Using y = mx + c solutions

Matching worked solutions and teaching prompts | 38 marks

AQA A Level 7408 and OCR A A Level H556 | Mathematical skills

Skills practised

Choose axes; compare equations with a straight line; plot data; interpret gradients, intercepts and their units.
AQA maths MS 2.2, 2.3, 3.1–3.4; selected applications 3.4.2.1; 3.5.1.1, 3.5.1.3, 3.5.1.6; 3.6.1.3.
OCR A maths M2.2, M2.3, M3.1–M3.4; selected applications 3.4.1(b), (c), (d)(i); 4.2.3(a); 4.2.4(a)(i); 4.3.2(a)–(c)(i); 5.3.1(a), (c)(ii).

An engineering laboratory commissions an inspection gantry. You will calibrate its spring load sensor, a portable battery supply and a mass–spring mount. The models you need are supplied. Use a calculator and ruler; give numerical results to a sensible precision.

1 Read a straight-line equation [3 marks]

Compare the physical equation with y = mx + c. First name the vertical and horizontal quantities; then identify their coefficients. In this familiar form m means gradient. In Questions 4–5, m means mass, so we call the graph gradient G.
Worked example: F = kx. For F against x, gradient = k and intercept = 0. For x against F, first write x = (1/k)F: gradient = 1/k, not k.
Find a gradient using changes: G = Δy/Δx. Its unit is the vertical-axis unit divided by the horizontal-axis unit. A straight line shows direct proportionality only if it passes through the origin.

(a) For V = E − rI, a graph has V vertically and I horizontally. State its gradient and vertical intercept. [1 mark]
(b) A force sensor has output S = αF + S_0. Output S is measured in volts and force F in newtons. State the SI unit of α. [1 mark]
(c) The graph of S against F is straight but has a non-zero vertical intercept. Is S directly proportional to F? Justify your answer. [1 mark]

Worked answers and marking

(a) Gradient = −r; vertical intercept = E. Both required. [1] The minus sign belongs to the gradient.
(b) α has unit V N^{-1}. [1] It is output change divided by force change, not a force.
(c) No: a direct-proportionality graph must pass through the origin. [1] Equivalent: S/F is not constant when S_0 is non-zero.

Teaching prompt Say which quantity is plotted against which. A coefficient becomes a gradient only after the axes and the equation have been matched.

2 Calibrate the spring load sensor [9 marks]

The spring obeys Hooke's law. A pointer gives a length reading d, with an arbitrary zero-force reading d_0. The actual extension is $d - d_0$. Table 1 gives the calibration data.

Model: $d = d_0 + F/k$, where F is force and k is stiffness. Use metres for d and d_0 when k is in N m^{-1}. The graph uses mm so the small movements are easy to plot.

Table 1

F / N	0	1.0	2.0	3.0	4.0	5.0
d / mm	2.0	6.2	9.8	14.2	17.8	22.0

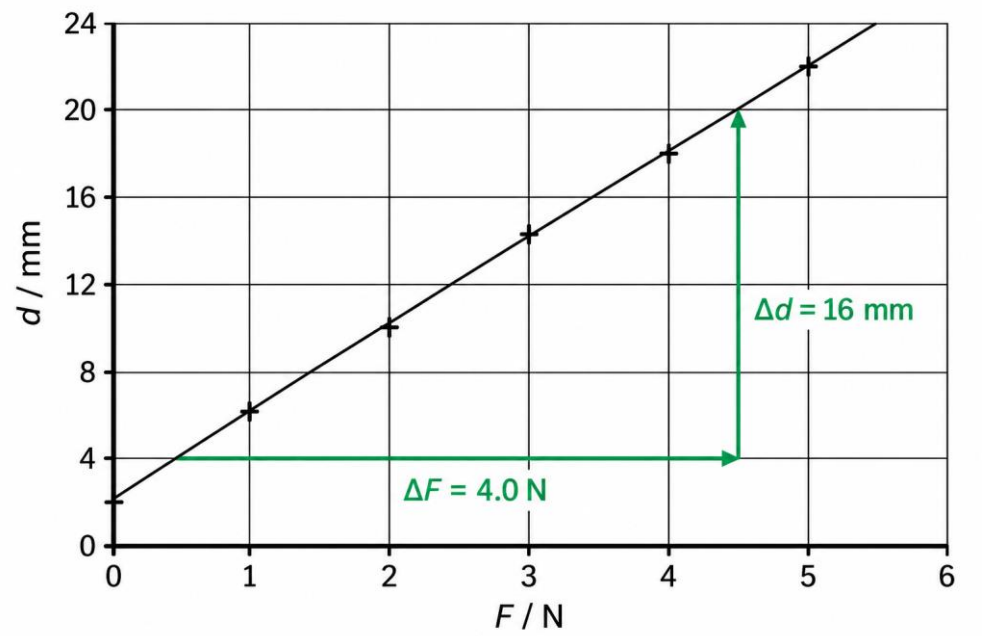


Figure 1 Pointer reading against applied force. Estimate between the coarse grid lines. Representative gradient triangle shown.

(a) Plot all six points from Table 1 on Figure 1 and draw a straight line of best fit. [2 marks]

(b) Use a large gradient triangle on your line to determine the gradient, including its unit. [2 marks]

(c) Use the model and your gradient to determine k in N m^{-1}. [2 marks]

(d) Estimate d_0 from the graph. Does the spring have this extension when F = 0? Explain. [2 marks]

(e) Predict the pointer reading for an applied force of 3.5 N. [1 mark]

Worked answers and marking

(a) Six points plotted within about ±0.5 mm in d and ±0.1 N in F. [1] A sensible straight fit spanning the data, not a dot-to-dot line. [1] Representative line: $d/\text{mm} \approx 4.0(F/\text{N}) + 2.0$.

(b) Example: from (0.5 N, 4.0 mm) to (4.5 N, 20.0 mm), $\Delta d = 16.0$ mm and $\Delta F = 4.0$ N. Large triangle shown. [1] G = 16.0/4.0 = 4.0 mm N^{-1}. [1] Accept about 3.8–4.2 mm N^{-1} from a reasonable fit.

(c) G = 4.0 mm N^{-1} = 0.0040 m N^{-1}. [1] Since G = 1/k, k = 1/0.0040 = 250 N m^{-1}. [1] Accept a value consistent with the pupil's fitted gradient (about 238–263 N m^{-1}).

(d) $d_0 \approx 2.0$ mm, accepting about 1.5–2.5 mm. [1] No: d_0 is the pointer's reference reading; extension = $d - d_0 = 0$ at F = 0. [1]

(e) From the line: $d \approx 4.0 \times 3.5 + 2.0 = 16.0$ mm. [1] Accept about 15.5–16.5 mm or a consistent read-off from the pupil's sensible fit.

Teaching prompt The plotted quantity is the pointer reading, not extension. Convert mm N^{-1} to m N^{-1} before taking the reciprocal for SI stiffness.

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3 Interpret the battery calibration [7 marks]

A portable battery powers the gantry's calibration instruments. Figure 2 is its fitted terminal-voltage graph. Treat emf E and internal resistance r as constant over the tested range. Use V = E − rI.

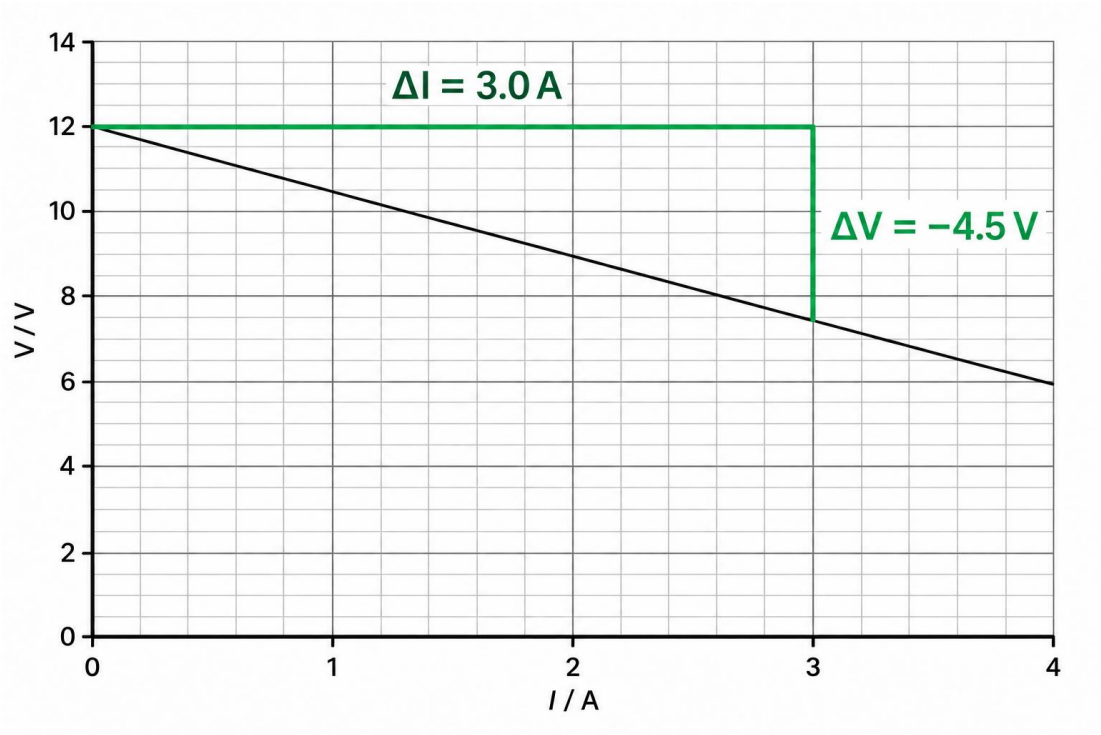


Figure 2 Terminal voltage V against current I. Representative gradient triangle shown.

(a) Determine the emf E from Figure 2. [1 mark]

(b) Use a large gradient triangle to calculate the signed gradient, including its unit. [2 marks]

(c) Hence determine the internal resistance r. [1 mark]

(d) Calculate the terminal voltage when the current is 2.40 A. [1 mark]

(e) At I = 2.0 A, a pupil calculates V/I = 4.5 Ω and calls this the internal resistance. Identify what this ratio represents and explain the error. [2 marks]

Worked answers and marking

(a) At I = 0, V = E = 12.0 V. [1]

(b) Use points such as (0 A, 12.0 V) and (3.0 A, 7.5 V): ΔV = −4.5 V and ΔI = 3.0 A. Triangle or clearly identified widely separated points. [1] G = −4.5/3.0 = −1.50 V A^{-1}. [1] Accept −1.45 to −1.55 V A^{-1} with valid graphical working.

(c) G = −r, hence r = 1.50 Ω. [1] The negative gradient does not make the resistance negative.

(d) V = 12.0 − 1.50 × 2.40 = 8.40 V. [1]

(e) V/I is the resistance of the external load. [1] Internal resistance is the magnitude of the V–I gradient, 1.50 Ω, or (E − V)/I; it is not terminal V divided by I. [1]

Teaching prompt Keep the signed gradient separate from the positive internal resistance. A downward line does not mean that terminal voltage itself is negative.

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4 Choose a useful graph for oscillations [6 marks]

The gantry camera sits on a mass–spring mount. An added mass m moves with a fixture of fixed mass m_0. For small oscillations, assume negligible damping and a spring of negligible mass. The supplied model is $T = 2\pi\sqrt{[(m + m_0)/k]}$, where T is period and k is stiffness.

A curved relationship may become linear after transforming one or both variables. Worked example: if $z = \sqrt{(Ax + B)}$, then $z^2 = Ax + B$. Plot z^2 against x to obtain gradient A and intercept B.

Table 2

m / kg	0.040	0.120	0.240	0.400
T / s	0.400	0.566	0.748	0.938
T^2 / s^2				

(a) Complete the T^2 row in Table 2. Give each value to three significant figures. [2 marks]

(b) Rearrange the supplied mount model into a straight-line equation. State what to plot on each axis and identify its gradient G and vertical intercept c. [2 marks]

(c) Rearrange your expression for G to make k the subject. [1 mark]

(d) Explain why a graph of T against m is not expected to be a straight line for this model. [1 mark]

Worked answers and marking

(a) T^2 = 0.160, 0.320, 0.560 and 0.880 s^2. [1] for the first pair and [1] for the second pair. Raw squares are 0.160000, 0.320356, 0.559504 and 0.879844.

(b) $T^2 = (4\pi^2/k)m + 4\pi^2 m_0/k$. Correct straight-line rearrangement. [1] Plot T^2 vertically against m horizontally; $G = 4\pi^2/k$ and $c = 4\pi^2 m_0/k$. Correct matching of axes and both coefficients. [1]

(c) $k = 4\pi^2/G$. [1]

(d) T depends on the square root of $m + m_0$, so it varies non-linearly with m; T^2, not T, is linear in m. [1]

Teaching prompt Square the period as well as the numerical coefficient. An axis labelled T^2 must contain squared period values and have unit s^2.

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5 Find the hidden moving mass [9 marks]

A second camera mount follows the same model as Question 4 but has different k and m_0. Figure 3 is a best-fit graph for this second mount; do not reuse the numerical values in Table 2. The added mass m is measured in kg.

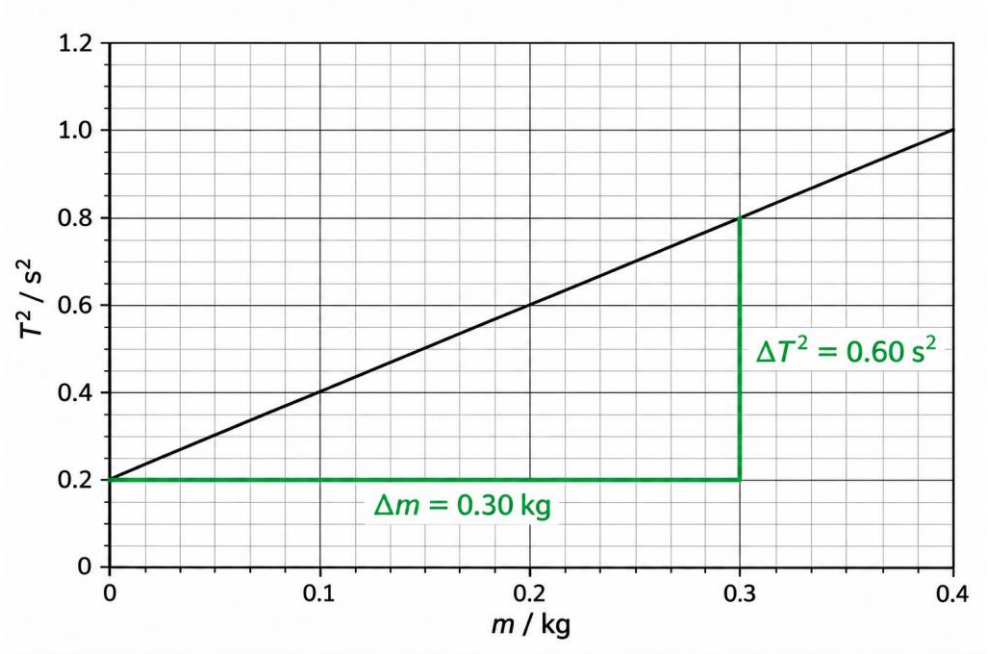


Figure 3 Squared period against added mass for the second mount. Representative gradient triangle shown.

(a) Use a large gradient triangle to determine G, including its unit. [2 marks]

(b) Calculate the stiffness k of this mount. [1 mark]

(c) Use the graph's intercept and gradient to determine m_0. [2 marks]

(d) The mount must have a period no greater than 0.800 s with an added camera mass of 0.250 kg. Calculate the period and decide whether it meets this requirement. [2 marks]

(e) A pupil forces the fitted line through the origin, saying that zero added mass means zero oscillating mass. Explain why this is unjustified and how it could affect k. [2 marks]

Worked answers and marking

(a) Example: $\Delta T^2 = 0.60\ s^2$ for $\Delta m = 0.30$ kg. Large triangle or clearly identified widely separated line points. [1] $G = 0.60/0.30 = 2.00\ s^2\ kg^{-1}$. [1] Accept about 1.9–2.1 $s^2\ kg^{-1}$.

(b) $k = 4\pi^2/2.00 = 19.7\ N\ m^{-1}$. [1] Accept a value consistent with the pupil's gradient. The unit kg s^{-2} is equivalent.

(c) $c \approx 0.20\ s^2$ and $c = Gm_0$, so $m_0 = c/G$. Correct relationship using the intercept. [1] $m_0 = 0.20/2.00 = 0.100$ kg. [1] Accept about 0.09–0.11 kg for a valid read-off.

(d) $T^2 = 2.00 \times 0.250 + 0.20 = 0.700\ s^2$; $T = \sqrt{0.700} = 0.837$ s. [1] It does not meet the requirement because 0.837 s > 0.800 s. [1] Use a consistent result from the pupil's valid graph/model.

(e) The fixture still has mass m_0 when the added mass is zero, so the model predicts a non-zero intercept and period. [1] Forcing the fit through zero changes its gradient and therefore the inferred $k = 4\pi^2/G$. [1] For these positive-intercept data a through-origin fit typically overestimates G and underestimates k.

Teaching prompt A zero added mass is not a zero total mass. T^2 is linear in m, and a non-zero intercept is part of this model rather than automatic evidence of a zero error.

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6 Transfer the method to an unfamiliar model [4 marks]

The laboratory also tests thin connecting wires of equal length and material at constant temperature. A supplied model for the measured resistance is $R = K/D^2 + R_c$. Here D is wire diameter in metres, K is constant and R_c is a constant contact resistance. The model is the only extra physics needed.

Choose an axis quantity that makes the variable part a single multiple of x. Do not assume that plotting the two original measured quantities will give a straight line.

(a) Choose a straight-line plot that would determine both K and R_c. State the vertical and horizontal quantities, the gradient and the vertical intercept. [2 marks]

(b) State the SI unit of K. [1 mark]

(c) For non-zero R_c, explain why R is not directly proportional to $1/D^2$, even though your chosen graph can be straight. [1 mark]

Worked answers and marking

(a) Write $R = K(1/D^2) + R_c$. A useful plot is R vertically against $1/D^2$ horizontally. [1] Its gradient is K and its vertical intercept is R_c. [1] Also accept $1/D^2$ against R with gradient 1/K and intercept $-R_c/K$ if all interpretations are consistent.

(b) K has unit $\Omega\ m^2$. [1] For R against $1/D^2$, gradient units are Ω divided by m^{-2}.

(c) The plot has a non-zero offset R_c, so it does not pass through the origin. [1] $R - R_c$ is proportional to $1/D^2$; R is not.

Teaching prompt Separate linearity from direct proportionality. Name the transformed horizontal quantity and check the dimensions of the recovered coefficient.

Final check Name the axes, rearrange, compare coefficients, check units and interpret the intercept.

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