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WORKSHEETS
ATTEMPT | CHECK | CORRECT | IMPROVE

Simultaneous equations and physical roots

35–40 minutes independent work + 15–20 minutes review | 38 marks

AQA A Level 7408 and OCR A A Level H556 | Mathematical skills

Skills practised and specification links

Construct simultaneous and quadratic equations; use common coordinates and clocks; interpret roots, intersections and the limits of motion models.
AQA maths MS 2.1–2.4, 3.1, 3.3; selected application 3.4.1.3.
OCR A maths M2.1–M2.4, M3.1, M3.3; selected applications 3.1.1(a)–(c), 3.1.2(a)(i).

An automated inspection rig checks when trolley T aligns with loading carriage C on separate parallel lanes. Alignment means the same longitudinal position, not a collision. Each trial resets the clock. Use a common origin O, positive along the track. Distances are in metres and times in seconds unless stated.

1 Define the event before solving [3 marks]

Use one origin and one clock for both objects. An alignment means that their longitudinal positions are equal at the same time. It need not mean equal velocities.

(a) Rearrange $x = 4.0 + 1.5t$ to make t the subject. [1 mark]

(b) Expand $(t - 2)^2$. [1 mark]

(c) Both objects move in the positive direction. T starts at $x = 0$ and C starts farther along the track. Explain why equal distances travelled do not establish an alignment. [1 mark]

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2 Eliminate an unknown from two linear models [6 marks]

Trial A: at t = 0, T is at x = 0 and C is at x = 16.0 m. T moves uniformly and reaches x = 12.0 m at t = 4.00 s. C moves uniformly at +1.00 m s^{-1}. Both models apply for 0 ≤ t ≤ 10.0 s.

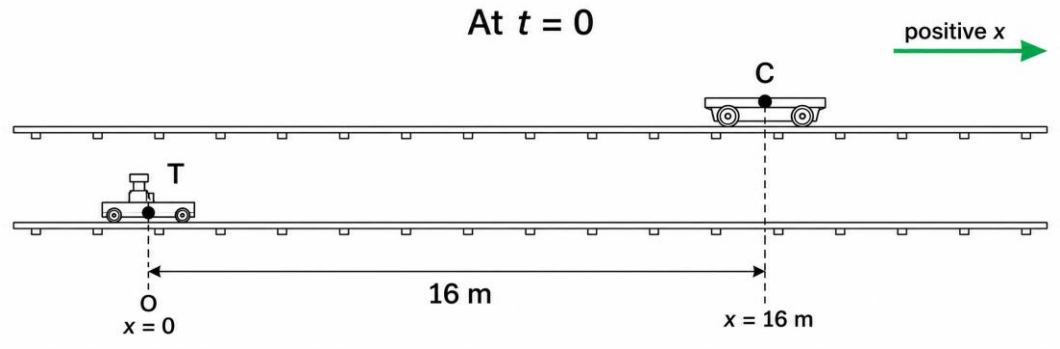


Figure 1 Separate parallel lanes; the dots define longitudinal position. Schematic.

(a) Find T's velocity. Write simultaneous equations for the alignment position x and clock time t. [2 marks]

(b) Eliminate x and solve for the alignment time. [1 mark]

(c) Find the alignment position. Check it by substituting your time into both original position models. [2 marks]

(d) Explain why setting $v_T = v_C$ would fail to find this alignment. [1 mark]

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3 Construct and solve a quadratic [8 marks]

Trial B: T starts from rest at x = 0 when t = 0 and has constant acceleration +1.00 m s^{-2}. C starts at x = 12.0 m and moves at constant velocity +1.00 m s^{-1}. The trial runs from t = 0 to 8.00 s.

For constant acceleration, $x = x_0 + u\Delta t + \frac{1}{2}a(\Delta t)^2$. For $At^2 + Bt + C = 0$, $t = (-B \pm \sqrt{(B^2 - 4AC)})/(2A)$. Keep both roots until the physical domain has been checked.

(a) Write a position model for each object. Hence form a quadratic equation for an alignment, with zero on one side. [2 marks]

(b) State the numerical coefficients A, B and C for your quadratic, using t in seconds. [1 mark]

(c) Use the quadratic formula, showing both signs, to find both roots. [2 marks]

(d) Select the root that describes an alignment in this trial. Explain why the other is rejected. [1 mark]

(e) At the valid alignment, calculate the common position and the velocity of T. [2 marks]

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4 Use a delayed clock correctly [8 marks]

Trial C uses one clock, started at t = 0. C has position $x_C = 2.00 + 2.00t$. T stays at x = 0 until t = 2.00 s, then starts from rest with constant acceleration +1.50 m s^{-2}. This acceleration continues until t = 8.00 s. Positions are in metres and clock time is in seconds.

Elapsed time since launch = clock time − launch time. Use that elapsed time in every term of T's motion model, and state when the model applies.

(a) Give T's elapsed time since launch and the valid clock-time interval for its accelerating model. [1 mark]

(b) Construct T's position model after launch, then write the alignment equation before expanding it. [2 marks]

(c) Expand and collect terms to obtain $3t^2 - 20t + 4 = 0$. Show your working. [1 mark]

(d) Solve this quadratic. Explain which root describes a real alignment in Trial C, and why rejecting a negative root is not a sufficient rule here. [3 marks]

(e) Use $v = u + a\Delta t$ to find T's velocity at the valid alignment. Decide which object is moving faster. [1 mark]

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5 Keep two genuine encounters [8 marks]

Trial D: for $0 \le t \le 5.00$ s, $x_T = 5.00t - 0.500t^2$ and $x_C = 4.00 + 2.00t$. Positions are in m. T slows uniformly; neither object reverses in this interval. Figure 2 shows these models. Use algebra for the numerical answers.

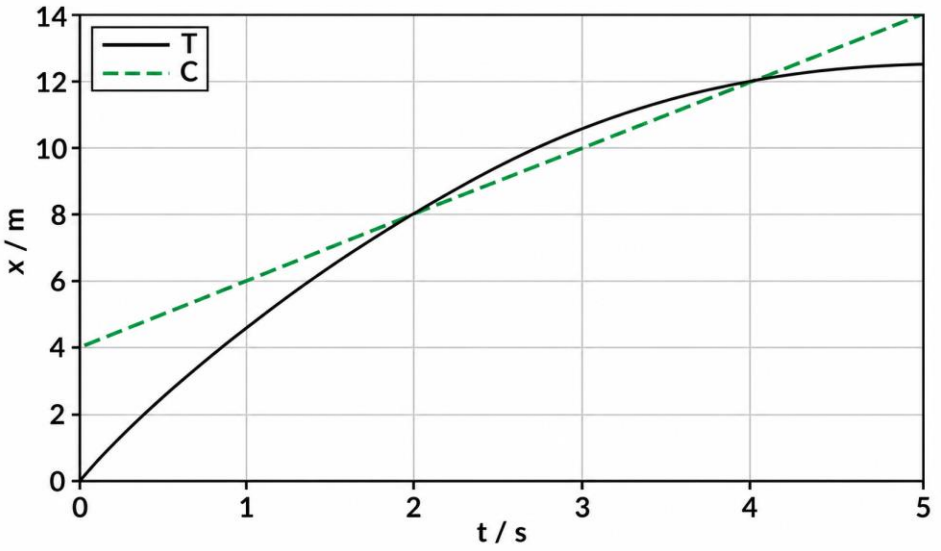


Figure 2 Original Trial D models. Both intersections are within the stated interval.

(a) Form and solve the alignment quadratic. Retain both roots. [2 marks]

(b) Check each root in both position models and state the position of each encounter. [2 marks]

(c) Use $v_T = 5.00 - 1.00t$ and $v_C = 2.00$ to explain which object passes the other at each encounter. [2 marks]

(d) In a repeat, a controller changes T's motion at $t = 3.50$ s. C is unchanged. Which original encounter remains predicted? Can the old equations establish an encounter at 4.00 s? Explain. [2 marks]

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6 Find a design that can align [5 marks]

Trial E: for $0 \le t \le 4.00$ s, $x_T = 4.00t - 0.500t^2$ and $x_C = 6.00 + 1.00t$, with x in m and t in s. Figure 3 compares these original models qualitatively. C's starting position d can be adjusted; its velocity stays at +1.00 m s^{-1}.

The discriminant is $D = B^2 - 4AC$. $D < 0$ gives no real roots; $D = 0$ gives one repeated root; $D > 0$ gives two distinct real roots. Check the physical domain as well.

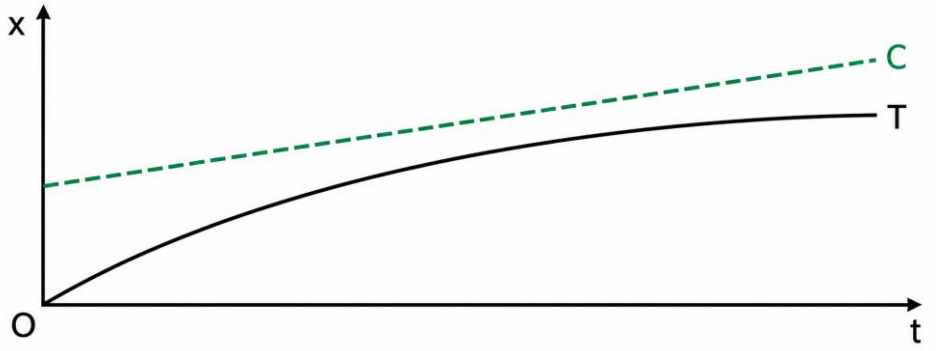


Figure 3 Qualitative only: use the equations for numerical results.

(a) Form the alignment quadratic for the original trial and calculate its discriminant. Explain how the result agrees with Figure 3. [2 marks]

(b) Now use $x_C = d + 1.00t$. Find d so that the objects align at t = 3.00 s. [1 mark]

(c) For your new d, use $v_T = 4.00 - 1.00t$ and compare positions at t = 2.00 s and 4.00 s. Explain whether the alignment at 3.00 s involves overtaking. [2 marks]

Final check Common origin and clock; correct event equation; both roots; launch and operating intervals; substitution into the original models.

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