Trigonometry and vectors for physics solutions

Matching worked solutions and teaching prompts | 34 marks

AQA A Level 7408 and OCR A A Level H556 | Mathematical skills

Skills practised and specification links

Choose trig ratios from the angle; resolve signed components; construct a vector triangle; solve force balances and calculate work by an angled force.
AQA maths MS 0.6, 2.3–2.4, 4.1–4.2, 4.4–4.5; selected applications 3.4.1.1, 3.4.1.7.
OCR A maths M0.6, M2.3–M2.4, M4.1–M4.2, M4.4–M4.5; 2.3.1(a)–(d); 3.2.1(c)–(e); 3.2.3(f); 3.3.1(a)–(b).

An inspection team uses cables and an access rail. Each stage is a separate test. Use a ruler, protractor and calculator in degree mode; g = 9.81 m s^{-2}. Figures are schematic: use the stated values.

1 Read the angle before choosing sin or cos [3 marks]

Locate the angle in a right triangle: adjacent = F cos θ; opposite = F sin θ. Retain component signs and check the quadrant after using inverse trig.

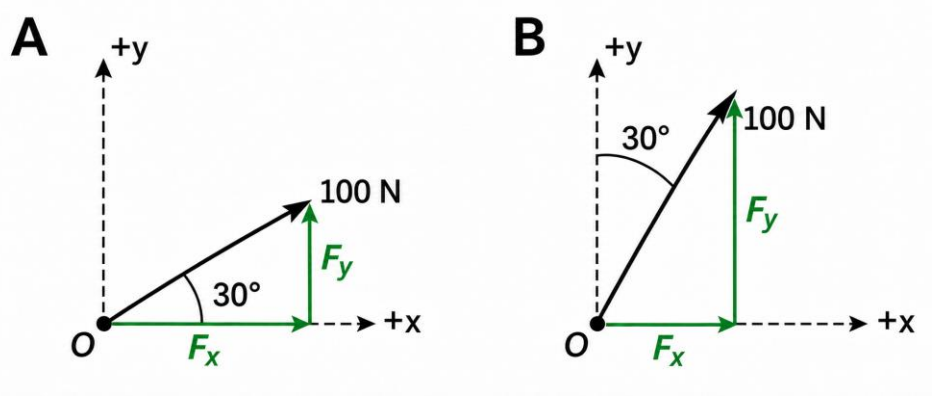


Figure 1 Two positioning forces: the same angle is measured from different axes.

(a) Calculate the horizontal component of the 100 N force in panel A. [1 mark]
(b) Calculate the horizontal component of the 100 N force in panel B. [1 mark]
(c) A technician reports a 112 N horizontal component for a 100 N force. Explain why this is impossible. [1 mark]

Worked answers and marking

(a) F_x = 100 cos 30° = +86.6 N. [1] The horizontal component is adjacent to this angle.
(b) F_x = 100 sin 30° = +50.0 N. [1] The horizontal component is opposite this angle.
(c) A perpendicular component cannot have a magnitude greater than the original vector; |sin θ| and |cos θ| are at most 1. [1]

Teaching prompt Cosine selects the component adjacent to the stated angle. A green triangle represents one force; it does not add independent forces.

2 Resolve the pod's support force [6 marks]

In a separate test, an inspection pod is held at rest by a support cable and a horizontal control rope. Cable tension T = 1.20 kN acts upward and left, at 25.0° to the vertical. The control rope pulls horizontally right with force H. The only other force is the pod's weight W. Take right as +x and up as +y.

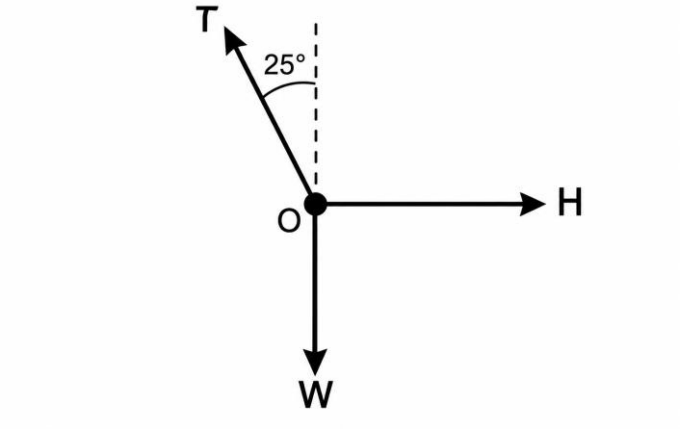


Figure 2 Free-body diagram of the pod; arrow lengths are not a force scale.

(a) Draw a labelled free-body diagram of the pod. Include all three forces and the 25.0° angle. [2 marks]

(b) Calculate the signed horizontal component T_x, in newtons. [1 mark]

(c) Calculate the signed vertical component T_y, in newtons. [1 mark]

(d) In a new setup, $T_y = 0.800T$. Find the acute angle β between the cable and the vertical. [1 mark]

(e) Explain why both perpendicular components cannot each have magnitude T. [1 mark]

Worked answers and marking

(a) T directed up-left along the cable, with 25.0° measured from upward vertical. [1] H horizontal right and W vertically down, labelled and acting on the pod, with no extra forces. [1] Arrows need not be to scale.

(b) $T_x = -1200 \sin 25.0° = -507$ N. [1] The negative sign indicates left; the angle is from vertical.

(c) $T_y = +1200 \cos 25.0° = +1088\ \text{N} \approx +1.09 \times 10^3$ N. [1]

(d) $\cos \beta = T_y/T = 0.800$, so $\beta = \cos^{-1}(0.800) = 36.9°$. [1] Do not use $\sin^{-1}$ for an angle from vertical.

(e) $T^2 = T_x^2 + T_y^2$; if both component magnitudes were T, their resultant would be $\sqrt{2}T$ rather than T. [1] Accept an equivalent correct triangle argument.

Teaching prompt Convert kN to N, establish the signs, and locate the angle. A component pair represents the original force; it does not add two new forces to it.

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3 Construct a resultant for two oblique forces [5 marks]

During a positioning trial, two horizontal-plane forces act on a ring: F_1 = 400 N along +x, and F_2 = 300 N at 120° anticlockwise from +x. Here +y is the perpendicular direction in the same plane. For part (a), use a scale of 1.0 cm to 50 N.

A vector may be translated without rotating it. Draw the second arrow from the head of the first, keeping its direction relative to +x. The resultant runs from the first tail to the final head.

(a) Draw a head-to-tail scale construction and use it to estimate the resultant's magnitude. [2 marks]

(b) A logger reports the resultant's perpendicular components as R_x = +250 N and R_y = +260 N. Calculate its magnitude from these components. [1 mark]

(c) Use the logger's components to find the resultant's direction relative to +x. [1 mark]

(d) Explain why adding 400 N and 300 N to obtain 700 N gives the wrong resultant magnitude here. [1 mark]

Worked answers and marking

(a) Draw F_1 as an 8.0 cm arrow right, then F_2 as a 6.0 cm arrow up-left at 120° from +x. Correct declared scale and head-to-tail arrangement. [1] The resultant from the first tail to final head is about 7.21 cm, giving 361 N (accept about 346–376 N from a sound construction). [1] Example construction coordinates in cm: O = (0,0), A = (8,0), B ≈ (5,5.20).

(b) $R = \sqrt{(250^2 + 260^2)} = 360.7$ N ≈ 361 N. [1] Pythagoras is applied to the two perpendicular components, not directly to F_1 and F_2.

(c) $\theta = \tan^{-1}(260/250) = 46.1°$ anticlockwise from +x, or 46.1° above +x. [1] Both components are positive, so the resultant is in the first quadrant.

(d) Adding magnitudes gives 700 N only if the two forces point in the same direction; these directions differ by 120°. [1]

Teaching prompt Oblique vector addition is tested by scale construction. The numerical check starts from two supplied perpendicular components. AQA restricts direct vector-addition calculations to right-angle vectors.

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4 Find the tensions in the platform supports [7 marks]

Figure 3 shows the upper suspension ring O. Treat the ring as a particle of negligible weight at rest. Upper cables A and B are at 40.0° and 45.0° to the horizontal. The lower link transmits ONE load force L to the ring, with components L_x = +250 N and L_y = −1200 N. Thus only three forces act on O: T_A, T_B and L. Take right and up as positive.

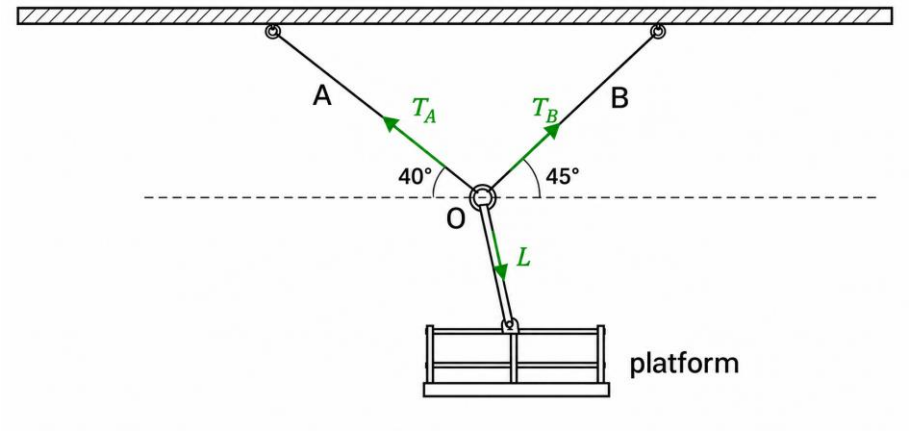


Figure 3 Forces on the ring are transmitted along its three connections; schematic.

(a) Write the horizontal equilibrium equation in terms of T_A and T_B. [1 mark]

(b) Write the vertical equilibrium equation. [1 mark]

(c) Use cos 45° = sin 45° to eliminate T_B, then calculate both tensions. [2 marks]

(d) Check your tensions by substituting them into both signed component balances. [1 mark]

(e) Available cables have permitted working tensions of 0.950 kN or 1.20 kN. Select the smallest rating suitable for both upper cables and justify it. [2 marks]

Worked answers and marking

(a) $-T_A \cos 40.0° + T_B \cos 45.0° + 250 = 0$. [1] Equivalent rearrangement accepted.

(b) $T_A \sin 40.0° + T_B \sin 45.0° - 1200 = 0$. [1]

(c) Rearrange the horizontal balance as $T_A \cos 40° - T_B \cos 45° = 250$. Add the vertical balance: $T_A(\cos 40° + \sin 40°) = 1450$, giving T_A = 1029 N ≈ 1.03 kN. [1] $T_B = (1200 - T_A \sin 40°)/\sin 45°$ = 761 N ≈ 0.761 kN. [1] Accept valid consistent follow-through or another correct simultaneous-equation method.

(d) Using guard digits: $\Sigma F_x = -788.4 + 538.4 + 250 \approx 0$ N; $\Sigma F_y = 661.6 + 538.4 - 1200 \approx 0$ N. Both balances checked. [1] Allow small residuals from rounding.

(e) The larger tension is about 1.03 kN, which exceeds 0.950 kN. [1] Choose the 1.20 kN rating for both cables; both calculated tensions are below it. [1] Use the given permitted working tensions directly.

Teaching prompt The external sideways effect is already included in the single lower-link load. Do not add a fourth force or count the platform's weight again on the ring. Equal tensions require suitable symmetry; these angles and loads are not symmetric.

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5 Resolve forces on the access rail [8 marks]

A separate 40.0 kg inspection carriage moves uphill at constant velocity along the straight frictionless rail in Figure 4, inclined at 18.0° to the horizontal. Its pulling force P acts at 10.0° above the uphill rail direction, away from the rail. The normal contact force has magnitude R. Air resistance is negligible.

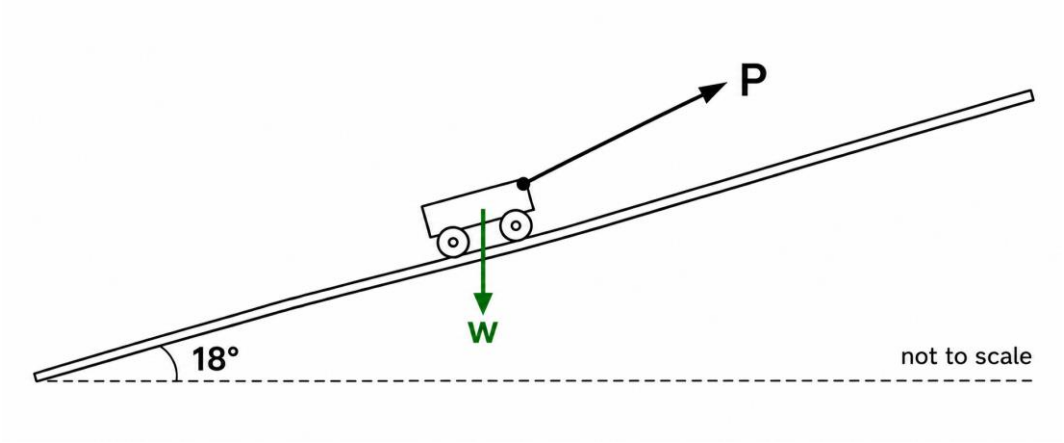


Figure 4 Weight direction for part (a); the contact force R is perpendicular to the rail.

(a) Sketch axes parallel and perpendicular to the rail, then a weight-component triangle. Find the weight components down the rail and into it. [2 marks]

(b) Use the balance along the rail to calculate P. [2 marks]

(c) Use the perpendicular balance to calculate R. Explain why R is less than mg cos 18°. [2 marks]

(d) Calculate the work done by P over 4.00 m uphill. Explain why P × 4.00 m would be incorrect. [2 marks]

Worked answers and marking

(a) The weight is vertical; its angle to the inward normal direction is 18.0°. A correct perpendicular resolution/triangle with the weight as the diagonal. [1] W = mg = 392.4 N; component down the rail = W sin 18° = 121.3 N, and into the rail = W cos 18° = 373.2 N. Both values. [1]

(b) Constant velocity means zero resultant along the rail: P cos 10° − mg sin 18° = 0. [1] P = 121.258/cos 10° = 123.1 N ≈ 123 N. [1]

(c) R + P sin 10° − mg cos 18° = 0: the pull has an outward component, so the contact force supplies less of the required outward balance. [1] R = 373.195 − 123.129 sin 10° = 351.8 N ≈ 352 N. [1]

(d) Only the pull component parallel to displacement does work: E = Ps cos 10°, not Ps, because P is not parallel to the rail. [1] E = 123.129 × 4.00 × cos 10° = 485 J. [1] The net force is zero, but this individual force still does positive work.

Teaching prompt Choose axes for this geometry. Weight components depend on the slope angle 18°; pull components depend on the angle 10° between the pull and rail. The work calculation uses the individual pulling force, not the zero resultant.

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6 Diagnose the vector mistakes [5 marks]

(a) A pupil draws a tension T and both its perpendicular components as three independent forces on the same object. Sketch a correct component representation and explain the error. [2 marks]

(b) Another force has $F_x = -120$ N and $F_y = +160$ N. A calculator gives $\tan^{-1}(160/-120) = -53.1°$. Give its correct direction anticlockwise from +x and explain the quadrant. [2 marks]

(c) A pupil proposes a "horizontal component" of the carriage's kinetic energy. Explain why this is not a valid vector resolution. [1 mark]

Worked answers and marking

(a) A correct representation replaces T by its two perpendicular components, or shows a labelled head-to-tail triangle as a geometric construction. [1] The components together represent T; counting T and the component pair as independent forces counts the same force twice. [1]

(b) Negative x and positive y put the force in quadrant II, up and left. [1] The direction is 180° − 53.1° = 126.9° anticlockwise from +x. [1] Equivalently it is 53.1° above the negative x-axis, but the requested +x reference must be answered.

(c) Kinetic energy is scalar and has no direction, so it is not resolved into directional vector components. [1] Components of velocity may be used to calculate contributions to total kinetic energy; that does not make energy a vector.

Teaching prompt State which force, component, direction or scalar you mean. A calculator supplies an angle value; the component signs identify the correct direction.

Final check Locate the angle, choose axes, retain component signs, name the direction and check the magnitude and units.

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