

Electron diffraction and wave–particle duality solutions

Matching worked solutions and teaching prompts | 38 marks

AQA AS Physics 7407 • First-year A Level 7408

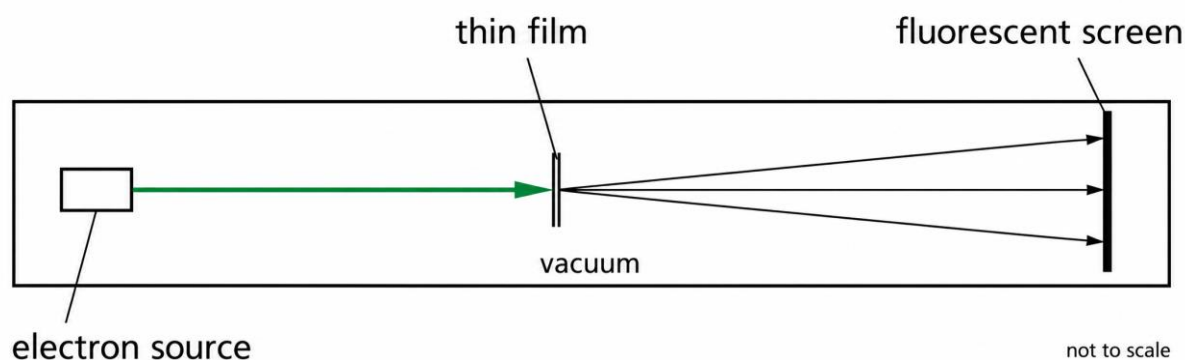
Specification points covered

AQA AS 7407 / first-year A Level 7408: 3.2.2.4 — electron diffraction, wave–particle duality, $\lambda = h/p = h/(mv)$, momentum changes, peer review and validation.

Supporting content: 3.2.2.1 — photoelectric evidence; 3.2.2.2 — electronvolts; 3.4.1.6 — momentum; 3.4.1.8 — kinetic energy. Instrument-specific relationships are supplied.

A thin-film quality check

A sensor manufacturer tests a very thin protective film before approving a production batch. Electrons pass through a test area containing many small, differently oriented crystals and form diffraction rings on a fluorescent screen. The team must choose a beam setting that gives measurable rings, then decide what the observations show about electrons.



Use $h = 6.63 \times 10^{-34} \text{ J s}$, $m_e = 9.11 \times 10^{-31} \text{ kg}$ and $e = 1.60 \times 10^{-19} \text{ C}$. $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$; $1 \text{ pm} = 10^{-12} \text{ m}$. Use the non-relativistic model throughout. Show calculations in blank spaces and explanations on ruled lines. Diagrams are schematic: do not measure them.

1. Describe the electron beam [2 marks]

(a) State what is meant by wave–particle duality. [1 mark]

Matter and electromagnetic radiation can show both wave-like and particle-like behaviour, depending on the experiment. [1] Accept an equivalent statement that entities such as electrons and photons have both wave and particle properties.

(b) In an initial low-energy test, electrons have de Broglie wavelength 180 pm. Calculate the momentum of one electron. [1 mark]

$p = h/\lambda = (6.63 \times 10^{-34})/(180 \times 10^{-12}) = 3.68 \times 10^{-24} \text{ kg m s}^{-1}$. [1] Convert picometres to metres before substituting.

Teaching prompt: de Broglie wavelength is h divided by momentum. It is not an electron size or the distance travelled per second.

Continue your physics journey

Videos, practice and support for your next step

www.physicsuk.co.uk



2. Choose the test-beam energy [7 marks]

The electron source accelerates electrons from rest through a potential difference U . Assume the kinetic energy gained is $E_k = eU$ and that there are no energy losses before the film.

(a) For $U = 1.20$ kV, calculate the electron speed and de Broglie wavelength. Give the wavelength in pm. [3 marks]

$E_k = eU = 1.60 \times 10^{-19} \times 1200 = 1.92 \times 10^{-16}$ J. [1] From $E_k = \frac{1}{2}m_e v^2$, $v = \sqrt{(2E_k/m_e)} = 2.05 \times 10^7$ m s⁻¹. [1] $\lambda = h/(m_e v) = 3.54 \times 10^{-11}$ m = 35.4 pm. [1] Keep guard digits between steps.

(b) The first test requires a wavelength between 30 pm and 40 pm, inclusive. Is 1.20 kV a suitable setting? Justify your decision. [1 mark]

Yes: 35.4 pm lies within the 30–40 pm range. [1] Accept a consistent decision from the candidate's wavelength in (a). This decides only the stated wavelength requirement.

(c) A technician says, "Doubling U halves the wavelength." Use $E_k = eU$ and $E_k = \frac{1}{2}m_e v^2$ to obtain an expression for λ in terms of U . Use it to correct the claim, stating the voltage multiplier that would halve λ . [3 marks]

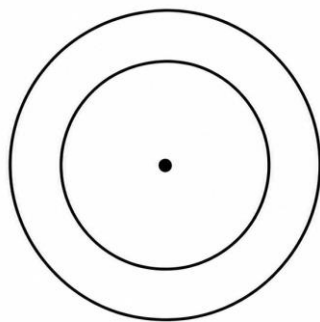
Combining the energy expressions gives $m_e v = \sqrt{(2m_e eU)}$. [1] Therefore $\lambda = h/\sqrt{(2m_e eU)}$, so λ is proportional to $1/\sqrt{U}$. [1] Doubling U multiplies λ by $1/\sqrt{2}$, not $\frac{1}{2}$; U must be multiplied by 4 to halve λ . [1] Accept an equivalent derivation using $p^2 = 2m_e E_k$.

Teaching prompt: Distinguish voltage, kinetic energy, speed and momentum. E_k is proportional to U , but v and p are proportional to $\sqrt{U}$.

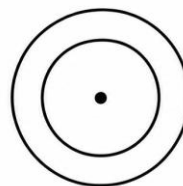


3. Interpret the diffraction pattern [8 marks]

The two screen patterns below are obtained with the same film containing many differently oriented crystals, the same film-to-screen distance and the same alignment, but different electron momenta. Corresponding rings are smaller in Pattern B. Their drawn size ratio is not a measured result.



Pattern A



Pattern B

schematic — not to scale

(a) Explain why a ring diffraction pattern provides evidence that electrons have wave properties. Identify where in the apparatus the diffraction occurs. [2 marks]

Diffraction occurs as the electrons interact with the regularly spaced atomic structure of the thin film. [1] The ring pattern results from interference of the diffracted electron waves; diffraction/interference are wave behaviour. [1] Merely naming the fluorescent screen, or saying that the film “acts like a grating” without wave reasoning, is insufficient.

(b) Which pattern corresponds to the greater electron momentum? Explain the full chain linking momentum to the observed ring sizes. [3 marks]

Pattern B. [1] From $\lambda = h/p$, greater momentum gives a smaller de Broglie wavelength. [1] With atomic spacing and screen distance unchanged, the shorter wavelength produces less diffraction/smaller diffraction angles, so corresponding rings have smaller diameters. [1] Do not use $v = f\lambda$ or an electron-frequency argument.

(c) For the beam giving Pattern A, the number of electrons reaching the film each second is doubled, but each electron has the same momentum as before. Predict the ring diameters and brightness. Assume the screen responds linearly and scattering probabilities are unchanged. [2 marks]

Ring diameters stay the same because p and hence λ are unchanged. [1] Rings become twice as bright because twice as many electrons reach corresponding parts of the screen each second under the stated linear response. [1] A brighter pattern does not by itself imply a longer wavelength.

(d) After a thin-film sample is changed, the ring diameters change even though the beam momentum is unchanged. A learner concludes that $\lambda = h/p$ has failed. Give one reason why this conclusion is not justified. [1 mark]

The new film may have different atomic spacings, changing diffraction angles even when λ is unchanged. [1] Also accept changed sample-to-screen distance or alignment if explained. Different films cannot be compared using ring diameter alone without controlling their structure and geometry.

Teaching prompt: Use “momentum increases”, “wavelength decreases” and “diffraction angle decreases”. Do not say that the wavelength slows down. Ring positions and intensity are different observations.



4. Keep the ring inside the measuring region [7 marks]

For this same film and fixed geometry, a calibration model relates the diameter D of one chosen ring to accelerating voltage U : $D\sqrt{U} = \text{constant}$. This relationship is supplied; it is not a law you need to recall. At $U = 1200 \text{ V}$, $D = 28.0 \text{ mm}$. Use U in volts consistently. The instrument can measure this ring reliably only when $24.0 \text{ mm} \leq D \leq 32.0 \text{ mm}$.

(a) Use the calibration model to calculate D at $U = 2700 \text{ V}$. [2 marks]

$D = 28.0 \times \sqrt{(1200/2700)} \text{ mm}$. [1] $D = 18.7 \text{ mm}$. [1] The diameter is proportional to $1/\sqrt{U}$, not $1/U$.

(b) During a check on the same film, D is measured as 21.0 mm . Calculate the accelerating voltage implied by the calibration model. [2 marks]

$21.0\sqrt{U} = 28.0\sqrt{1200}$, so $U = 1200 \times (28.0/21.0)^2$. [1] $U = 2.13 \times 10^3 \text{ V} = 2.13 \text{ kV}$. [1] This ring is outside the reliable diameter range, even though a voltage can be inferred from the ideal model.

(c) Only three preset voltages are available: 675 V , 1200 V and 2700 V . Calculate D at 675 V , then use all three settings to choose the only preset that meets the stated measurement requirement. [2 marks]

At 675 V , $D = 28.0\sqrt{(1200/675)} = 37.3 \text{ mm}$. [1] Select 1200 V : its 28.0 mm diameter is within $24.0\text{--}32.0 \text{ mm}$; 37.3 mm at 675 V is too large and 18.7 mm at 2700 V is too small. [1] Use the stated reliable range, not an assumption that higher voltage is always better.

(d) The team moves the screen farther from the film. Explain why the original calibration cannot then be used without a new check. [1 mark]

At the same diffraction angle, a greater film-to-screen distance produces a larger ring diameter. [1] The constant in $D\sqrt{U}$ therefore depends on the geometry; the original constant no longer applies.

Teaching prompt: A calibration belongs to a particular ring, sample and geometry. Rearranging a square-root relation requires squaring the diameter ratio.



5. Electrons that lose energy in the film [6 marks]

In a separate low-energy investigation, an electron reaches the film with kinetic energy 400 eV and loses 100 eV in an inelastic collision. Ignore recoil energy in the calculation. The instrument can select electrons by their kinetic energy after the film.

(a) Calculate the de Broglie wavelength, in pm, of this electron after the collision. [3 marks]

Remaining $E_k = 400 - 100 = 300 \text{ eV} = 4.80 \times 10^{-17} \text{ J}$. [1] $p = \sqrt{2m_e E_k} = 9.35 \times 10^{-24} \text{ kg m s}^{-1}$ (or first find $v = 1.03 \times 10^7 \text{ m s}^{-1}$). [1] $\lambda = h/p = 7.09 \times 10^{-11} \text{ m} = 70.9 \text{ pm}$. [1] Use the remaining energy, not the lost energy.

(b) Before the collision the electron wavelength was 61.4 pm. Determine the percentage increase in wavelength. Explain why it is not equal to the percentage decrease in kinetic energy. [2 marks]

Percentage increase = $(70.8955 - 61.3973)/61.3973 \times 100 = 15.5\%$ (about 15.5% using rounded values). [1] λ is proportional to $1/\sqrt{E_k}$, not E_k or $1/E_k$, so a 25% energy decrease does not give a 25% wavelength increase. [1]

(c) The team wants to compare diffraction using only electrons with the original 400 eV kinetic energy. Explain why the instrument should reject the 300 eV electrons. [1 mark]

They have a different (longer) wavelength and so would not test the film at the intended single incident wavelength/energy; selecting the original-energy electrons avoids mixing different-wavelength contributions. [1] Do not assume every scattered electron retains its original energy.

Teaching prompt: Conservation of energy comes before de Broglie substitution. Some electron–film interactions are inelastic; a monoenergetic incident beam need not remain monoenergetic after the sample.



6. Evaluate the model and an alternative probe [8 marks]

The team reviews the electron model and considers whether a proton beam could replace the electron beam. No detailed knowledge of proton diffraction equipment is required.

(a) When electrons are sent through the film one at a time, individual localised detections build up a diffraction pattern. Explain why neither a purely classical particle model nor a purely classical continuous-wave model describes both observations. [2 marks]

The overall diffraction/interference pattern requires wave behaviour, which a purely classical particle model cannot explain. [1] The individual localised detection events show particle-like behaviour rather than a continuous spread of detected energy. [1] These are complementary observations; do not claim that each electron travels along a measured sinusoidal path.

(b) The team compares its evidence with the photoelectric effect. Explain how the photoelectric effect supports the particle nature of electromagnetic radiation. Include the threshold frequency in your explanation. [2 marks]

Radiation transfers energy in photons, with one photon transferring hf in one interaction with an electron. [1] Emission requires hf at least equal to the work function, giving a threshold frequency; below it, increasing intensity does not cause emission in the single-photon model. [1] Electron diffraction is the complementary evidence for particles having wave properties.

(c) The team claims its data support $\lambda = h/p$. Suggest two ways that peer review and independent testing could strengthen confidence in this claim. [2 marks]

Publish sufficient method, calibration, uncertainty and data details for other specialists to scrutinise the reasoning and identify errors. [1] Independent teams repeat the test with their own equipment and check agreement within uncertainties and across a range of momenta. [1] Accept equivalent distinct points. Review is not proof by authority, and repeated readings by the same operator are not independent replication.

(d) A proposed proton beam has the same de Broglie wavelength as the electron beam. State how the two momenta compare, then calculate v_p/v_e . Use $m_p = 1.67 \times 10^{-27}$ kg and the supplied electron mass. [2 marks]

The momenta are equal because $p = h/\lambda$ and λ is the same. [1] $m_p v_p = m_e v_e$, so $v_p/v_e = m_e/m_p = (9.11 \times 10^{-31})/(1.67 \times 10^{-27}) = 5.46 \times 10^{-4}$. [1] The proton is much slower, not faster. Equal wavelength does not imply equal speed or kinetic energy.

Teaching prompt: "Wave" and "particle" identify behaviours supported by particular experiments. Use the measured evidence and the limits of the classical models, not claims that an electron is sometimes literally a tiny water wave.

