spring force on the trolley is 200 N .

(a) Buffer A has spring constant 1000 N m^{-1} and an allowed compression of 0.200 m . Calculate the spring force and stored elastic energy at that compression. [2 marks]

Spring force = $kx = 1000 \times 0.200 = 200 \text{ N}$. [1] Stored elastic energy = $\frac{1}{2}kx^2 = \frac{1}{2} \times 1000 \times (0.200)^2 = 20.0 \text{ J}$. [1] Elastic energy is not Fx for a force rising from zero; the mean spring force over the compression is half the final force.

(b) Determine the trolley's speed if it reaches 0.200 m compression in A. Hence decide whether A stops the trolley within its allowed compression. [3 marks]

Initial kinetic energy = 24.0 J . Resistance transfers $4.00 \times 0.200 = 0.800 \text{ J}$ during compression. [1] Remaining kinetic energy = $24.0 - 20.0 - 0.800 = 3.20 \text{ J}$. [1] Speed = $\sqrt{(2 \times 3.20/12.0)} = 0.730 \text{ m s}^{-1}$, so A does not stop it within 0.200 m . [1] Reaching the 200 N force limit is not sufficient if kinetic energy remains.

(c) Buffer B has spring constant 600 N m^{-1} and an allowed compression of 0.300 m . Without solving for its exact stopping compression, use force and energy calculations to decide whether B stops the trolley before the compression limit while keeping the spring force below 200 N . [3 marks]

At 0.300 m , the spring force would be $600 \times 0.300 = 180 \text{ N} < 200 \text{ N}$. [1] The elastic-energy-plus-resistance transfer available up to that point is $\frac{1}{2} \times 600 \times (0.300)^2 + 4.00 \times 0.300 = 27.0 + 1.20 = 28.2 \text{ J}$. [1] This exceeds the initial 24.0 J , so the first stop occurs before 0.300 m ; because spring force rises with compression, it remains below 180 N and hence below 200 N . B meets both criteria in this model. [1] The trolley cannot actually store/transfer all 28.2 J on this first compression; that is the capacity at the limit, used to show that the limit is not reached.

Teaching prompt: Check force and energy separately. A softer spring with more available travel can stop the trolley while keeping the peak spring force smaller. This conclusion concerns the first stop, not rebound control.

