

Stationary waves on a string • Required Practical 1 — solutions

Matching worked solutions and teaching prompts | 36 marks

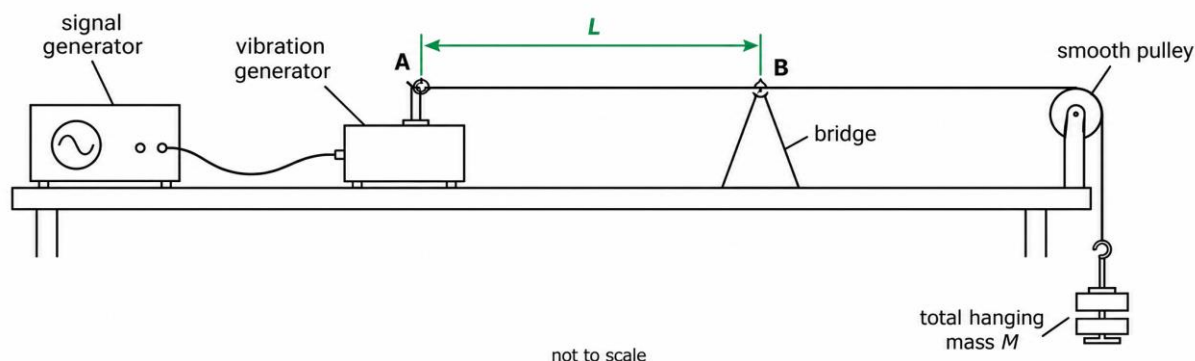
AQA AS Physics 7407 • First-year A Level 7408

Specification points covered

AQA AS 7407 / first-year A Level 7408: 3.3.1.3 — stationary waves, nodes, antinodes, harmonics; first-harmonic frequency and Required Practical 1: effects of length, tension and mass per unit length. Supporting content: 3.3.1.1 — $v = f\lambda$, wavelength and phase; 3.1.2 — absolute and percentage uncertainties and combining uncertainties.

A resonance tester for instrument cords

A sensor manufacturer is developing a resonance tester for tension in lightweight instrument cords. The team first validates its method on a bench rig, then checks whether a replacement cord can meet a frequency target without exceeding the tester's tension limit.



Use $g = 9.80 \text{ m s}^{-2}$ and $T = Mg$; M includes the hanger and all added masses. For the first harmonic, $f_1 = (1/(2L))\sqrt{(T/\mu)}$, where μ is mass per unit length. Assume uniform cords, negligible bridge/pulley friction and a stationary hanging mass. With small drive amplitude, A and B are approximate nodes. Diagrams are not to scale.

1. Prepare the resonance tester [2 marks]

Measure the vibrating length L from attachment A to bridge B, not along the whole cord.

(a) Calculate μ in kg m^{-1} for an unstretched uniform cord sample of length 4.00 m and mass 3.20 g. [1 mark]

$\mu = m/\text{length} = (3.20 \times 10^{-3})/4.00 = 8.00 \times 10^{-4} \text{ kg m}^{-1}$. [1] Convert grams to kilograms. The length here is the weighed sample length, not the vibrating span L .

(b) The total hanging mass is 0.200 kg. Calculate the tension T in the cord. [1 mark]

$T = Mg = 0.200 \times 9.80 = 1.96 \text{ N}$. [1] Include the hanger in M ; tension is a force, not a mass.

Teaching prompt: Distinguish the weighed sample length, the vibrating span and the total cord length. A is only approximately a node because its drive amplitude is small.

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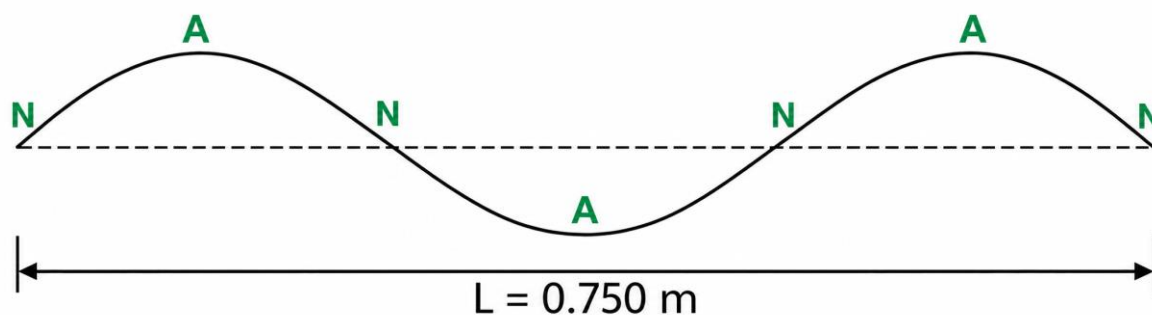
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2. Identify a resonance [8 marks]

With $L = 0.750$ m, the driver is set to 99.0 Hz. The snapshot shows the third harmonic at an instant of maximum displacement. Use the measured drive frequency in calculations.

(a) On the diagram, label every node N and every antinode A. [2 marks]



snapshot at maximum displacement — not to scale

Four nodes, including both ends and the two internal crossings: all correctly identified. [1] Three antinodes at the centres of the loops: all correctly identified. [1] The dashed line is the equilibrium position; not every point on it is a node.

(b) Determine the wavelength of each travelling wave that forms this pattern, then calculate the wave speed. [2 marks]

Three half-wavelengths fit into L , so $\lambda = 2L/3 = (2 \times 0.750)/3 = 0.500$ m. [1] $v = f\lambda = 99.0 \times 0.500 = 49.5$ m s⁻¹. [1] L is not one full wavelength.

(c) State the phase relationship between two particles within the same loop. Then state the phase difference between particles at the centres of adjacent loops. [2 marks]

Particles within one loop oscillate in phase. [1] Particles in adjacent loops oscillate in antiphase, with a phase difference of 180° or π rad. [1] "Opposite amplitudes" is not a phase description. The phase comparison excludes the nodes, which do not oscillate in the ideal model.

(d) Explain how a wave from the driver and a reflected wave form this stationary-wave pattern. [2 marks]

Two waves of the same frequency (and wavelength), travelling in opposite directions, overlap; their amplitudes are similar. [1] Superposition produces fixed positions of cancellation (nodes) and maximum oscillation amplitude (antinodes). [1] The string does not stop moving everywhere: only nodes remain at zero displacement in the ideal model.

Teaching prompt: A node has zero amplitude at all times, not merely zero displacement at one instant. Within a loop all moving particles are in phase, even though their amplitudes differ.

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3. Use a length investigation [7 marks]

Using the same cord and total hanging mass, the team changes L and retunes the driver each time to give one loop: the first harmonic. The table contains rounded measurements.

Vibrating length L / m	Frequency f_1 / Hz	$1/L$ / m^{-1}
0.320	77.2	3.13
0.400	61.8	2.50
0.500	49.4	2.00
0.625	39.6	1.60
0.800	30.9	1.25
1.000	24.7	1.00

(a) Complete the two missing values of $1/L$ in the table. [1 mark]

For $L = 0.625$ m, $1/L = 1.60 \text{ m}^{-1}$; for $L = 0.800$ m, $1/L = 1.25 \text{ m}^{-1}$. Both correct. [1] The reciprocal of 0.320 m is 3.125 m^{-1} , shown rounded to 3.13 in the table.

(b) Plot all six values of f_1 against $1/L$ on the grid. Draw a single straight line of best fit; do not join successive points. [3 marks]

Coordinates ($1/L$ in m^{-1} , f_1 in Hz): (1.00, 24.7), (1.25, 30.9), (1.60, 39.6), (2.00, 49.4), (2.50, 61.8), (3.13, 77.2). Three to five points correctly plotted to within half a small grid division. [1] All six correct. [1] A single reasonable straight best-fit line through/very close to the origin, balancing the tiny rounding scatter. [1] A suitable reference line is $f_1 \approx 24.7 \times (1/L)$. Do not require an exactly zero intercept from real measurements.

(c) Use a large triangle on your best-fit line to determine the gradient. Show the values you use and give the unit. [2 marks]

For example, use line coordinates (1.00 m^{-1} , 24.7 Hz) and (3.00 m^{-1} , 74.1 Hz): gradient = $(74.1 - 24.7)/(3.00 - 1.00)$. [1] Gradient $\approx 24.7 \text{ Hz m} = 24.7 \text{ m s}^{-1}$. [1] Accept approximately 24.2–25.2 m s^{-1} from a reasonable plotted line. Use a triangle spanning at least half the occupied horizontal range, not two adjacent data points.

(d) Use the first-harmonic relationship to obtain the wave speed from your gradient. [1 mark]

For the first harmonic $\lambda = 2L$, so $f_1 = (v/2)(1/L)$. Thus the gradient is $v/2$ and $v = 2 \times 24.7 = 49.4 \text{ m s}^{-1}$. [1] Accept a consistent speed from the candidate's gradient; the gradient itself is not the wave speed.

Teaching prompt: The x variable is $1/L$, not L . Graph gradient units are Hz divided by m^{-1} , giving m s^{-1} . The gradient is half the wave speed.



4. Approve a cord for the tester [6 marks]

Use $v = 49.4 \text{ m s}^{-1}$ and $T = 1.96 \text{ N}$ in part (a), even if your graph result differs. For parts (b) and (c), use the weighed μ values stated, not your answer to (a).

(a) The model gives $v = \sqrt{T/\mu}$. Infer μ from the graph result and compare it with the weighed value $8.00 \times 10^{-4} \text{ kg m}^{-1}$. [2 marks]

Rearranging gives $\mu = T/v^2$. [1] $\mu = 1.96/(49.4)^2 = 8.03 \times 10^{-4} \text{ kg m}^{-1}$, about 0.4% above the weighed value and very close to it. [1] Numerical closeness alone does not prove the model; uncertainties would be needed for a formal agreement claim.

(b) For the original cord, $\mu = 8.00 \times 10^{-4} \text{ kg m}^{-1}$. Calculate the tension needed for first-harmonic frequency 66.0 Hz when $L = 0.750 \text{ m}$. [2 marks]

$T = \mu(2Lf_1)^2$. [1] $T = (8.00 \times 10^{-4})(2 \times 0.750 \times 66.0)^2 = 7.84 \text{ N}$. [1] Doubling first-harmonic frequency at fixed L and μ needs four times the tension, not twice.

(c) A replacement cord has $\mu = 1.80 \times 10^{-3} \text{ kg m}^{-1}$. It must resonate at a first-harmonic frequency of 33.0 Hz with $L = 0.750 \text{ m}$. The tester's maximum permitted tension is 4.00 N. Decide, with a calculation, whether this cord meets that requirement. [2 marks]

Required $T = (1.80 \times 10^{-3})(2 \times 0.750 \times 33.0)^2 = 4.41 \text{ N}$. [1] Reject it for this requirement: 4.41 N exceeds the 4.00 N limit. [1] This is a tension-limit decision, not evidence that the string cannot resonate at all.

Teaching prompt: Calculate the tension required for the stated length and frequency before comparing with the limit. Use total tension, not an increase in tension.



5. Plan the complete Required Practical 1 [6 marks]

Parts (a)–(c) scaffold a six-mark-style practical method. Write enough detail for another team to investigate all three variables with this rig. Use first-harmonic resonances throughout.

(a) Describe how to investigate the effect of vibrating length L on frequency, including how L is measured, how a resonance is identified and what is kept constant. [2 marks]

Measure L along the equilibrium cord from the small-amplitude driver attachment to the bridge; move the bridge to obtain several lengths. Adjust the signal-generator frequency to find one clear loop with approximate end nodes and maximum central amplitude; record L and f_1 at each setting. [1] Use the same uniform cord and keep the total hanging mass/tension unchanged, with the hanger freely suspended. [1] “Keep the length of wire constant” contradicts this investigation; “keep mass constant” must specify the hanging mass, not the vibrating cord mass.

(b) Describe how to investigate the effect of tension. Identify the quantity changed, the quantities kept constant and a suitable graph to test the expected relationship. [2 marks]

Vary the total suspended mass M , calculate $T = Mg$ and retune to the first harmonic each time; record T and f_1 for several masses. [1] Keep vibrating length and cord (hence μ) fixed; plot f_1^2 against T , expecting a straight line through/close to the origin. [1] Recheck L and that the hanger is stationary and clear of the floor. A graph of f_1 against T is not expected to be straight.

(c) Describe how to determine μ for different cords and investigate its effect on frequency. Include how you would improve confidence in the frequency readings. [2 marks]

Use a balance to weigh a long measured sample of each uniform cord and calculate $\mu = \text{sample mass/sample length}$; use the same vibrating L and total hanging mass for the different cords. [1] Find the first-harmonic frequency for each cord, retuning independently several times and calculating a mean; compare f_1 with μ (for example f_1^2 against $1/\mu$). [1] Keep the drive amplitude small and use the same resonance criterion. Using a long sample reduces percentage uncertainties in sample mass and length.

Teaching prompt: Credit operational instructions, not “change it and repeat”. In each investigation vary one of L , T and μ while controlling the other two. Frequency is the measured dependent variable.

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6. Evaluate the calibration [7 marks]

The tester will infer cord properties from measured frequency and length. The team checks its uncertainty budget and a software mass entry.

(a) The endpoint readings are $x_A = (0.125 \pm 0.001)$ m and $x_B = (0.875 \pm 0.001)$ m. Calculate L , its absolute uncertainty and its percentage uncertainty using a worst-case approach. [2 marks]

$L = x_B - x_A = 0.750$ m; absolute uncertainties add on subtraction, giving $\Delta L = 0.002$ m, so $L = (0.750 \pm 0.002)$ m. [1] Percentage uncertainty = $(0.002/0.750) \times 100 = 0.267\% \approx 0.27\%$. [1] Do not subtract the two uncertainties or use 0.001 m for the span.

(b) For $f_1 = (1/(2L))\sqrt{T/\mu}$, the percentage uncertainties in T and μ are 2.0% and 3.0%. Use your length uncertainty from (a) to calculate the worst-case percentage uncertainty in the predicted first-harmonic frequency. [2 marks]

Percentage uncertainty in f_1 = percentage uncertainty in L + $\frac{1}{2}$ (percentage uncertainty in T) + $\frac{1}{2}$ (percentage uncertainty in μ). [1] = $0.267 + \frac{1}{2}(2.0) + \frac{1}{2}(3.0) = 2.77\% \approx 2.8\%$. [1] The exact factor 2 has no uncertainty. Worst-case percentage contributions add even when a quantity is in the denominator.

(c) The software calculates $\mu = T/(2Lf_1)^2$. A technician enters only 150 g of added masses and omits the 50 g hanger. L and f_1 are measured correctly. Calculate the percentage error in the inferred μ and state whether repeating the readings removes this error. [2 marks]

The entered total mass is 150 g instead of 200 g, so entered T and inferred μ are both $150/200 = 0.75$ of their correct values: μ is underestimated by 25%. [1] Repetition does not remove this systematic error; include the hanger's mass/correct the entry. [1] The reference for percentage error is the correct total mass, not the added mass.

(d) Suggest one specific precaution against injury from falling masses that still allows the cord to have the intended tension during a reading. [1 mark]

Place a catch tray just below the freely suspended hanger, without letting the hanger rest on it during the reading. [1] Also accept keeping feet clear or securing the pulley/clamp to prevent a fall, with a relevant falling-mass explanation. A tray supporting the hanger changes the tension and is not acceptable.

Teaching prompt: Name the source of a systematic error and its direction. Repeated readings can reduce random variation, but cannot correct an omitted hanger mass.

