

Electrical power and motor efficiency — solutions

Matching worked solutions and teaching prompts | 36 marks

AQA AS Physics 7407 • First-year A Level 7408

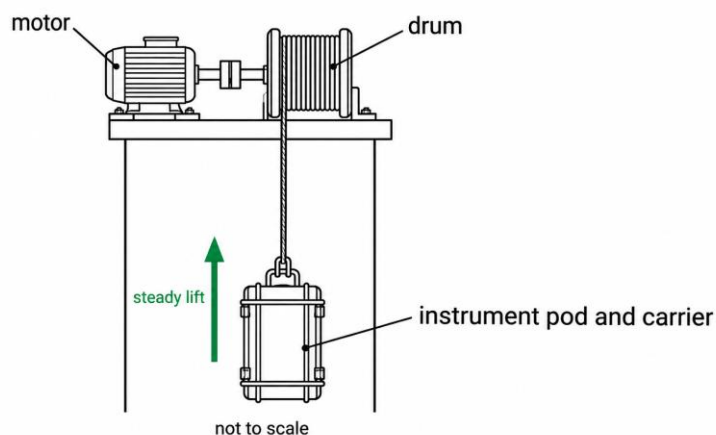
Specification points covered

AQA AS 7407 / first-year A Level 7408: 3.5.1.1 — current, charge and potential difference; 3.5.1.4 — $E = IVt$ and $P = IV = I^2R = V^2/R$.

3.4.1.7 — work, $P = Fv$, efficiency and investigating a motor lifting a mass; 3.4.1.8 — kinetic and gravitational potential energy. Supporting: 3.4.1.5 — resultant force.

A winch for cave-monitoring instruments

A cave ecology team uses a portable winch to retrieve a sealed monitoring pod. It must check energy use, choose a replacement motor and plan retrievals without exhausting the battery. The motor turns a drum which pulls a rope attached to the pod and its carrier.



Use $g = 9.81 \text{ m s}^{-2}$. Unless stated otherwise, neglect rope mass, air resistance and guide friction. The winch includes the motor and drive mechanism. Useful output means energy transferred to the pod and carrier. Show calculation working in the blank spaces.

1. Connect electrical power to energy [3 marks]

Begin with the electrical relationships used by the winch team.

(a) Use $V = W/Q$ and $I = Q/t$ to derive $P = IV$ for a steady current and potential difference. [2 marks]

From $V = W/Q$, $W = VQ$, so $P = W/t = VQ/t$. [1] Since $I = Q/t$, $P = VI = IV$. [1] Power is energy transferred per unit time; potential difference is energy transferred per unit charge.

(b) A 3.00Ω test resistor is connected across 12.0 V. Calculate the power dissipated in it. [1 mark]

$P = V^2/R = (12.0)^2/3.00 = 48.0 \text{ W}$. [1] This expression uses the potential difference across that resistor.

Teaching prompt: The power equations apply to a specified component. Never insert a voltage measured across another part of the circuit.

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2. Measure a steady retrieval [8 marks]

The total mass of the pod and carrier is 6.00 kg. During a steady part of a retrieval it rises vertically through 4.00 m in 8.00 s at constant speed. The potential difference across the winch motor is 12.0 V and its current is 4.00 A throughout this interval.

(a) Calculate the rope tension and the useful mechanical output power during the steady lift. [2 marks]

Constant speed means zero resultant force, so tension $T = mg = 6.00 \times 9.81 = 58.9 \text{ N}$. [1] $v = 4.00/8.00 = 0.500 \text{ m s}^{-1}$, so useful power $= Tv = 58.86 \times 0.500 = 29.4 \text{ W}$. [1] Equivalently use mgh/t .

(b) Calculate the electrical energy supplied to the winch in this interval and its efficiency as a percentage. [2 marks]

$E_{\text{in}} = VIt = 12.0 \times 4.00 \times 8.00 = 384 \text{ J}$. [1] Useful energy $= mgh = 6.00 \times 9.81 \times 4.00 = 235.44 \text{ J}$, so efficiency $= 235.44/384 \times 100 = 61.3\%$. [1] A consistent power ratio $29.43/48.0$ gives the same result. Initial and final speeds are equal, so net kinetic energy change is zero.

(c) A learner uses $P = Fv$ with F equal to the resultant force and concludes that the winch has zero useful output power. Explain the error. [2 marks]

The resultant force is zero because the pod moves at constant velocity, but the upward rope tension is not zero. [1] For the power transferred by the rope, F must be the rope tension; its positive work increases gravitational potential energy while gravity does negative work on the moving pod. [1] Resultant-force power describes the rate of change of kinetic energy, not the winch output.

(d) Calculate the rate of energy dissipation in the winch and describe one physical route by which this energy reaches the surroundings. [2 marks]

Dissipation rate $=$ electrical input power $-$ useful output power $= 12.0 \times 4.00 - 29.43 = 18.6 \text{ W}$. [1] Accept heating of motor windings, heating due to friction in the bearings/gears, or sound that ultimately transfers energy to the surroundings. [1] Energy is conserved; it is not destroyed.

Teaching prompt: Specify the force responsible for a transfer before using $P = Fv$. Zero net force does not imply zero work by each individual force.



3. Include the connecting leads [6 marks]

Return to the same steady lift. The two connecting leads have a combined resistance of $0.200\ \Omega$. They carry 4.00 A , and the motor still has 12.0 V across it. The source has negligible internal resistance; only the leads are being added to the energy accounting.

(a) Calculate the total heating power in the two leads. [1 mark]

$P_{\text{leads}} = I^2 R = (4.00)^2 \times 0.200 = 3.20\text{ W}$. [1] The resistance stated is already the total for both leads; do not double it.

(b) Determine the source potential difference and the power supplied by the source. [2 marks]

The leads have a total voltage drop $IR = 4.00 \times 0.200 = 0.800\text{ V}$, so source pd = $12.0 + 0.800 = 12.8\text{ V}$. [1] Source power = $12.8 \times 4.00 = 51.2\text{ W}$. [1] Check: $51.2\text{ W} = 48.0\text{ W}$ into the winch + 3.20 W into the leads.

(c) Calculate the overall efficiency from the source to the rising pod. Explain why this differs from the efficiency in Question 2(b). [2 marks]

Overall efficiency = useful output/source power = $29.43/51.2 \times 100 = 57.5\%$. [1] The denominator now includes the extra electrical energy dissipated in the leads, so the overall efficiency is lower even though the winch itself operates as before. [1] State the system boundary whenever quoting efficiency.

(d) Another learner obtains lead heating power from $(12.0)^2/0.200$. Explain why this substitution is invalid. [1 mark]

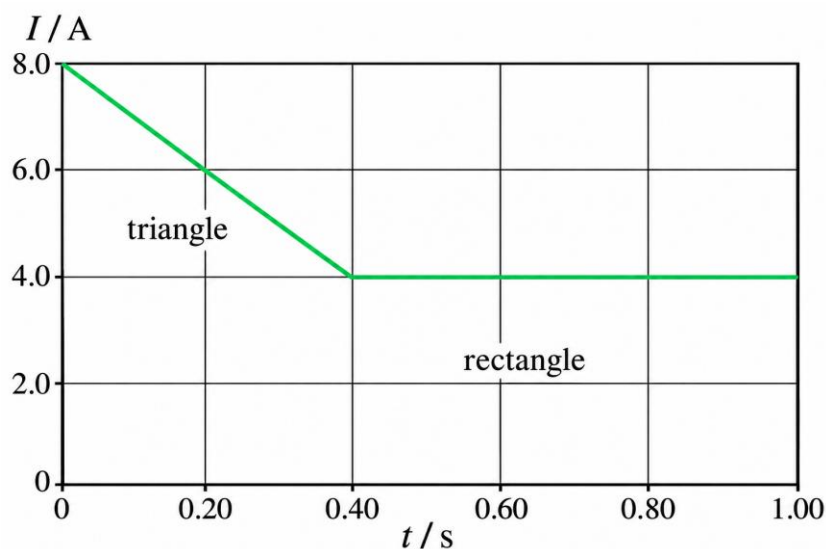
The 12.0 V is across the motor, not across the leads. The leads have only 0.800 V across them; $(0.800)^2/0.200 = 3.20\text{ W}$. [1] A potential difference and resistance in $P = V^2/R$ must refer to the same resistive component.

Teaching prompt: $I^2 R$ correctly models heating in these resistive leads. Do not model the whole running motor as a simple fixed resistor or assume all IV becomes heating.



4. Account for the start of a lift [7 marks]

In this separate start-up test the motor pd is constant at 12.0 V. Current falls linearly from 8.0 A at $t = 0$ to 4.0 A at 0.40 s, then stays at 4.0 A until 1.00 s. The 6.00 kg load starts from rest; at 1.00 s its rise is 0.400 m and its speed is 0.500 m s⁻¹. Count both its gravitational potential and kinetic energy gains as useful output.



(a) Use the graph to determine the charge passing through the motor in the first 1.00 s, then the electrical energy supplied to it. [2 marks]

Area under I–t graph = charge = rectangle + triangle = $4.0 \times 1.00 + \frac{1}{2} \times 0.40 \times (8.0 - 4.0) = 4.80$ C.
 [1] Energy input = $VQ = 12.0 \times 4.80 = 57.6$ J. [1] The average current is 4.80 A, not 6.0 A from an unweighted average of the endpoints.

(b) Calculate the useful energy transferred to the pod and carrier during this start-up interval and the average efficiency of the winch. [2 marks]

Useful energy = $mgh + \frac{1}{2}mv^2 = 6.00 \times 9.81 \times 0.400 + \frac{1}{2} \times 6.00 \times (0.500)^2 = 23.544 + 0.750 = 24.294$ J ≈ 24.3 J. [1] Efficiency = $24.294/57.6 \times 100 = 42.2\%$. [1] Use the input energy over the same interval.

(c) Explain why using only mgh for useful output energy is incomplete in this start-up test, even though it was sufficient in the steady lift. [1 mark]

The pod gains kinetic energy as well as gravitational potential energy during start-up. During the steady interval its speed is unchanged, so its net kinetic energy change is zero. [1] Kinetic energy is part of the stated useful output, not an additional dissipative loss.

(d) A data logger uses the final current, 4.0 A, for the whole 1.00 s. Determine the error in its calculated electrical input energy and state whether it overestimates or underestimates the input. [2 marks]

Logger energy = $12.0 \times 4.0 \times 1.00 = 48.0$ J, so error magnitude = $57.6 - 48.0 = 9.60$ J. [1] It underestimates the input by omitting the extra charge represented by the triangle above 4.0 A during the first 0.40 s. [1] Using too small an input would overestimate the efficiency.

Teaching prompt: Area under current–time gives charge, not energy. Multiply by the constant motor pd to obtain input energy. Compare energies over the same time interval and boundary.



5. Investigate efficiency experimentally [7 marks]

The team makes a bench version of the winch. Parts (a)–(c) scaffold a six-mark-style method to measure its steady-lifting efficiency using a known mass, ruler, timer, ammeter and voltmeter. The voltage and current are approximately constant during the chosen interval.

(a) Describe the electrical meter connections and how you would establish the moving mass and vertical rise. [2 marks]

Place the ammeter in series with the motor and the voltmeter in parallel directly across the motor terminals. [1] Measure/include the total moving mass (load plus carrier/hook) and use a vertical ruler with two position markers to obtain the rise of the same point on the load. [1] Do not include a stationary clamp in the moving mass.

(b) Describe how you would collect the measurements so that mgh is a suitable measure of useful output energy. [2 marks]

Let the load reach steady speed before the first marker and time its travel between the two markers, recording the motor pd and current during that same interval. [1] Check that the start and finish speeds are equal/constant, and use a sufficiently long measured rise with clear markers or electronic timing to reduce relative timing uncertainty. [1] Do not time from switching on if using mgh alone.

(c) Describe the calculation and repeat strategy. Identify one systematic error that repeating the lift would not remove, and how to reduce it. [2 marks]

Calculate efficiency = $mgh/(VIt)$, repeating comparable steady lifts to assess random variation and calculate a representative mean. [1] Identify a specific systematic error with a matched remedy: for example omitting the carrier mass → measure total moving mass; a zero error in a meter → check/correct zero or calibrate; measuring voltage at the source instead of the motor → connect the voltmeter at the motor terminals. [1] Repetition alone does not remove such a bias.

(d) State one specific precaution against injury or equipment damage in this experiment. [1 mark]

Secure the winch/stand and provide a catch tray below the load to prevent a falling mass striking feet or damaging the bench. [1] Accept keeping hands clear of the rotating drum, limiting current to the motor rating or another relevant specific precaution.

Teaching prompt: A useful practical method states what is measured, where it is measured, which motion interval is used and how each identified error is reduced.



6. Choose the motor and plan retrievals [5 marks]

The field team requires each steady 4.00 m lift of the 6.00 kg load to take no more than 8.00 s. Use the stated simplified models; do not add unprovided battery or motor behaviour.

(a) A replacement winch operates at 12.0 V and 3.20 A with motor-and-drive efficiency 72.0%. Neglect its lead losses and assume constant useful power and speed during the measured lift. Calculate its lift time and decide whether it meets the deadline, despite its higher efficiency. [3 marks]

Input power = $12.0 \times 3.20 = 38.4 \text{ W}$; useful output = $0.720 \times 38.4 = 27.648 \text{ W}$. [1] Time = $mgh/P_{\text{out}} = 235.44/27.648 = 8.52 \text{ s}$. [1] It misses the 8.00 s deadline. Higher efficiency does not guarantee greater useful power when the input powers differ. [1] The original winch has the greater useful power, 29.43 W.

(b) A separate field measurement gives 430 J from the source for each complete 4.00 m retrieval by the original winch and leads, including start-up and stopping. Use this cycle value and ignore other energy use. The battery has 12.0 kJ available, but 10.0% must remain as a reserve. Determine the maximum number of complete retrievals. [2 marks]

Energy available for retrievals = $12.0 \times 10^3 \times 0.900 = 10800 \text{ J}$. [1] Number = $10800/430 = 25.116\dots$, so at most 25 complete retrievals. [1] Round down: 26 would exceed the usable budget. After 25 cycles, 1250 J remains, just above the required 1200 J reserve. Use the stated complete-cycle energy rather than the earlier steady-interval input.

Teaching prompt: Efficiency is a ratio, not an output-power rating. For a limited energy budget, reserve energy first and round a maximum count of completed operations down.

