

Projectile motion solutions

AQA AS and A level Physics 7407 and 7408

Conveyor transfer and inspection systems

Matching questions with worked solutions and teaching prompts 36 marks

Specification points covered

3.4.1.4 Selected projectile motion: independent horizontal and vertical motion, uniform-acceleration calculations and qualitative effects of air resistance.

3.4.1.1 Selected vector skills: resolve velocity into perpendicular components and combine components to find a resultant velocity.

3.4.1.3 Selected motion skills: displacement, mean velocity, constant acceleration and velocity-time graphs.

1 Transferring beads to a lower receiver

An automated inspection line transfers small ceramic calibration beads between conveyors and receivers. In the first arrangement, each bead leaves a horizontal belt at O with speed 2.40 m s^{-1} (Figure 1). The receiver mouth is 0.800 m below O and accepts beads crossing that level at horizontal displacements from 0.900 m to 1.050 m . Treat each bead as a particle. Until Section 3, neglect air resistance and take $g = 9.81 \text{ m s}^{-2}$. Take right and up as positive.

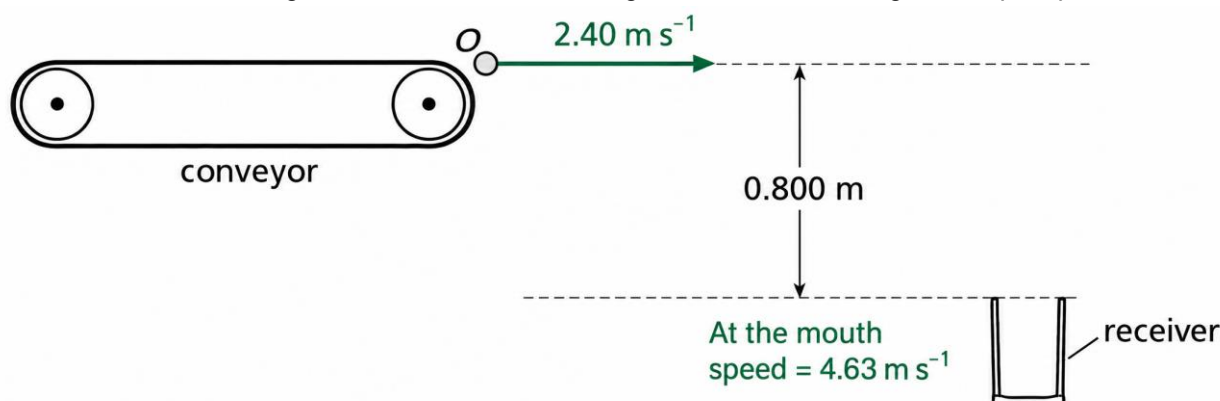


Figure 1 Not to scale

1(a) State the horizontal and vertical components of the acceleration of a bead immediately after it leaves the horizontal belt. [1 mark]

Answer and mark points

$a_x = 0$ and $a_y = -9.81 \text{ m s}^{-2}$, using right and up as positive. One mark for both components. B1

Teaching prompt

The bead does not retain a forward force from the belt after contact ends. A horizontal velocity can exist with zero horizontal acceleration.

1(b) Calculate the time taken for the bead to reach the level of the receiver mouth. [2 marks]

Answer and mark points

Vertical motion: $u_y = 0$, $y = -0.800 \text{ m}$ and $a_y = -9.81 \text{ m s}^{-2}$. Use $y = u_y t + \frac{1}{2}a_y t^2$. M1

$-0.800 = -\frac{1}{2} \times 9.81 \times t^2$ gives $t = 0.403855 \text{ s} = 0.404 \text{ s}$. A1

Teaching prompt

Use vertical displacement and initial vertical velocity to determine flight time. The initial speed 2.40 m s^{-1} is horizontal, not vertical.

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1 Checking the lower receiver

1(c) Determine the horizontal displacement when the bead reaches the mouth level. Decide whether the bead enters the opening. [2 marks]

Answer and mark points

Horizontal displacement $x = u_x t = 2.40 \times 0.403855 = 0.969 \text{ m}$. M1

The mouth accepts $0.900 \text{ m} \leq x \leq 1.050 \text{ m}$. Since 0.969 m lies inside this interval, the bead enters the opening. A1

Teaching prompt

A receiver has a finite opening. A calculated landing displacement must be compared with both edges.

1(d) Calculate the vertical component of the velocity just before the bead reaches the mouth level. Include its sign. [1 mark]

Answer and mark points

$v_y = u_y + a_y t = 0 - 9.81 \times 0.403855 = -3.96 \text{ m s}^{-1}$. A1 The negative sign means downward, not a negative speed.

Teaching prompt

With up positive, v_y decreases from zero to a negative value. The downward speed increases.

1(e) Determine the resultant speed and the angle of the velocity below the horizontal at the mouth level. [2 marks]

Answer and mark points

$v_x = 2.40 \text{ m s}^{-1}$. Speed $v = \sqrt{(v_x^2 + v_y^2)} = \sqrt{(2.40^2 + 3.96182^2)} = 4.63 \text{ m s}^{-1}$. A1

Angle below the horizontal: $\theta = \tan^{-1}(3.96182/2.40) = 58.8^\circ$. A1 Accept equivalent vector methods.

Teaching prompt

Velocity is a vector. The resultant speed is neither the vertical component alone nor the sum of the component magnitudes.

1(f) The horizontal release speed increases by 20%, with all heights unchanged. State whether the flight time changes. Calculate the new horizontal displacement at mouth level and decide whether the bead still enters the opening. [2 marks]

Answer and mark points

The flight time remains 0.404 s : the initial vertical velocity, vertical displacement and gravitational acceleration are unchanged. B1

New horizontal speed $= 1.20 \times 2.40 = 2.88 \text{ m s}^{-1}$. New $x = 2.88 \times 0.403855 = 1.16 \text{ m}$, beyond 1.050 m , so the bead misses the opening on the far side. A1

Teaching prompt

The components share the same flight time, but the horizontal speed does not determine the vertical fall time.

1(g) A bead of twice the mass leaves with the same horizontal velocity from the same height. Explain whether its landing position changes in this model. [2 marks]

Answer and mark points

The only force in free flight is weight mg , so the acceleration is $mg/m = g$, independent of mass. B1

The initial vertical motion and fall height are unchanged, so the flight time is unchanged. With the same u_x , the horizontal displacement is still 0.969 m . B1

Teaching prompt

Doubling mass doubles weight, but also doubles inertia. Do not infer a larger gravitational acceleration.

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2 Clearing the inspection baffle

In inspection mode, an inclined belt releases each bead at 4.00 m s^{-1} and 30.0° above the horizontal (Figure 2). Reset $x = 0$ and $y = 0$ at this release point O. A baffle is at $x = 0.600 \text{ m}$ and its top is $y = 0.180 \text{ m}$. The required clearance above its top is at least 10.0 mm . A different receiver has its mouth level with O and accepts horizontal displacements from 1.300 m to 1.500 m .

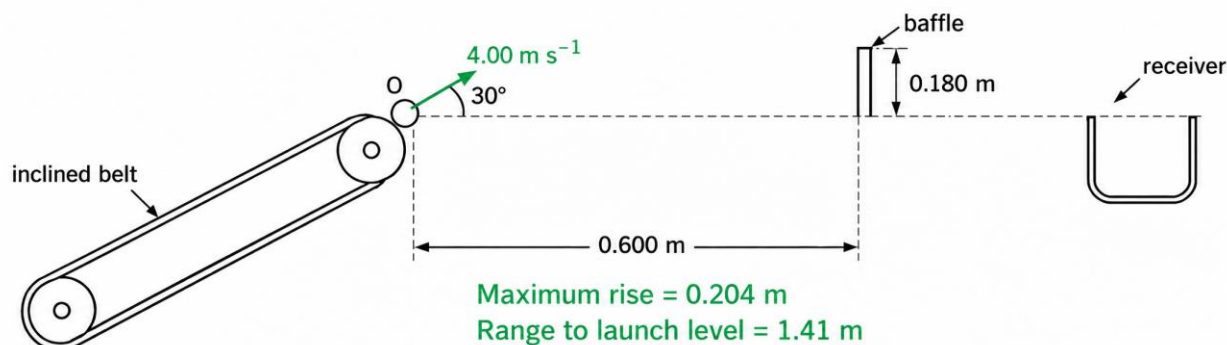


Figure 2 Not to scale

2(a) Calculate the horizontal and vertical components of the initial velocity in inspection mode. [2 marks]

Answer and mark points

$$u_x = u \cos 30.0^\circ = 4.00 \cos 30.0^\circ = 3.46 \text{ m s}^{-1}. \text{ A1}$$

$$u_y = u \sin 30.0^\circ = 4.00 \sin 30.0^\circ = +2.00 \text{ m s}^{-1}. \text{ A1}$$

Teaching prompt

The angle is measured from the horizontal, so cosine gives u_x and sine gives u_y .

2(b) Calculate the maximum rise above O. Explain why the velocity is not zero at the highest point. [2 marks]

Answer and mark points

$$\text{At the highest point } v_y = 0. \text{ From } v_y^2 = u_y^2 - 2gh, h = 2.00^2 / (2 \times 9.81) = 0.204 \text{ m}. \text{ A1}$$

Only the vertical component is zero. The horizontal component is still $+3.46 \text{ m s}^{-1}$, so the bead continues moving horizontally. B1

Teaching prompt

At the apex, $v_y = 0$ but $a_y = -g$. Neither the total velocity nor the acceleration is zero.

2(c) Calculate the height of the bead above O when it passes the baffle. Decide whether the bead meets the minimum clearance requirement. [3 marks]

Answer and mark points

$$\text{Time to the baffle: } t = 0.600 / (4.00 \cos 30.0^\circ) = 0.173205 \text{ s}. \text{ M1}$$

$$y = u_y t - \frac{1}{2}gt^2 = 2.00(0.173205) - \frac{1}{2}(9.81)(0.173205)^2 = 0.199260 \text{ m}. \text{ A1}$$

$$\text{Clearance} = 0.199260 - 0.180 = 0.019260 \text{ m} = 19.3 \text{ mm} > 10.0 \text{ mm}, \text{ so the requirement is met. A1}$$

Teaching prompt

The maximum height alone does not establish clearance at a particular horizontal position. Find the time to that position first.

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2 Positioning the receiver and camera

2(d) Calculate the time at which the bead returns to the level of O and its horizontal displacement then. Decide whether it enters the inspection receiver. **[2 marks]**

Answer and mark points

At the return to $y = 0$, $0 = u_y t - \frac{1}{2}gt^2$. Choose the nonzero root $t = 2u_y/g = 0.407747$ s. M1

$x = u_x t = 3.46410 \times 0.407747 = 1.41$ m. This is between 1.300 m and 1.500 m, so the bead enters the opening. A1

Teaching prompt

The $t = 0$ root is the release event, not the return. The inclined receiver is level with O, unlike the lower receiver in Section 1.

2(e) An engineer changes the release angle to 45.0° but keeps the speed at 4.00 m s^{-1} . Calculate the horizontal displacement on returning to the level of O. Decide whether this adjustment is suitable for the unchanged receiver. **[2 marks]**

Answer and mark points

At 45.0° , $u_x = u_y = 4.00/\sqrt{2} = 2.82843 \text{ m s}^{-1}$. Return time $= 2u_y/g = 0.576642$ s. M1

$x = u_x t = 1.63$ m, beyond the 1.500 m far edge. The adjustment is not suitable for this receiver. A1 Accept a valid equivalent range calculation, but do not require memorisation of a range formula.

Teaching prompt

A greater range is not automatically useful. The design must deliver the bead into the stated opening.

2(f) Return to the 30.0° setting. An inspection camera triggers when the vertical component of velocity is -1.20 m s^{-1} . Calculate the horizontal displacement and height of the bead relative to O at that instant. **[3 marks]**

Answer and mark points

From $v_y = u_y - gt$, $-1.20 = 2.00 - 9.81t$, so $t = 3.20/9.81 = 0.326198$ s. M1

Horizontal displacement $x = 3.46410 \times 0.326198 = 1.13$ m. A1

Height $y = 2.00t - \frac{1}{2}(9.81)t^2 = +0.130$ m. A1 An equivalent height calculation is $y = (2.00^2 - 1.20^2)/(2 \times 9.81)$.

Teaching prompt

A negative vertical velocity indicates descent; the bead can still be above its release level.

3 Testing the model with a camera

The horizontal arrangement from Section 1 is tested using a side-on camera. Table 1 gives measured bead positions; uncertainties are small compared with the observed changes. The table is experimental evidence, not values calculated from the ideal model.

Table 1

Time t / s	0.000	0.080	0.160	0.240	0.320
Position x / m	0.000	0.187	0.366	0.537	0.700

3(a) Use Table 1 to calculate the mean horizontal velocity during the first time interval and during the last time interval. **[2 marks]**

Answer and mark points

First interval: mean $v_x = (0.187 - 0)/(0.080 - 0) = 2.3375 = 2.34 \text{ m s}^{-1}$. A1

Last interval: mean $v_x = (0.700 - 0.537)/(0.320 - 0.240) = 2.0375 = 2.04 \text{ m s}^{-1}$. A1

Teaching prompt

Use the displacement change within each specified interval, not the displacement since release. These are interval means, not exact instantaneous velocities.

Using the mark scheme

B1 is an independent statement mark, M1 a method or reasoning mark, and A1 an answer mark. Each denotes one mark; do not exceed the part total.

Accept equivalent correct methods and consistent signed-component conventions. Follow through an earlier numerical error when the later physics is correct, except where it creates an inconsistent physical conclusion.

A correct numerical answer with a suitable unit can receive full calculation credit unless an explanation, working or derivation is explicitly required. Where a part requires a decision, it must be supported by the relevant result and constraint.

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3 Graphs and the measurement method

3(b) Explain what these results show about the model of negligible air resistance. Suggest a physical cause of the observed change in horizontal velocity. [2 marks]

Answer and mark points

The mean horizontal velocity decreases between equal time intervals, outside the stated negligible measurement uncertainty. This is inconsistent with the constant horizontal velocity predicted by the ideal model. B1

Air resistance has a horizontal component opposite the bead's horizontal motion, giving a backward horizontal resultant force and reducing v_x . B1 Do not attribute the horizontal change to gravity: weight acts vertically.

Teaching prompt

Use the measured trend as evidence, then link force direction to horizontal acceleration. A changed range alone would not identify the cause.

3(c) Return to the ideal horizontal-release model in Section 1. On Figure 3, draw the horizontal and vertical velocity graphs from $t = 0$ to $t = 0.40$ s. State what the gradient of the vertical-velocity graph represents. [3 marks]

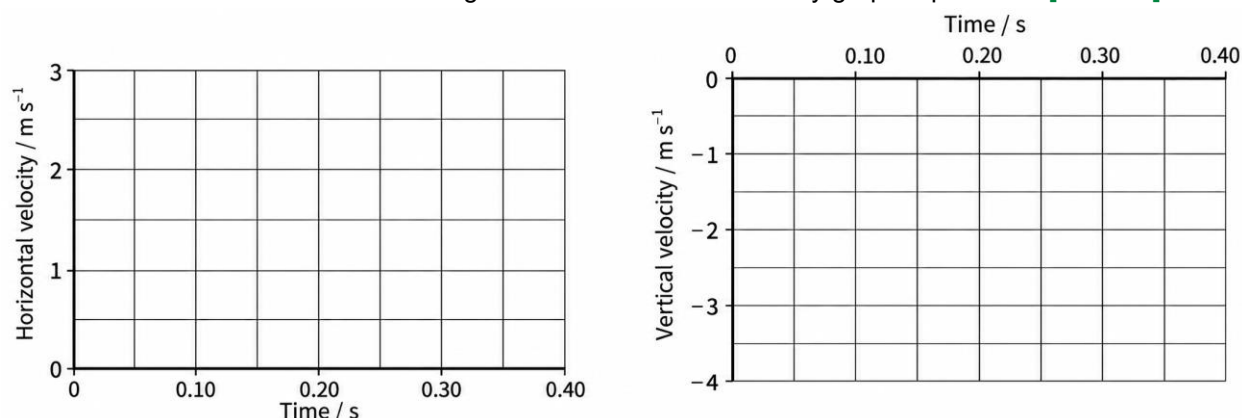


Figure 3 Reference axes; use the exact coordinates below

Coordinates for the model graphs

Time t / s	0.00	0.10	0.20	0.30	0.40
v_x / m s^{-1}	2.40	2.40	2.40	2.40	2.40
v_y / m s^{-1}	0.000	-0.981	-1.962	-2.943	-3.924

Answer and mark points

Horizontal graph: a horizontal line at $+2.40 \text{ m s}^{-1}$ for $0 \leq t \leq 0.40$ s. B1

Vertical graph: a straight line from $(0, 0)$ to $(0.40 \text{ s}, -3.924 \text{ m s}^{-1})$. Allow a careful endpoint near -3.9 to -4.0 m s^{-1} . B1

Its gradient is the vertical acceleration, $\Delta v_y / \Delta t = -9.81 \text{ m s}^{-2}$. B1 The bead is still above the mouth level at 0.40 s; impact is at about 0.404 s.

Teaching prompt

Say horizontal velocity stays constant and vertical velocity decreases linearly. Avoid "the velocity is accelerating". A negative v_y means downward motion.

3(d) Describe a video method to test whether the horizontal velocity stays constant during flight. Include how the camera and scale should be positioned, how times are determined, and how the measurements would test the prediction. [3 marks]

Answer and mark points

Fix a side-on camera perpendicular to the flight plane. Put a calibrated scale in that plane to convert bead-centre positions to metres and minimise parallax. B1

Use a known high frame rate f . For equal frame-interval counts N , calculate each elapsed time using $\Delta t = N/f$. B1

Measure horizontal displacements over these equal intervals and calculate $\Delta x / \Delta t$. Constancy within measurement uncertainty supports the model; a systematic change contradicts it. Repeat identical releases to check reproducibility. B1

Teaching prompt

Count frame intervals, not frames. Repetition does not correct parallax or a misplaced scale.

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