Radians sine cosine and phase

35–40 minutes independent work + 15–20 minutes review | 38 marks

AQA A Level 7408 and OCR A A Level H556 | Mathematical skills

Skills practised and specification links

Use radians and angular frequency; choose sine/cosine from initial conditions; draw periodic traces; interpret phase, inverse-trig branches and squared functions.
AQA maths MS 0.6, 2.2–2.3, 3.1–3.2, 3.12, 4.5, 4.7; selected applications 3.6.1.1–3.6.1.3.
OCR A maths M0.6, M2.2–M2.3, M3.1–M3.2, M3.12, M4.5, M4.7; 5.2.1(a)–(c); 5.3.1(a), (b), (d), (g); 5.3.2(a).

An inspection scanner moves a camera head across a surface. Each stage is a separate ideal test. Ignore damping; use a ruler and calculator. Check calculator mode and show working. Times in equations are in seconds.

1 Connect radians to the timing disc [4 marks]

Radian angle θ = s/r (arc/radius, same length units). One turn = 2π rad = 360°.

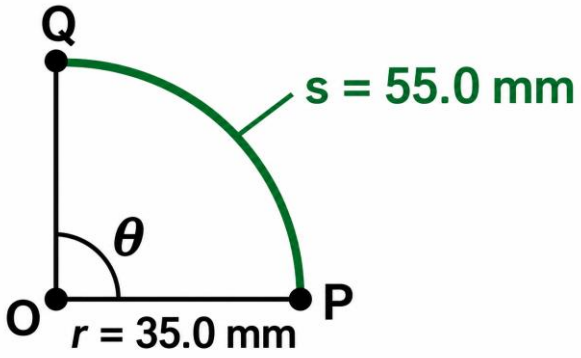


Figure 1 A marked arc on the scanner's circular timing disc; schematic.

(a) Express one quarter of a complete turn in radians. [1 mark]

(b) Convert 135° to radians, giving an exact multiple of π. [1 mark]

(c) Use the values in Figure 1 to calculate θ. Decide whether it is approximately a quarter turn. [2 marks]

2 Set the drive speed and calculator mode [6 marks]

The timing disc turns uniformly at f = 2.00 revolutions per second. One complete disc turn sets one complete scanner cycle. Let ω be the angular speed of the disc and the angular frequency of the scanner. Use $\omega = 2\pi f = 2\pi/T$.

(a) Calculate the period T of one scanner cycle. [1 mark]

(b) Calculate ω and explain why it is not numerically equal to f. [2 marks]

(c) Calculate the disc's angular advance in 0.0750 s, in radians. [1 mark]

(d) A pupil enters cos(ωt) at t = 0.0750 s in degree mode and obtains almost 1. Explain the error and give the correct value of cos(ωt). [2 marks]

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

3 Choose an equation from the starting motion [8 marks]

Two separate test heads have amplitude A = 6.00 mm and angular frequency $\omega = 4\pi$ rad s^{-1}. Displacement is measured from equilibrium, positive to the right. Head P starts at its positive extreme at t = 0 and then moves left. Head Q starts at equilibrium at t = 0, moving left. Keep A positive.

Test the starting value AND the direction just after t = 0. For example, cos 0 = 1 but sin 0 = 0; for a small positive radian angle, sine is positive. A correct starting displacement alone does not establish the correct equation.

(a) Write a displacement equation for head P, with x_P in metres and t in seconds. [2 marks]

(b) Write a displacement equation for head Q, with x_Q in metres and t in seconds. [2 marks]

(c) A technician proposes $x_Q = A \sin(\omega t)$. Explain why testing only t = 0 fails to reveal its error. [1 mark]

(d) Use the correct equation for Q to calculate its displacement at t = 0.0750 s. [1 mark]

(e) Find the first positive time when P reaches $x_P = -3.00$ mm. Identify the quarter-cycle interval containing that time. [2 marks]

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

4 Read one trace and draw another [7 marks]

Figure 2 is a recording for head 1 in a new test. Its motion is sinusoidal. In part (d), add a trace for head S, which has the same amplitude and period but crosses equilibrium moving in the positive direction at t = 0.

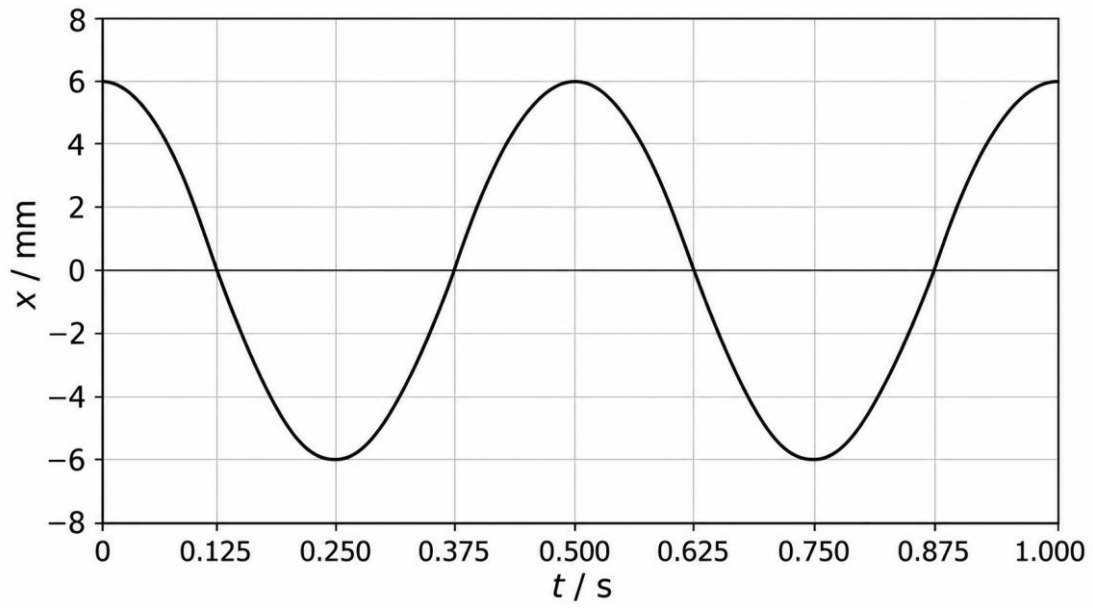


Figure 2 Displacement of head 1; use the linear time scale.

t / s	0	0.125	0.250	0.375	0.500
x_1 / mm					

(a) Read the amplitude and period of head 1 from the graph. [2 marks]

(b) Write an equation for head 1, with x_1 in millimetres and t in seconds. [1 mark]

(c) Complete the table with the first cycle's quarter-cycle displacements for head 1. [1 mark]

(d) Add head S to Figure 2 over the full 0–1.00 s interval. Use a dashed curve and label it S. [2 marks]

(e) Explain why this displacement–time graph is not a picture of the scanner's shape at one instant. [1 mark]

Continue your physics journey

Videos, practice and support for your next step

www.physicsuk.co.uk

5 Synchronise the two scanner heads [8 marks]

Use head 1 and head S from Figure 2. The controller records each positive maximum of S 0.125 s after the corresponding maximum of head 1. Their common period is 0.500 s. For calculations in this section use $x_1 = 6.00\cos(4\pi t)$ and $x_S = 6.00\sin(4\pi t)$, with displacements in mm.

Phase difference = 2π × time delay / period. State which signal leads or lags. An inverse-trig key returns one principal angle; check the other branches and restrict answers to the requested time interval.

(a) Calculate the phase difference in radians and in degrees. [2 marks]

(b) State which head leads and justify the direction of the lead. [1 mark]

(c) Find all times in $0 \le t < 0.500$ s when $x_S = +3.00$ mm. Show both phase solutions. [3 marks]

(d) A trigger is enabled only for $0.180 \le t \le 0.250$ s and requires $x_S = +3.00$ mm while S moves in the negative direction. Which event triggers it? Explain. [2 marks]

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

6 Interpret a squared sine in an energy model [5 marks]

In a separate free-oscillation calibration, the spring-mounted head starts at positive maximum displacement and oscillates without energy loss. Its displacement is $x = A\cos(\omega t)$, with $\omega = 4\pi$ rad s^{-1}. The total mechanical energy E is constant. Use the supplied relation $K/E = \sin^2(\omega t)$ for its kinetic-energy fraction. Here $\theta = \omega t$.

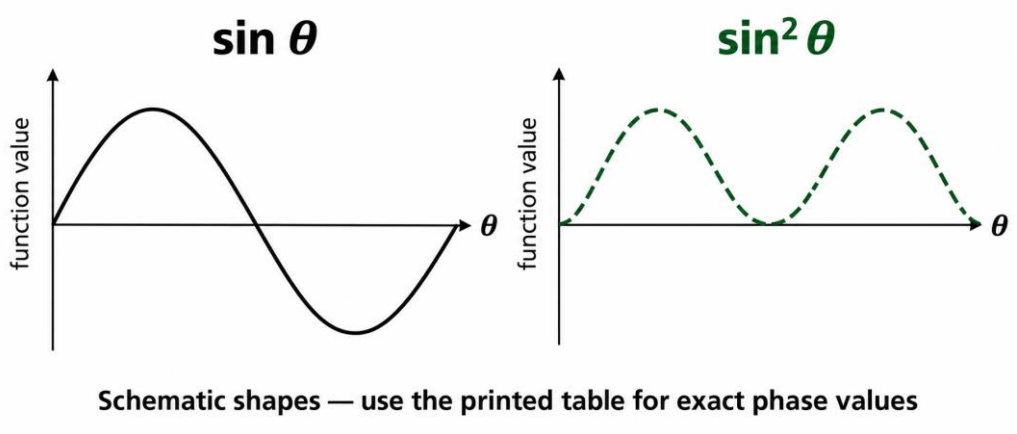


Figure 3 Qualitative function shapes. Use the table and equations for exact phase values.

θ / rad	0	π/2	π	3π/2	2π
sin θ	0	+1	0	−1	0
$\sin^2$ θ	0	1	0	1	0

(a) Calculate K/E when θ = 0.800 rad. [1 mark]

(b) Find the repeat time of K/E. Explain why it is half the displacement period. [2 marks]

(c) At θ = 2.00 rad, a pupil enters $\sin(\theta^2)$ and obtains −0.757. Give the correct bracketed calculation and fraction, then explain why the negative value is impossible for K/E. [2 marks]

Final check Check angle units, initial value and direction, phase branches, the time window and calculator brackets.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk