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SOLUTIONS
ATTEMPT | CHECK | CORRECT | IMPROVE

Simultaneous equations and physical roots solutions

Matching worked solutions and teaching prompts | 38 marks

AQA A Level 7408 and OCR A A Level H556 | Mathematical skills

Skills practised and specification links

Construct simultaneous and quadratic equations; use common coordinates and clocks; interpret roots, intersections and the limits of motion models.
AQA maths MS 2.1–2.4, 3.1, 3.3; selected application 3.4.1.3.
OCR A maths M2.1–M2.4, M3.1, M3.3; selected applications 3.1.1(a)–(c), 3.1.2(a)(i).

An automated inspection rig checks when trolley T aligns with loading carriage C on separate parallel lanes. Alignment means the same longitudinal position, not a collision. Each trial resets the clock. Use a common origin O, positive along the track. Distances are in metres and times in seconds unless stated.

1 Define the event before solving [3 marks]

Use one origin and one clock for both objects. An alignment means that their longitudinal positions are equal at the same time. It need not mean equal velocities.

(a) Rearrange $x = 4.0 + 1.5t$ to make t the subject. [1 mark]

(b) Expand $(t - 2)^2$. [1 mark]

(c) Both objects move in the positive direction. T starts at $x = 0$ and C starts farther along the track. Explain why equal distances travelled do not establish an alignment. [1 mark]

Worked answers and marking

(a) $t = (x - 4.0)/1.5$. [1] Subtract the initial position before dividing by the speed.

(b) $(t - 2)^2 = t^2 - 4t + 4$. [1] The middle term must be retained.

(c) Their different starting positions remain part of the position equations. Equal travel distances leave the initial separation unchanged; alignment instead requires $x_T = x_C$ at the same time. [1]

Teaching prompt Ask what the unknown describes: position, elapsed time or clock time. Write the event condition before choosing an equation.

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2 Eliminate an unknown from two linear models [6 marks]

Trial A: at t = 0, T is at x = 0 and C is at x = 16.0 m. T moves uniformly and reaches x = 12.0 m at t = 4.00 s. C moves uniformly at +1.00 m s^{-1}. Both models apply for 0 ≤ t ≤ 10.0 s.

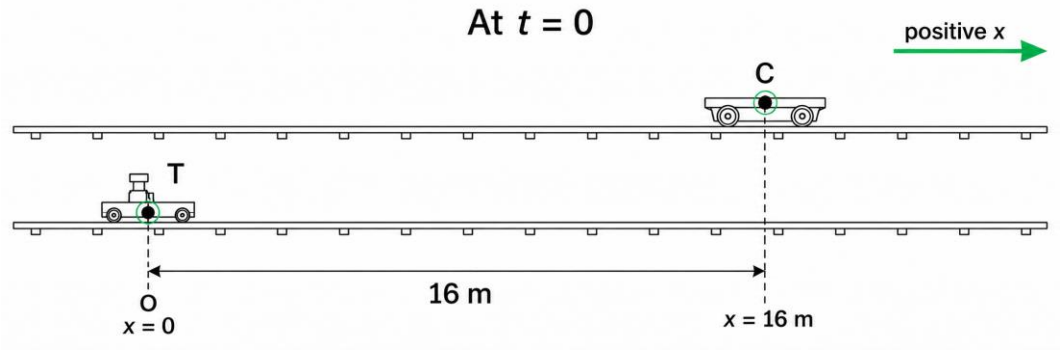


Figure 1 Separate parallel lanes; the dots define longitudinal position. Schematic.

(a) Find T's velocity. Write simultaneous equations for the alignment position x and clock time t. [2 marks]

(b) Eliminate x and solve for the alignment time. [1 mark]

(c) Find the alignment position. Check it by substituting your time into both original position models. [2 marks]

(d) Explain why setting $v_T = v_C$ would fail to find this alignment. [1 mark]

Worked answers and marking

(a) $v_T = (12.0 - 0)/4.00 = +3.00$ m s^{-1}. [1] At alignment, x = 3.00t and x = 16.0 + 1.00t, with x in m and t in s. Both equations. [1]

(b) 3.00t = 16.0 + 1.00t; 2.00t = 16.0, so t = 8.00 s. [1] Equating the two expressions is elimination of their common unknown x.

(c) x = 24.0 m. [1] T: 3.00 × 8.00 = 24.0 m. C: 16.0 + 1.00 × 8.00 = 24.0 m. Explicit check of both. [1]

(d) Their velocities are constantly +3.00 and +1.00 m s^{-1} and never equal, yet their positions become equal at 8.00 s. Equal velocities describe a different condition. [1]

Teaching prompt x is position relative to O, not the distance travelled by each object. Eliminate x, find t, then substitute back into BOTH original models.

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3 Construct and solve a quadratic [8 marks]

Trial B: T starts from rest at x = 0 when t = 0 and has constant acceleration +1.00 m s^{-2}. C starts at x = 12.0 m and moves at constant velocity +1.00 m s^{-1}. The trial runs from t = 0 to 8.00 s.

For constant acceleration, $x = x_0 + u\Delta t + \frac{1}{2}a(\Delta t)^2$. For $At^2 + Bt + C = 0$, $t = (-B \pm \sqrt{(B^2 - 4AC)})/(2A)$. Keep both roots until the physical domain has been checked.

(a) Write a position model for each object. Hence form a quadratic equation for an alignment, with zero on one side. [2 marks]

(b) State the numerical coefficients A, B and C for your quadratic, using t in seconds. [1 mark]

(c) Use the quadratic formula, showing both signs, to find both roots. [2 marks]

(d) Select the root that describes an alignment in this trial. Explain why the other is rejected. [1 mark]

(e) At the valid alignment, calculate the common position and the velocity of T. [2 marks]

Worked answers and marking

(a) $x_T = 0.500t^2$ and $x_C = 12.0 + 1.00t$. Correct models including the ½ factor. [1] Equating and collecting gives $0.500t^2 - 1.00t - 12.0 = 0$. [1] Positions in m and time in s.

(b) A = 0.500, B = −1.00, C = −12.0. [1] These are the numerical coefficients in the stated metre–second convention; A is not the trolley's acceleration.

(c) $t = [1.00 \pm \sqrt{(1.00 + 24.0)}]/1.00 = 1.00 \pm 5.00$. Correct substitution including both signs. [1] t = +6.00 s and −4.00 s. Both roots. [1]

(d) Use t = 6.00 s, within 0–8.00 s. The −4.00 s root extrapolates the models to a time before the trial starts, so it does not describe an event in this trial. [1] A negative root is not automatically meaningless in every possible problem.

(e) At t = 6.00 s: $x_T = 0.500 \times 6.00^2 = 18.0$ m; $x_C = 12.0 + 6.00 = 18.0$ m. Common position. [1] $v_T = u + at = 0 + 1.00 \times 6.00 = +6.00$ m s^{-1}. [1]

Teaching prompt Check signs after collecting terms. A root can solve the algebra while lying outside the physical time interval. Root selection needs a reason tied to the trial.

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4 Use a delayed clock correctly [8 marks]

Trial C uses one clock, started at t = 0. C has position $x_C = 2.00 + 2.00t$. T stays at x = 0 until t = 2.00 s, then starts from rest with constant acceleration +1.50 m s^{-2}. This acceleration continues until t = 8.00 s. Positions are in metres and clock time is in seconds.

Elapsed time since launch = clock time − launch time. Use that elapsed time in every term of T's motion model, and state when the model applies.

(a) Give T's elapsed time since launch and the valid clock-time interval for its accelerating model. [1 mark]
(b) Construct T's position model after launch, then write the alignment equation before expanding it. [2 marks]
(c) Expand and collect terms to obtain $3t^2 - 20t + 4 = 0$. Show your working. [1 mark]
(d) Solve this quadratic. Explain which root describes a real alignment in Trial C, and why rejecting a negative root is not a sufficient rule here. [3 marks]
(e) Use $v = u + a\Delta t$ to find T's velocity at the valid alignment. Decide which object is moving faster. [1 mark]

Worked answers and marking

(a) $\Delta t = t - 2.00$, with $2.00 \le t \le 8.00$ s. Both the elapsed-time expression and valid domain. [1]

(b) $x_T = ½ \times 1.50 \times (t - 2.00)^2 = 0.750(t - 2.00)^2$. Correct model. [1] Alignment: $0.750(t - 2.00)^2 = 2.00 + 2.00t$. [1]

(c) $0.750(t^2 - 4t + 4) = 2 + 2t$; $0.750t^2 - 5t + 1 = 0$. Multiplying by 4 gives $3t^2 - 20t + 4 = 0$. Expansion and collection shown. [1]

(d) $t = [20 \pm \sqrt{(400 - 48)}]/6 = (10 \pm 2\sqrt{22})/3$. Roots: 0.206 s and 6.46 s. Both values. [1] The 6.46 s root lies within 2.00–8.00 s and is valid. [1] Check using the unrounded root: $x_T = 0.750(6.4603\ldots - 2)^2 = 14.9206\ldots$ m and $x_C = 2 + 2(6.4603\ldots) = 14.9206\ldots$ m. The other root is positive but occurs before launch, when T is stationary at x = 0, not following the quadratic model. At that clock time C is at about 2.41 m, so no alignment occurs. [1]

(e) $v_T = 1.50(6.4603\ldots - 2.00) = +6.69$ m s^{-1}, greater than C's +2.00 m s^{-1}, so T is moving faster. [1] Using 1.50t would incorrectly count the waiting time as acceleration.

Teaching prompt The smaller root is positive and still invalid. Test the original piecewise motion: $x_T = 0$ before launch; the accelerating expression applies only from 2.00 s.

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5 Keep two genuine encounters [8 marks]

Trial D: for $0 \le t \le 5.00$ s, $x_T = 5.00t - 0.500t^2$ and $x_C = 4.00 + 2.00t$. Positions are in m. T slows uniformly; neither object reverses in this interval. Figure 2 shows these models. Use algebra for the numerical answers.

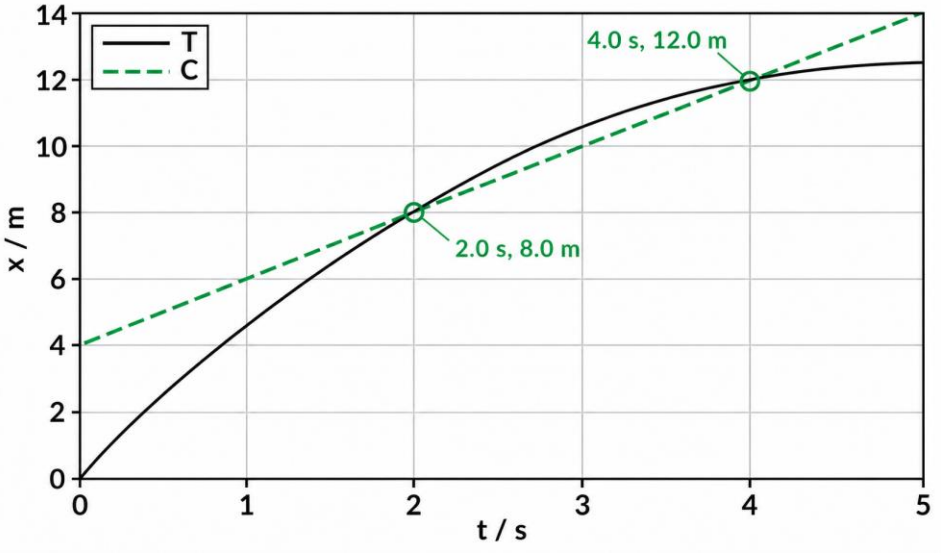


Figure 2 Original Trial D models. Both intersections are within the stated interval.

(a) Form and solve the alignment quadratic. Retain both roots. [2 marks]

(b) Check each root in both position models and state the position of each encounter. [2 marks]

(c) Use $v_T = 5.00 - 1.00t$ and $v_C = 2.00$ to explain which object passes the other at each encounter. [2 marks]

(d) In a repeat, a controller changes T's motion at t = 3.50 s. C is unchanged. Which original encounter remains predicted? Can the old equations establish an encounter at 4.00 s? Explain. [2 marks]

Worked answers and marking

(a) $5t - 0.500t^2 = 4 + 2t$ gives $t^2 - 6t + 8 = 0$. Correct equation. [1] $(t - 2)(t - 4) = 0$, so t = 2.00 s and 4.00 s. Both roots. [1]

(b) At 2.00 s: $x_T = 10 - 2 = 8.00$ m and $x_C = 4 + 4 = 8.00$ m. [1] At 4.00 s: $x_T = 20 - 8 = 12.0$ m and $x_C = 4 + 8 = 12.0$ m. [1] Each mark requires both original-model checks and the corresponding common position.

(c) At 2.00 s, $v_T = +3.00$ m s^{-1} exceeds $v_C = +2.00$ m s^{-1}; T catches and passes C from behind. [1] At 4.00 s, $v_T = +1.00$ m s^{-1} is less than v_C, so C catches and passes T. [1] T's velocity decreases linearly; it remains positive until 5.00 s.

(d) The encounter at 2.00 s remains predicted because both original models still apply then. [1] The 4.00 s root is beyond the change: without T's new motion model it is not a justified prediction. [1] Do not conclude that an alignment at 4.00 s is impossible; the new motion could produce one.

Teaching prompt Both positive roots are genuine in the original trial. Their velocities explain the order of passing. A later control change removes a prediction, not necessarily the possibility of an event.

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6 Find a design that can align [5 marks]

Trial E: for $0 \le t \le 4.00$ s, $x_T = 4.00t - 0.500t^2$ and $x_C = 6.00 + 1.00t$, with x in m and t in s. Figure 3 compares these original models qualitatively. C's starting position d can be adjusted; its velocity stays at +1.00 m s^{-1}.

The discriminant is $D = B^2 - 4AC$. $D < 0$ gives no real roots; $D = 0$ gives one repeated root; $D > 0$ gives two distinct real roots. Check the physical domain as well.

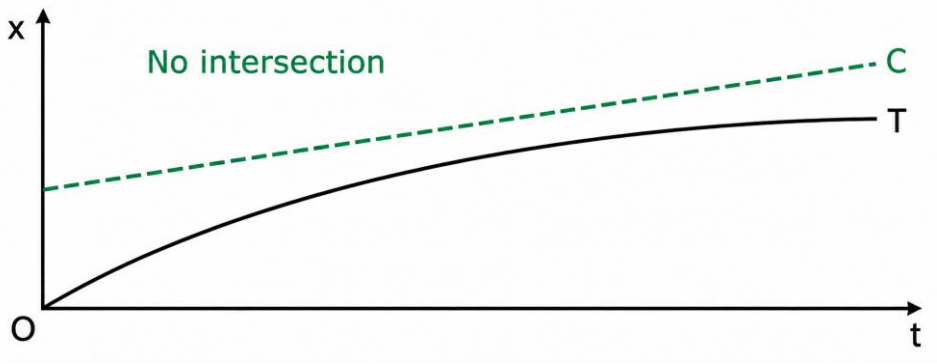


Figure 3 Qualitative only: use the equations for numerical results.

(a) Form the alignment quadratic for the original trial and calculate its discriminant. Explain how the result agrees with Figure 3. [2 marks]

(b) Now use $x_C = d + 1.00t$. Find d so that the objects align at t = 3.00 s. [1 mark]

(c) For your new d, use $v_T = 4.00 - 1.00t$ and compare positions at t = 2.00 s and 4.00 s. Explain whether the alignment at 3.00 s involves overtaking. [2 marks]

Worked answers and marking

(a) $0.500t^2 - 3.00t + 6.00 = 0$; $D = (-3.00)^2 - 4 \times 0.500 \times 6.00 = -3.00$. Correct equation and discriminant. [1] $D < 0$ gives no real alignment time, consistent with C's position trace remaining above T's without an intersection. [1]

(b) $x_T(3.00) = 4.00 \times 3.00 - 0.500 \times 3.00^2 = 7.50$ m. $d + 3.00 = 7.50$, so $d = 4.50$ m. [1]

(c) At t = 3.00 s both velocities are +1.00 m s^{-1}. [1] At 2.00 s T is at 6.00 m and C at 6.50 m; at 4.00 s T is at 8.00 m and C at 8.50 m. T is behind on both sides of the alignment, so they align momentarily without either overtaking. [1] Equivalently the new quadratic is $0.500(t - 3)^2 = 0$: the two position traces touch at a repeated root.

Teaching prompt A repeated root can describe a tangency, not a crossing. Equal positions and equal velocities occur together in this design, but they are different conditions.

Final check Common origin and clock; correct event equation; both roots; launch and operating intervals; substitution into the original models.

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