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WORKSHEETS
ATTEMPT | CHECK | CORRECT | IMPROVE

Using ln and log

35–40 minutes independent work + 15–20 minutes review | 42 core marks + 3 optional

AQA A Level 7408 and OCR A A Level H556 | Mathematical skills

Skills practised

Use ln, log and their inverses; apply logarithm laws; rearrange exponential equations; plot and interpret transformed data.
AQA maths MS 0.5, 2.5, 3.3, 3.4, 3.10, 3.11; applications 3.7.4.4 and 3.8.1.3 (selected content).
OCR A maths M0.5, M2.5, M3.3, M3.4, M3.10, M3.11; applications 6.1.3(b), (c), (e); 6.4.3(e)(i), (f)(i).

The maths is taught step by step. Physics equations and quantities are supplied. Use a scientific calculator where requested; otherwise reason from powers. Complete Questions 1–9 before the optional charging extension.

1 A logarithm tells you the power [4 marks]

A logarithm answers: what power of the base gives this number?
$10^3 = 1000$ means $\log_{10}(1000) = 3$. On this sheet, log means $\log_{10}$.
The number e is approximately 2.718. Natural logarithms use base e: $e^x = a$ means $\ln(a) = x$.
Worked examples: $\log(0.01) = -2$ because $10^{-2} = 0.01$; $\ln(e^2) = 2$.

(a) Write $10^4 = 10000$ as an equivalent logarithm statement. [1 mark]

(b) Find log(0.001) without a calculator. Show the power of ten you use. [1 mark]

(c) Find $\ln(e^{-3})$ without a calculator. [1 mark]

(d) A detector display uses the numbers $\log(r/r_{ref})$, where r and r_{ref} are count rates. The ratio r/r_{ref} rises from 10 to 1000. By how much does the displayed number increase? [1 mark]

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2 See what ln does [3 marks]

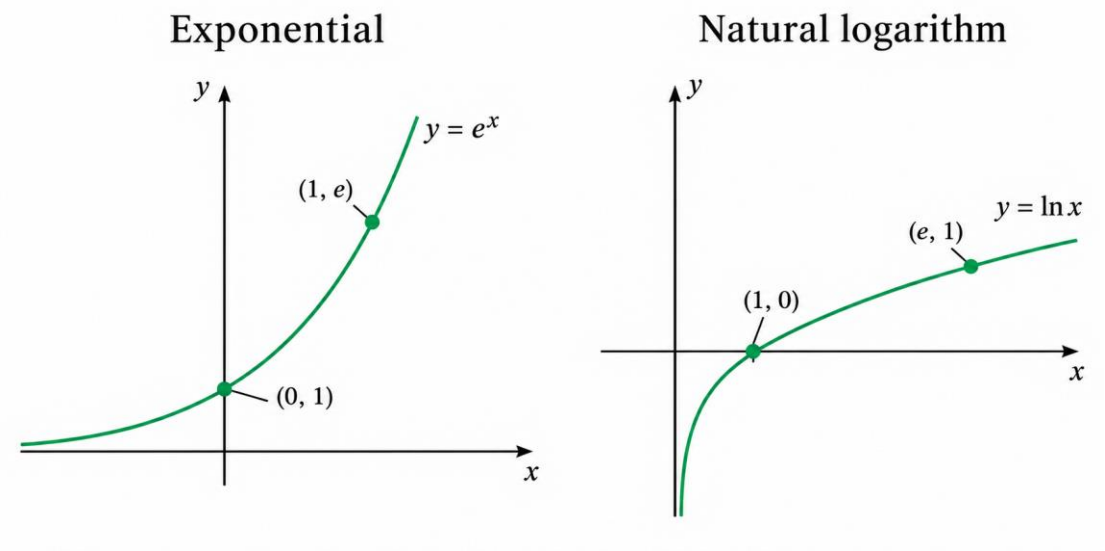

Figure 1 Schematic inverse functions. Use the labelled coordinates; do not measure the curves.

(a) Use the labelled points in Figure 1 to state ln(1) and ln(e). [1 mark]

(b) Which of these are valid real inputs to ln: −0.2, 0, 0.2 and 2? [1 mark]

(c) Explain why ln(0.2) is negative even though 0.2 is positive. [1 mark]

3 Use and undo the calculator functions [4 marks]

Undo ln with e^x; undo log with 10^x. Use the appropriate second function on your calculator. Keep unrounded values until the final answer.
Worked example: ln x = 0.70 gives $x = e^{0.70} = 2.01$. Using 1/ln(0.70) would not undo ln.

Table 1

Expression	Output	Expression	Output
ln(0.35)		log(0.35)	
$e^{1.40}$		$10^{-1.40}$	

(a) Complete Table 1. Give each numerical output to three significant figures. [2 marks]
(b) Solve ln x = −1.60. Give x to three significant figures. [1 mark]

(c) Solve log x = 1.70. Give x to three significant figures. [1 mark]

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4 Use the laws of logarithms [3 marks]

For positive a and b: $\ln(ab) = \ln a + \ln b$; $\ln(a/b) = \ln a - \ln b$; $\ln(a^n) = n \ln a$.
The same laws work for log when every logarithm has the same base. There is no corresponding rule for splitting a sum or difference.
Worked example: $\ln(3x) = \ln 3 + \ln x$ for $x > 0$.

(a) For $x > 0$, expand $\ln(5x^2/2)$ into separate logarithms. [2 marks]

(b) A pupil writes $\ln(8 + 2) = \ln 8 + \ln 2$. Explain the mistake. [1 mark]

5 Release an unknown exponent [5 marks]

Worked example: $20e^{-0.50x} = 5$. First divide by 20: $e^{-0.50x} = 0.25$. Then take ln: $-0.50x = \ln(0.25)$. Finally $x = -\ln(0.25)/0.50 = 2.77$.
Check: substituting $x = 2.77$ gives approximately 5. Do not take ln of a negative exponent by itself; take ln of the whole positive exponential.

(a) Solve $45e^{-0.30x} = 9$. Show the isolated exponential and the logarithm step. [2 marks]

(b) Solve $10^{0.40x} = 30$. Use log and show your working. [2 marks]

(c) For $e^{-0.20x} = 0.60$, a pupil obtains $x = -2.55$. Correct x and check the exponent's sign. [1 mark]

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6 Apply ln to a capacitor timing circuit [6 marks]

A hospital instrumentation team tests a capacitor timing circuit on a bench. The capacitor voltage falls as it discharges. All the physics you need is supplied below.

$V = V_0\,e^{-t/\tau}$, where V_0 is the initial voltage, V the later voltage, t the elapsed time and τ the time constant.
Also $\tau = RC$, where R is resistance and C capacitance.
For a remaining fraction $f = V/V_0$: $e^{-t/\tau} = f$, so $-t/\tau = \ln f$ and $t = -\tau \ln f$. The voltage ratio has no unit.
Worked example: for $\tau = 4.0$ s and $V/V_0 = 0.50$, $t = -4.0 \ln(0.50) = 2.77$ s.

(a) The test starts at $V_0 = 12.0$ V and $\tau = 10.0$ s. Find the time for V to fall to 3.00 V. Start with V/V_0, then $\ln(V/V_0)$, then t. [2 marks]

(b) For the same circuit, find the time for the voltage to fall to 10.0% of its initial value. [1 mark]

(c) A different circuit falls from 12.0 V to 4.80 V in 15.0 s. Determine its time constant τ. [2 marks]

(d) For a circuit with a constant τ, compare the time for V to fall from 12 V to 6 V with the time to fall from 6 V to 3 V. Explain using the voltage ratios. [1 mark]

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7 Transfer the method to radioactive decay [6 marks]

The same team monitors a radioactive reference sample at fixed detector geometry. A constant background adds to the detected count rate. The mean total count rates are 418 counts s^{-1} at t = 0 and 118 counts s^{-1} at t = 12.0 h. The background is 18 counts s^{-1}. Treat the background-corrected means as following the model below.

$r = r_0 e^{-\lambda t}$, where r is the net count rate, r_0 its initial net value and λ the decay constant. Use net count rate = total count rate − background count rate.

The net count rate is proportional to activity in this fixed setup; it is not the activity itself. Keep λ and t in matching time units.

Use the same steps: divide to get r/r_0, take ln, then isolate the unknown. At the half-life T, $r/r_0 = 0.5$.

(a) Calculate the net count rate at each time. [1 mark]

(b) Use your net count rates to determine λ in h^{-1}. Show the logarithm step. [2 marks]

(c) Use $0.5 = e^{-\lambda T}$ to find the half-life T. Keep your unrounded λ from part (b). [1 mark]

(d) How long after the initial reading will the net count rate be 20.0% of its initial value? [2 marks]

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8 Turn exponential data into a straight line [5 marks]

Figure 2 compares a remaining fraction f with its natural logarithm. These are schematic shapes. The right-hand plot uses ordinary linear axes; negative ln values lie below zero.

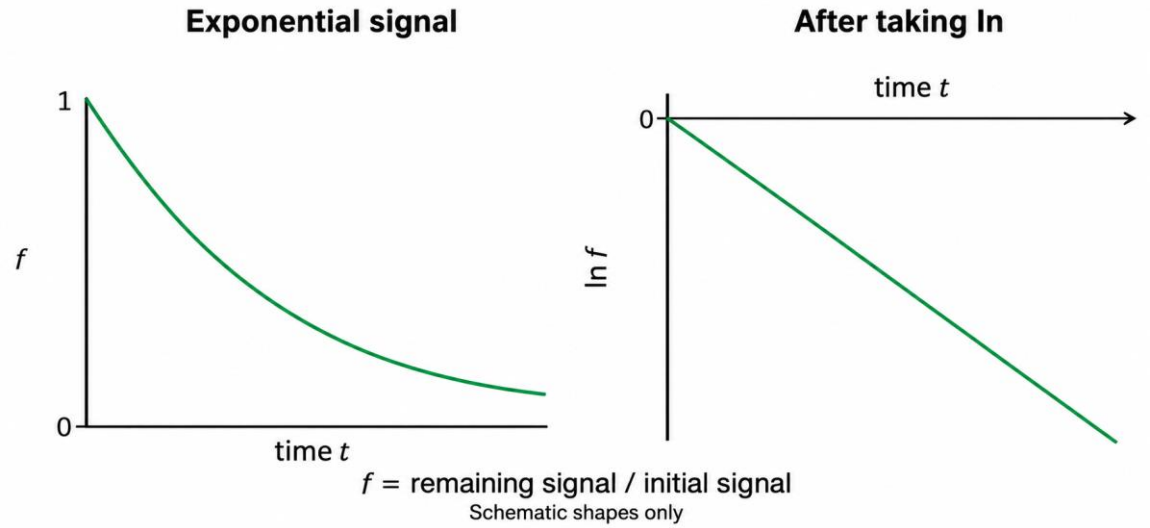

Figure 2 The logarithm of an exponentially decreasing fraction decreases linearly.

Taking ln undoes e to a power. For $f = e^{-kt}$, $\ln f = -kt$. This has the straight-line form $y = mx + c$, with $y = \ln f$, $x = t$, $m = -k$ and $c = 0$.

A separate capacitor test gives Table 2. Its model is $V = V_0\, e^{-t/\tau}$, with $V_0 = 9.00$ V. Its τ is unknown. Use this table, not Figure 2, for numerical work.

Scale check: a logarithmic axis can place the labels 0.001, 0.01, 0.1, 1 at equal distances. Their calculated log values −3, −2, −1, 0 go on an ordinary linear axis.

Table 2

t / s	V / V	$\ln(V/V_0)$
0	9.00	0.000
10	5.46	
20	3.31	
30	2.01	
40	1.22	

(a) Complete the four missing $\ln(V/V_0)$ values in Table 2 to three decimal places. [2 marks]

(b) Starting from the capacitor model, obtain an equation for $\ln(V/V_0)$ against t. Identify its gradient in terms of τ. [2 marks]

(c) Describe how V and $\ln(V/V_0)$ each change as time increases. [1 mark]

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9 Plot and interpret the transformed data [6 marks]

Use your completed Table 2 for Figure 3. The printed axes are ordinary linear axes: each horizontal grid division is 5 s and each downward vertical grid division is a decrease of 0.25 in $\ln(V/V_0)$.

(a) Plot the five points from Table 2 on Figure 3 and draw a straight line of best fit. [2 marks]

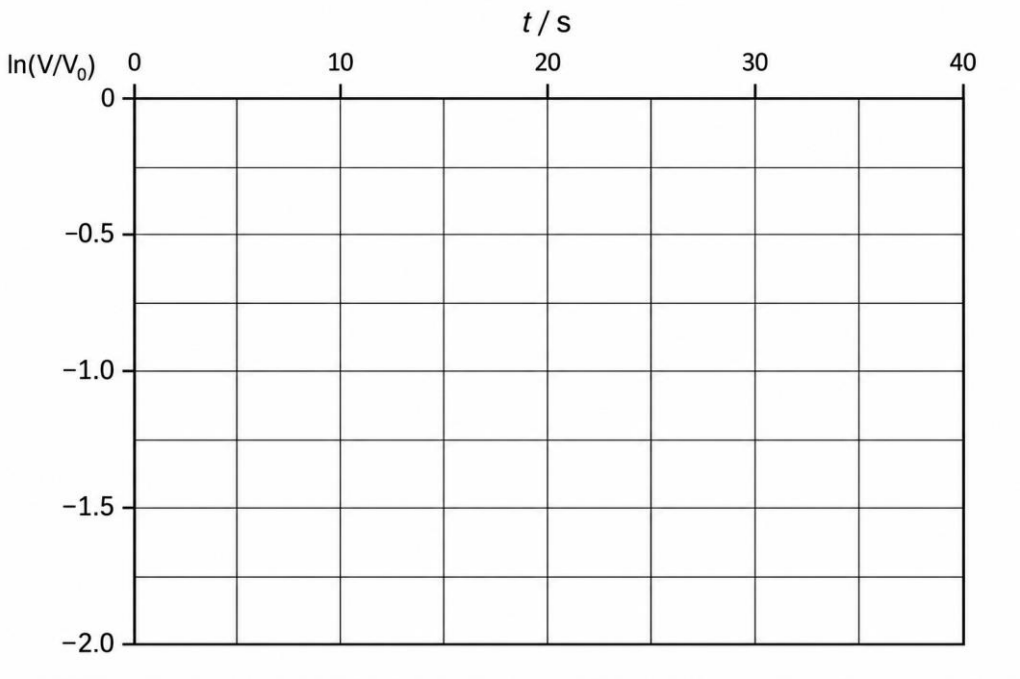

Figure 3 Plot the values from Table 2.

(b) Use a large gradient triangle to determine the gradient, including its unit. Hence determine τ. [2 marks]

(c) Another pupil plots $\log(V/V_0)$ instead of $\ln(V/V_0)$ and obtains gradient $g = -0.0217\ s^{-1}$. They calculate $\tau = -1/g = 46.1$ s. Explain the error and find the correct τ. Use $\ln x \approx 2.303 \log x$. [2 marks]

10 Optional extension on charging [3 marks]

Allow about 5 extra minutes. During charging the supplied model is $V = V_{max}(1 - e^{-t/\tau})$, where V_{max} is the final steady voltage.

(a) Rearrange the charging model to make t the subject. Isolate the exponential before taking ln. [2 marks]

(b) For $\tau = 8.00$ s, calculate the time to reach 75.0% of the final voltage. [1 mark]

Review check Identify the base, undo its logarithm, isolate the exponential and check the sign.

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