

Stress, strain and Young modulus solutions

AQA AS and A level Physics 7407 and 7408

Matching questions with worked solutions and teaching prompts 36 marks

Specification points covered

- 3.4.2.1 Selected bulk properties of solids: tensile stress and strain, Hooke's law, elastic strain energy and stress-strain curves.
- 3.4.2.2 The Young modulus: calculations, stress-strain gradients and a simple measurement method; Required practical 4.
- 3.1.2 Selected measurement skills: percentage uncertainties and their combination, including powers.

1 The precision inspection platform

A precision inspection platform hangs from two identical vertical steel rods (Figure 1). The platform and payload have a combined mass of 24.0 kg. Their centre of mass is midway between the rods. The platform is horizontal and at rest, with no other support. Ignore the rods' weights. Take $g = 9.81 \text{ m s}^{-2}$.

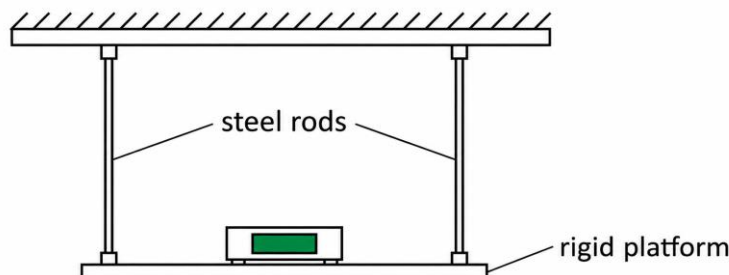


Figure 1 Not to scale

1(a) State what is meant by tensile stress and tensile strain in one of the rods. [2 marks]

Answer and mark points

Tensile stress is tensile force divided by the cross-sectional area perpendicular to the force. B1
Tensile strain is extension divided by original length; strain has no unit. B1

Teaching prompt

Extension is the increase in length, not the final length. Stress has unit Pa; strain is dimensionless.

1(b) Determine the tensile force in each rod. Explain why the two forces are equal. [2 marks]

Answer and mark points

The centrally placed combined load and identical symmetric supports give equal tensile forces. Vertical equilibrium gives $2F = mg$. M1
 $F = (24.0 \times 9.81)/2 = 117.72 \text{ N} = 118 \text{ N per rod}$. A1

Teaching prompt

Use the total platform-plus-payload mass once. Each rod carries half the total weight, not the full weight.



1 Rod measurements

Table 1

Quantity for each original rod	Value
Original length L	0.800 m
Diameter d	1.00 mm
Measured extension ΔL at full load	0.600 mm

1(c) Use Table 1 to calculate the tensile stress in each rod. [2 marks]

Answer and mark points

$$A = \pi d^2/4 = \pi(1.00 \times 10^{-3})^2/4 = 7.854 \times 10^{-7} \text{ m}^2. \text{ M1}$$

$$\sigma = F/A = 117.72/(7.854 \times 10^{-7}) = 1.50 \times 10^8 \text{ Pa} = 150 \text{ MPa}. \text{ A1}$$

Teaching prompt

Area depends on diameter squared. Convert mm to m before squaring; do not use diameter as radius.

1(d) Calculate the tensile strain in each rod. [1 mark]

Answer and mark points

$$\epsilon = \Delta L/L = (0.600 \times 10^{-3})/0.800 = 7.50 \times 10^{-4}. \text{ A1 The result is dimensionless.}$$

Teaching prompt

Use the same unit for extension and original length. A strain of 0.000750 is not 0.750%.

1(e) Determine the Young modulus of the steel. [1 mark]

Answer and mark points

$$E = \sigma/\epsilon = (1.499 \times 10^8)/(7.50 \times 10^{-4}) = 2.00 \times 10^{11} \text{ Pa}. \text{ A1 Accept equivalent use of } FL/(A\Delta L).$$

Teaching prompt

Young modulus is a material property in the linear elastic region. It is not the rod spring constant.

1(f) The maximum permitted working stress is 200 MPa. For accurate inspection, neither rod may extend by more than 0.500 mm. A technician says that meeting the working-stress limit is sufficient to approve the rods. Evaluate this claim using your results. [2 marks]

Answer and mark points

The working-stress condition is satisfied: $150 \text{ MPa} < 200 \text{ MPa}$. M1

The alignment condition is not satisfied: $0.600 \text{ mm} > 0.500 \text{ mm}$. Reject the claim and this original rod design for accurate inspection. A1

Teaching prompt

A design can meet a permitted stress and still deform too much for its intended purpose.

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2 Choosing a rod design

The engineer considers alternative rods for the same 24.0 kg combined load. Use $E = 2.00 \times 10^{11}$ Pa for steel. All proposed materials obey Hooke's law under the stated loads. The extension limit remains 0.500 mm.

2(a) Both rods are made 1.60 m long, with their diameter, material and tensile force unchanged. Predict the new extension of each rod and justify your prediction using the Young modulus relationship. **[2 marks]**

Answer and mark points

From $\Delta L = FL/(AE)$, extension is proportional to original length when F , A and E are unchanged. M1
The length doubles, so the extension doubles to 1.20 mm. A1

Teaching prompt

A longer rod is less stiff even when the material Young modulus is unchanged.

2(b) Instead, both original rods are replaced by steel rods of length 0.800 m and diameter 1.20 mm. Determine the extension of each replacement rod. **[1 mark]**

Answer and mark points

$A = \pi(1.20 \times 10^{-3})^2/4 = 1.131 \times 10^{-6} \text{ m}^2$.
 $\Delta L = (117.72 \times 0.800)/[(2.00 \times 10^{11})(1.131 \times 10^{-6})] = 4.16 \times 10^{-4} \text{ m} = 0.416 \text{ mm}$. A1
Accept 0.417 mm from scaling the measured 0.600 mm extension by $1/(1.20)^2$.

Teaching prompt

A 20% increase in diameter gives a 44% increase in area, not a 20% increase.

2(c) Determine the minimum diameter of a steel rod of length 0.800 m that meets the 0.500 mm extension limit under the same tensile force. Give a suitable diameter to three significant figures. **[2 marks]**

Answer and mark points

At the limiting extension, $A = FL/(E\Delta L) = (117.72 \times 0.800)/[(2.00 \times 10^{11})(0.500 \times 10^{-3})] = 9.4176 \times 10^{-7} \text{ m}^2$. M1
 $d = \sqrt{4A/\pi} = 1.0950 \times 10^{-3} \text{ m}$. A suitable three-significant-figure diameter is 1.10 mm. A1
Accept an equivalent diameter-ratio calculation. Do not round down to 1.09 mm: it would not meet the limit.

Teaching prompt

The diameter requires a square root. Keep guard digits before choosing a diameter that meets the constraint.

2(d) An alloy has a Young modulus of 8.00×10^{10} Pa and a breaking stress of 900 MPa. The steel has a breaking stress of 700 MPa. Calculate the extension if both original rods are replaced by alloy rods of the same original dimensions. Explain whether the larger breaking stress makes the alloy a better choice for maintaining alignment. **[2 marks]**

Answer and mark points

For the same F , L and A , ΔL is inversely proportional to E . The alloy extension is approximately $0.600 \times (2.00 \times 10^{11})/(8.00 \times 10^{10}) = 1.50 \text{ mm}$. M1
The alloy fails the 0.500 mm alignment limit. Its greater breaking stress indicates greater strength, not greater stiffness; its lower E produces a larger strain and extension at the same stress. A1

Teaching prompt

Do not use stronger to mean stiffer. Link the lower E to greater extension and the alignment limit.

2(e) For the two 1.20 mm diameter steel rods in 2(b), calculate the total elastic strain energy stored at full load, relative to their unstretched states. **[2 marks]**

Answer and mark points

For one rod, $W = \frac{1}{2}F\Delta L$. The total for two identical rods is $2 \times \frac{1}{2}F\Delta L$. M1
Total $W = 117.72 \times 4.1634 \times 10^{-4} = 4.90 \times 10^{-2} \text{ J}$. A1
Accept 0.0490–0.0491 J, including consistent use of the rounded extension in 2(b).

Teaching prompt

Include both rods. Using $F\Delta L$ for one rod would omit the factor $\frac{1}{2}$.

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3 Checking the supplier data

The laboratory tests a fresh specimen from the same batch of steel. Figure 2 shows its loading curve. Laboratory test stresses can exceed the permitted working stress for the platform.

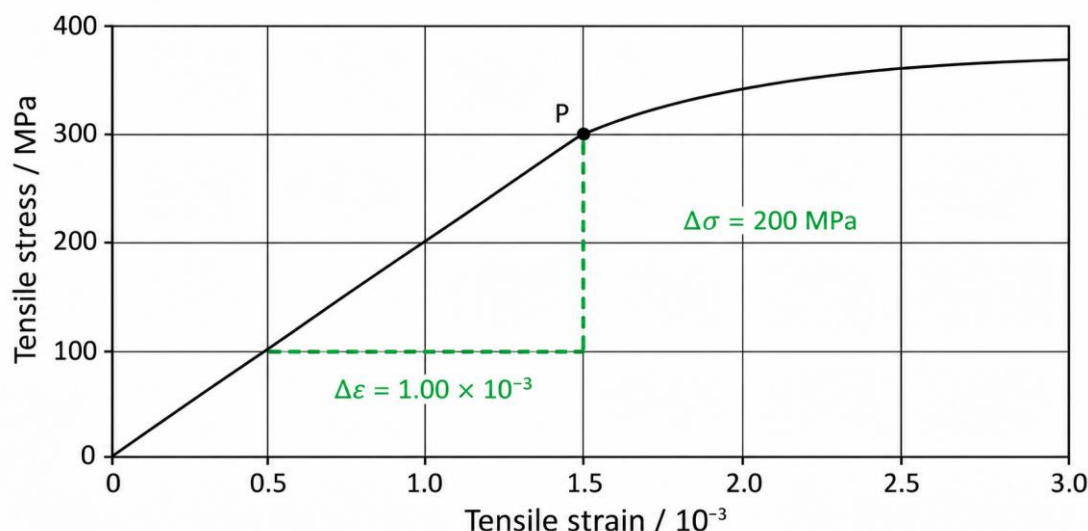


Figure 2 Gradient construction for 3(a)

3(a) Determine the Young modulus from Figure 2. Show clearly which graph values you use. **[2 marks]**

Answer and mark points

Use two widely separated points in the initial straight section, for example $(\epsilon, \sigma) = (0.500 \times 10^{-3}, 100 \times 10^6 \text{ Pa})$ and $(1.50 \times 10^{-3}, 300 \times 10^6 \text{ Pa})$. M1

$$E = \Delta\sigma/\Delta\epsilon = (200 \times 10^6)/(1.00 \times 10^{-3}) = 2.00 \times 10^{11} \text{ Pa. A1}$$

Allow a sensible read-off range of $1.90\text{--}2.10 \times 10^{11} \text{ Pa}$ with an appropriate initial-gradient method.

Teaching prompt

The displayed horizontal value 1.0 means a strain of 0.001. Convert MPa to Pa.

3(b) Point P is at the end of the straight section. A technician labels P as the elastic limit. Explain why this loading graph alone does not justify that label. **[2 marks]**

Answer and mark points

P identifies the limit of proportionality: beyond P stress is no longer proportional to strain. This is not automatically the elastic limit. M1

Unloading evidence is needed to establish whether the specimen returns to its original length or retains a permanent extension. A1

Teaching prompt

Nonlinear does not necessarily mean plastic. The unloading behaviour distinguishes elastic recovery from permanent deformation.

3(c) The laboratory compares the elastic strain energy stored per unit volume. Starting from $W = \frac{1}{2}F\Delta L$, show that $W/(AL) = \frac{1}{2}\sigma\epsilon$, where σ is tensile stress and ϵ is tensile strain. Use Figure 2 to calculate this energy per unit volume at a strain of 1.00×10^{-3} . **[3 marks]**

Answer and mark points

$$W/(AL) = \frac{1}{2}(F/A)(\Delta L/L) = \frac{1}{2}\sigma\epsilon. \text{ M1}$$

From the graph at $\epsilon = 1.00 \times 10^{-3}$, $\sigma = 200 \text{ MPa} = 2.00 \times 10^8 \text{ Pa}$. M1

$$W/(AL) = \frac{1}{2}(2.00 \times 10^8)(1.00 \times 10^{-3}) = 1.00 \times 10^5 \text{ J m}^{-3}. \text{ A1}$$

This is energy per unit volume, not the total energy in a rod. The area is within the linear elastic region.

Teaching prompt

Divide by original specimen volume AL . The derived result explains the area under the stress–strain graph.

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4 Confirming the material experimentally

Before approving the rods, the laboratory checks the steel using a long thin test wire from the same batch. Table 2 gives one set of measurements and their absolute uncertainties. The force and extension are measured from the same small-preload reference.

Table 2

Quantity	Measured value	Absolute uncertainty
Diameter d	0.400 mm	± 0.002 mm
Original gauge length L	1.200 m	± 0.001 m
Added tensile force F	30.0 N	± 0.2 N
Extension ΔL	1.44 mm	± 0.04 mm

4(a) Calculate the percentage uncertainty in the cross-sectional area of the test wire. Explain why it is not equal to the percentage uncertainty in its diameter. **[2 marks]**

Answer and mark points

Percentage uncertainty in $d = (0.002/0.400) \times 100 = 0.500\%$. M1

Since A is proportional to d^2 , percentage uncertainty in $A = 2 \times 0.500 = 1.00\%$. A1

Teaching prompt

The $\pi/4$ factor is exact; the squared measured diameter doubles its percentage contribution.

4(b) Use $E = FL/(A\Delta L)$ and the uncertainties in Table 2 to calculate the maximum percentage uncertainty in the measured Young modulus. **[2 marks]**

Answer and mark points

Add percentage uncertainties for the product/quotient, using the area contribution from 4(a): $(0.2/30.0) \times 100 + (0.001/1.200) \times 100 + 1.00 + (0.04/1.44) \times 100$. M1

Maximum percentage uncertainty = $0.667 + 0.0833 + 1.00 + 2.778 = 4.53\%$, normally reported as 4.5%. A1

Do not use quadrature for this maximum-uncertainty task. Do not double the area uncertainty again.

Teaching prompt

The extension measurement contributes about 2.8 percentage points, the largest term here. Repetition alone does not remove a zero error.

Using the mark scheme

B1 is an independent statement mark; M1 is a method or reasoning mark; A1 is an answer mark. Each denotes one mark. Each part is capped at its stated total.

Accept correct equivalent methods. Where an earlier numerical error is carried forward, award later method credit when the physics remains sound; do not credit an impossible or inconsistent physical argument.

A correct final numerical answer with a suitable unit can receive full calculation credit unless the prompt explicitly requires working, a derivation or explanation.

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4 The extended practical response

4(c) Write a method to determine the Young modulus of the steel using a long thin wire and known loads. Explain how you would measure the necessary quantities, use a graph to obtain the Young modulus, and improve the reliability of the result. Include a suitable safety precaution. **[6 marks]**

You may organise your method under: apparatus and initial measurements; loading and extension readings; graph and analysis; reliability and safety.

Example of a complete method

Clamp a long thin steel wire securely to a rigid support and attach a mass hanger. Use a small preload to straighten it and remove slack. Establish a reference reading; keep the preload small enough that its extension is negligible compared with the uncertainty in the original gauge length.

Measure the straight original gauge length L between the fixed clamp and a marker with a metre rule. Check a micrometer for zero error. Measure the original diameter at several positions, taking two perpendicular readings at each position, and calculate a mean diameter d . Calculate $A = \pi d^2/4$.

Increase the added mass in small known steps, including the hanger consistently. Calculate the added force $F = \Delta mg$. After each step, allow oscillations to stop and use a travelling microscope, vernier displacement scale or another sufficiently high-resolution displacement instrument to measure extension from the reference. Read at eye level where appropriate.

Take several force–extension pairs within the linear elastic region. Repeat readings and unload to check that the extension returns to the reference and the clamp has not slipped. Keep temperature as constant as practical.

Plot F vertically against extension ΔL horizontally, with both quantities in SI units. Find the gradient G of a best-fit line using a large triangle. Since $G = EA/L$, calculate $E = GL/A$. The reversed axes are also valid if $E = L/(AG)$ is used.

A long thin wire increases the extension for a given force and reduces its percentage reading uncertainty, but a very thin wire has a greater percentage diameter uncertainty and may exceed its elastic range. Wear eye protection, keep clear of the line of the stretched wire, and place a tray close below the hanger to catch falling masses.

Best fit levelled marking guidance

5–6 marks: a coherent, workable method with appropriate diameter, original-length and extension measurements; several known loads; and a correctly used graph to determine E . Includes justified quality controls and a relevant safety measure. Award 6 when the measurement-to-result chain is complete and precise.

3–4 marks: a mainly workable procedure with several relevant measurements and some analysis, but an important measurement, graph relationship or control is incomplete. Award 4 for the more coherent response.

1–2 marks: relevant isolated measurements or loading steps, with major gaps preventing a dependable E determination. 0 marks: no relevant physics.

Accept other valid apparatus, including a reference-wire arrangement. Do not require this example verbatim. Do not award six separate ticks without considering whether the proposed method works.

Teaching prompt: A graph must lead to the correct E relationship. More repeats do not repair an incorrect diameter or a slipping clamp.

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