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WORKSHEETS
ATTEMPT | CHECK | CORRECT | IMPROVE

Proportionality ratios and scaling

35–40 minutes independent work + 15–20 minutes review | 43 marks

AQA A Level 7408 and OCR A A Level H556 | Mathematical skills

Skills practised

Use factors, powers and percentages; compare models by ratios; identify what must remain fixed.
AQA maths MS 0.1, 0.3, 0.5, 2.1–2.3, 4.3; selected applications 3.4.1.7–3.4.1.8; 3.4.2.1; 3.5.1.3–3.5.1.4; 3.7.2.2.
OCR A maths M0.1, M0.3, M0.5, M2.1–M2.3, M4.3; selected applications 3.2.4(a); 3.3.2(a); 3.3.3(a); 4.2.4(a)(i), (b); 4.2.5(a), (b); 5.4.1(b); 5.4.2(b).

A remote environmental station is being redesigned. Compare conductors, power losses, battery life and detector positions; then transfer the method to a planetary probe. Models are supplied. Use a calculator; give ratios without units and physical results with units. Keep unrounded factors until the final step.

1 Turn a change into a factor [6 marks]

First divide the new value by the original value. If $y = Cx^n$ and C is unchanged, $y_2/y_1 = (x_2/x_1)^n$. For several changing variables, multiply their factors with the correct powers.
Percentage change = (factor − 1) × 100%. A factor of 1.30 means 130% of the original, an increase of 30%.
Worked example: $E_k = \frac{1}{2}mv^2$. If speed stays fixed and mass increases by 20%, $E_{k2}/E_{k1} = 1.20$. The constant ½ cancels, and the speed factor is 1.

(a) For $y \propto x^2$, x becomes 1.50 times its original value. State the factor by which y changes. [1 mark]

(b) For $y \propto 1/x$, x becomes 0.80 times its original value. State the factor by which y changes. [1 mark]

(c) The station's inspection trolley is redesigned with 0.80 times its original mass and 1.50 times its original speed. Find the kinetic-energy factor and percentage increase. Use $E_k = \frac{1}{2}mv^2$. [2 marks]

(d) A sensor has the same usable battery energy after an upgrade, but its constant power draw increases by 25%. Find its operating-time factor and percentage decrease. Use $E = Pt$. [2 marks]

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2 Choose the conductor geometry [7 marks]

The station needs a longer conductor. A and B are uniform round metal conductors of the same material at the same temperature. Their total conducting lengths and diameters are shown in Figure 1. Ignore insulation and connector resistance. Conductor A has resistance 0.800 Ω.

Use $R = \rho_e L/A_c$ and $A_c = \pi d^2/4$. Here ρ_e is resistivity and A_c is cross-sectional area. Form ratios before inserting values; use consistent units in each ratio.

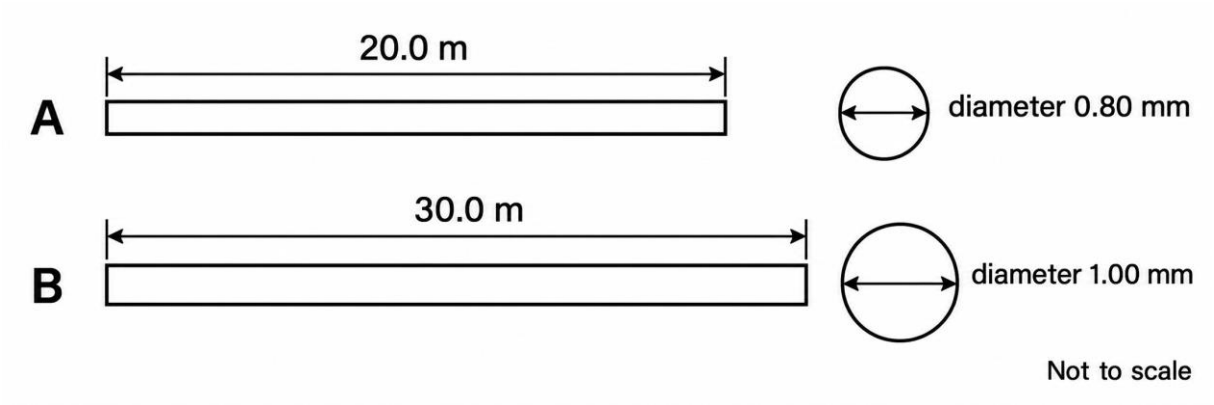


Figure 1 Candidate conductors and their circular cross-sections.

(a) Calculate A_{cB}/A_{cA}, the cross-sectional-area ratio. [1 mark]

(b) Obtain R_B/R_A using the length and diameter ratios. Show how the factors combine. [2 marks]

(c) Calculate the resistance of conductor B. [1 mark]

(d) A third conductor must be 1.50 times as long as A but have half its resistance. Find its required diameter as a factor of A's diameter, for the same material and temperature. [2 marks]

(e) Explain why both material and temperature were specified before cancelling ρ_e in part (b). [1 mark]

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3 Compare mass and electrical losses [7 marks]

Continue with conductors A and B from Question 2. Their density ρ_m is the same. Each conductor is tested separately on a laboratory supply; these are controlled comparison tests, not predictions of the station's operating current.

For a uniform conductor, $m = \rho_m A_c L$. At fixed density, mass scales as Ld^2. Electrical heating power can be written as $P = I^2R$ or $P = V^2/R$. Choose the form that contains the quantity held constant.

(a) Calculate the mass ratio m_B/m_A. [1 mark]

(b) Explain why B has a slightly lower resistance than A, yet a much greater mass. Use the way length and area enter the two models. [2 marks]

(c) Each conductor carries a controlled current of 0.600 A. Find the ratio of heating powers P_B/P_A. [1 mark]

(d) Calculate the heating power in B during this constant-current test. [1 mark]

(e) In a new test, the same potential difference is maintained directly across each conductor. Calculate the new P_B/P_A and explain why the direction of the change is different. [2 marks]

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4 Position an optical detector [7 marks]

An optical calibration source is used to check the station's detector. The supplied model is $s = CP_s/r^2$, where s is background-corrected detector signal, P_s is source power and r is distance from the emitting point. C stays constant for the same detector and orientation. Assume a point source, negligible absorption and no detector saturation.

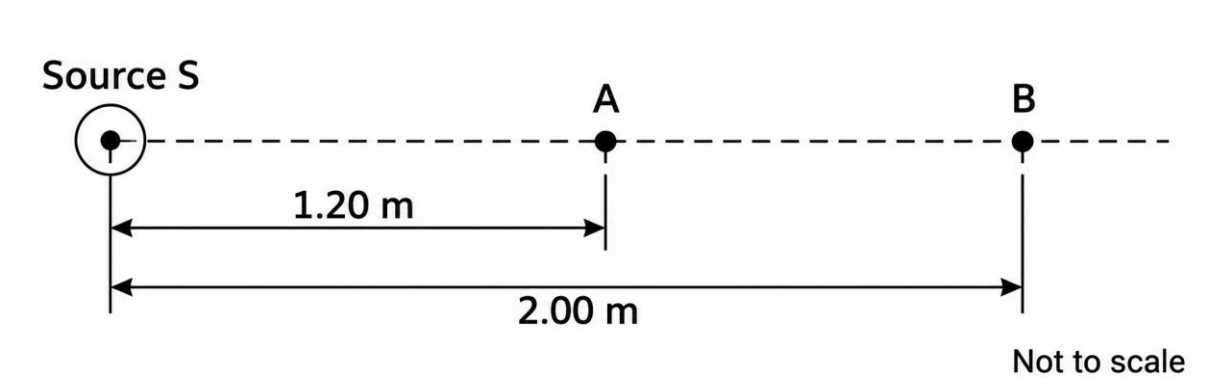


Figure 2 A and B are positions of the same detector, used one at a time.

(a) With source power fixed, calculate s_B/s_A for the two positions in Figure 2. [1 mark]

(b) Starting at A and keeping source power fixed, find how far the detector must move directly away from the source to halve its signal. [2 marks]

(c) The detector is now at B and source power is 1.50 times its original value. Compare this signal with the original signal at A by calculating their ratio. [2 marks]

(d) In a separate fixed-power test, distances of 1.00 m, 1.50 m and 2.00 m give background-corrected signals of 72, 32 and 18 arbitrary units. Test numerically whether these data fit the inverse-square model. [2 marks]

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5 Select a complete station design [10 marks]

The longer route is 1.50 times A's length. Candidate B is unchanged from Question 2. Candidate C has diameter 1.40 times A's diameter. Initially both use A's metal at A's temperature. Use fixed-current heating tests to compare cable losses. The table gives the battery energy and the measured total power draw for the proposed station; use these values directly for operating time.

Design limits: conductor mass ≤ $3.00m_A$; constant-current conductor loss ≤ $0.800P_A$; operating time ≥ 27.0 h. In the loss limit, P_A means the heating power of conductor A at the same test current. The upgraded station's measured power draw is separate from this conductor test.

Quantity	Original station	Upgraded station
Usable battery energy / W h	480	600
Constant total power draw / W	20.0	22.0
Operating time / h	24.0	To be determined

(a) Calculate R_C/R_A, showing how the simultaneous length and diameter changes combine. [2 marks]

(b) Calculate m_C/m_A and the percentage increase in conductor mass. [2 marks]

(c) Use an energy ratio and a power ratio to find the upgraded operating time from the original 24.0 h. Decide whether the time limit is met. [2 marks]

(d) For a separate C variant, a supplier uses a material with 0.80 times A's resistivity but the same C dimensions. Explain why the old R_C/R_A cannot simply be reused, and calculate the new ratio. [2 marks]

(e) Return to the original same-metal candidates B and C. Choose the conductor that meets both the mass and fixed-current loss limits. Justify your choice with both ratios. [2 marks]

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6 Transfer the ratio to a planetary probe [6 marks]

A probe studies a spherical planet of radius R = 3000 km. Its surface gravitational field strength has magnitude g_0 = 8.00 N kg^{-1}. Outside the planet use g = GM/r^2, where r is distance from its centre O. The planet's mass stays fixed. Figure 3 defines the probe's altitude h.

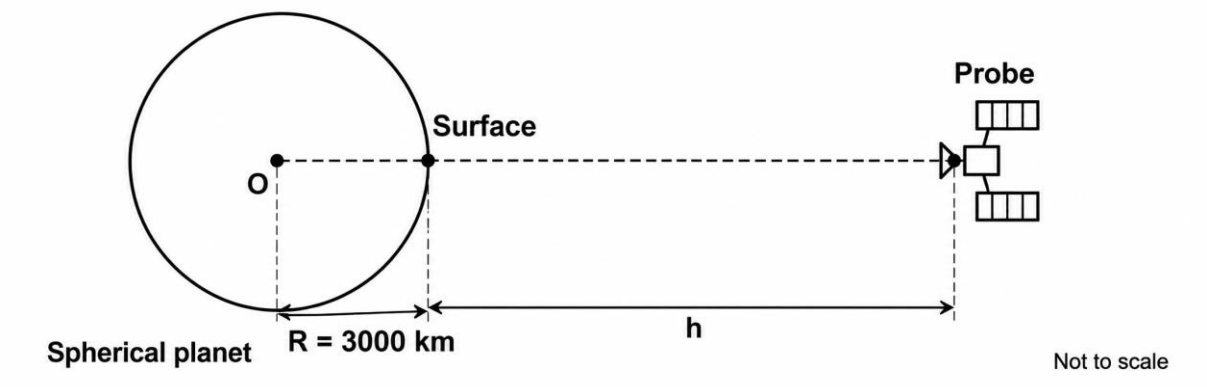


Figure 3 Altitude is measured from the surface, rather than from O.

(a) Calculate the gravitational field strength at an altitude of 1500 km, using a ratio to the surface value. [2 marks]

(b) Find the altitude at which the field strength is one ninth of its surface value. [2 marks]

(c) A pupil says "doubling altitude always quarters the field strength". Test the claim by comparing h = R with h = 2R, and explain the error. [2 marks]

Final check State what is fixed, form a new/original ratio, apply every power, then check the direction of the change.

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