Gradients tangents and areas

35–40 minutes independent work + 15–20 minutes review | 39 marks

AQA A Level 7408 and OCR A A Level H556 | Mathematical skills

Skills practised

Interpret axes and units; use secants and tangents; calculate signed areas and estimate curved areas.
AQA maths MS 0.1, 2.3, 3.1, 3.5–3.8, 4.3; selected applications 3.4.1.3, 3.4.1.6–3.4.1.7; 3.7.4.1, 3.7.4.3.
OCR A maths M0.1, M2.3, M3.1, M3.5–M3.8, M4.3; selected applications 3.1.1(a)–(d); 3.5.1(b)–(e); 6.1.1(a); 6.1.2(a)–(c).

A battery-powered inspection shuttle is tested on a straight track. Study its motion, a separate braking pulse and a capacitor buffer for its sensor electronics. Each test has its own time origin. Use a ruler and calculator; show graphical working and give units.

1 Read the axes first [3 marks]

A gradient is change in the vertical quantity divided by change in the horizontal quantity. Its unit is vertical unit ÷ horizontal unit. Example: a temperature–time gradient has unit K s^{-1}; it is a temperature-change rate, not an acceleration.
An area has unit vertical unit × horizontal unit. Use a secant between two curve points for an average rate, and a tangent at one point for an instantaneous rate. Check what the units mean before calculating.

(a) State the physical quantity and SI unit obtained from the gradient of displacement x against time t. [1 mark]

(b) Give the unit of an area on a force–time graph, and write it in base SI units. [1 mark]

(c) A technician wants the shuttle's acceleration at one instant from a curved velocity–time graph. State the graphical method and the unit of the result. [1 mark]

2 Compare average and instantaneous velocity [5 marks]

Trial A records the shuttle's displacement x from its starting point. Positive x is along the track. Use Figure 1; the curve represents the measured motion.

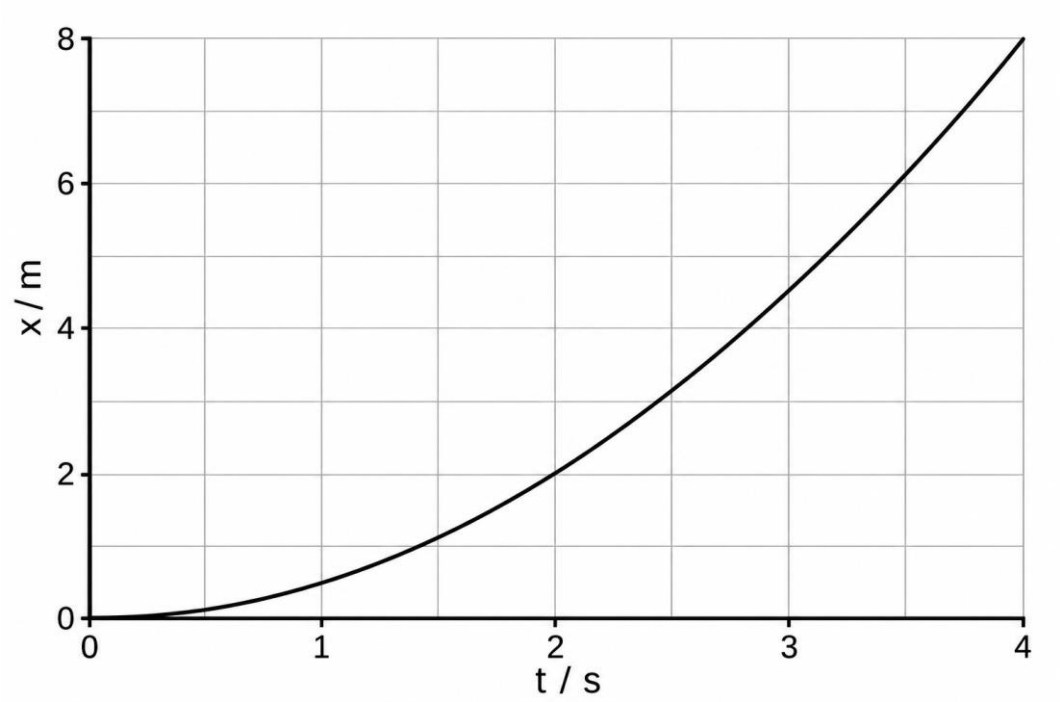


Figure 1 Displacement during the first acceleration trial.

(a) Calculate the average velocity from t = 0 to t = 4.0 s. [1 mark]

(b) Draw a tangent to the curve at t = 3.0 s. [1 mark]

(c) Use a large gradient triangle on your tangent to find the instantaneous velocity at t = 3.0 s. [2 marks]

(d) Explain why the instantaneous velocity in part (c) is greater than the average in part (a). [1 mark]

Continue your physics journey

Videos, practice and support for your next step

www.physicsuk.co.uk

3 Keep the sign when finding an area [11 marks]

Trial B is a separate manoeuvre. Figure 2 shows signed velocity v. Positive velocity is forward along the track; negative velocity is backward.

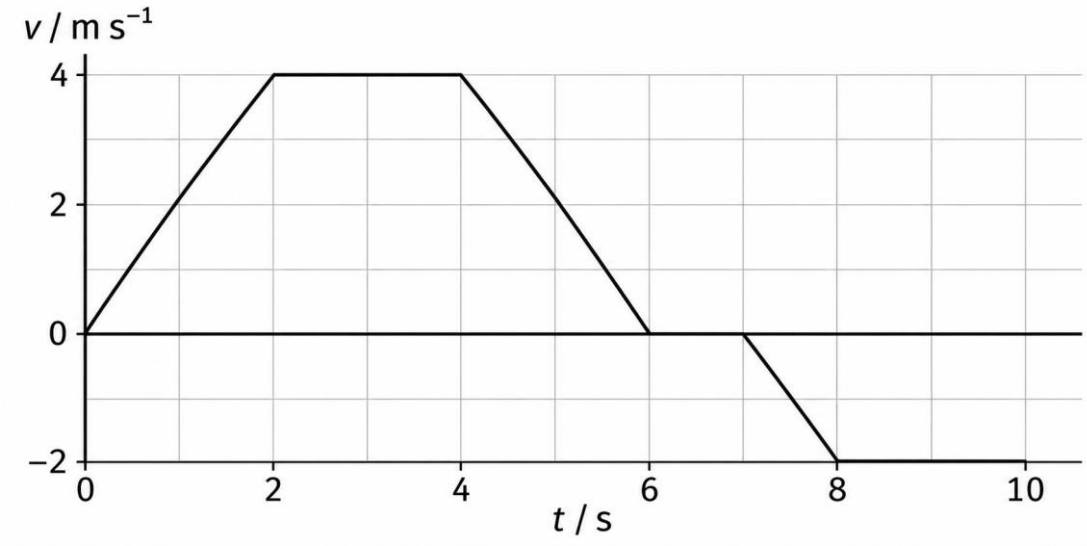


Figure 2 Forward travel, a pause and reverse travel.

(a) Use areas to find the displacement from t = 0 to t = 6.0 s. [2 marks]

(b) Find the signed displacement from t = 6.0 s to t = 10.0 s. [2 marks]

(c) Hence find both the total displacement and total distance travelled from t = 0 to t = 10.0 s. [2 marks]

(d) Describe the motion from t = 6.0 s to t = 10.0 s. Include the pause, direction and how speed changes in each moving section. [3 marks]

(e) Calculate the acceleration from t = 7.0 s to t = 8.0 s. Explain why its negative sign does not mean that the shuttle is slowing down. [2 marks]

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

4 Estimate the effect of a braking pulse [8 marks]

In trial C, the 10.0 kg shuttle starts at +6.00 m s^{-1}. Figure 3 shows the resultant horizontal force during a 1.00 s braking pulse. Forward is positive. The table gives the exact force readings to use in your estimate.

Signed trapezium contribution = ½(F_1 + F_2)Δt. Use four intervals of width 0.25 s. Retain the force signs.

t / s	0	0.25	0.50	0.75	1.00
F / N	−120	−67.5	−30.0	−7.5	0

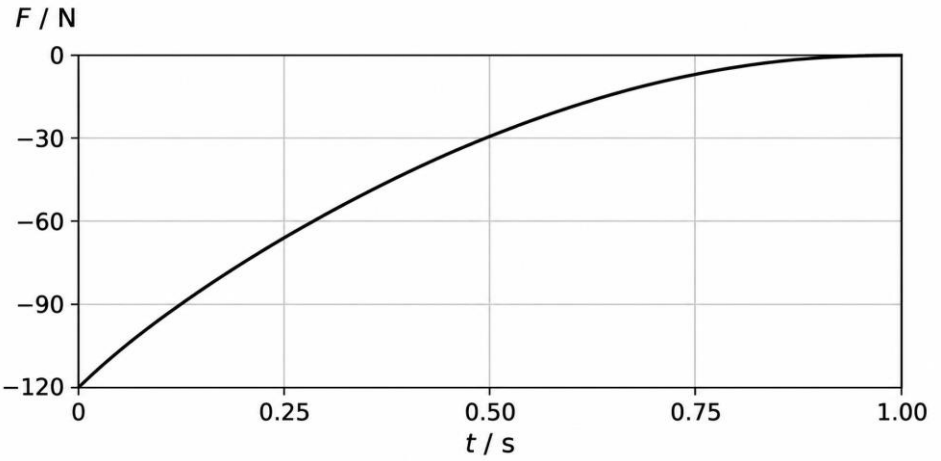


Figure 3 Resultant braking force; use the table values for numerical areas.

(a) State what the signed area between this force curve and F = 0 represents, with its unit. [1 mark]

(b) Use four trapezia and the table to estimate the signed impulse over 0–1.00 s. [2 marks]

(c) Calculate the initial momentum, then use your impulse to estimate the final momentum. [2 marks]

(d) Estimate the final velocity, including its direction. [1 mark]

(e) A pupil uses the initial force for the whole second. Calculate this rectangular impulse estimate and explain why it overestimates the magnitude of the braking effect. [2 marks]

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

5 Transfer area reasoning to electrical energy [8 marks]

A capacitor buffer supplies brief pulses to the shuttle's sensor electronics. It is separate from the main traction-energy store. Figure 4 shows voltage V against charge Q for an ideal capacitor of constant capacitance C. Stored energy gained equals the area under V against Q. Use C = Q/V and 1 mC = 10^{-3} C.

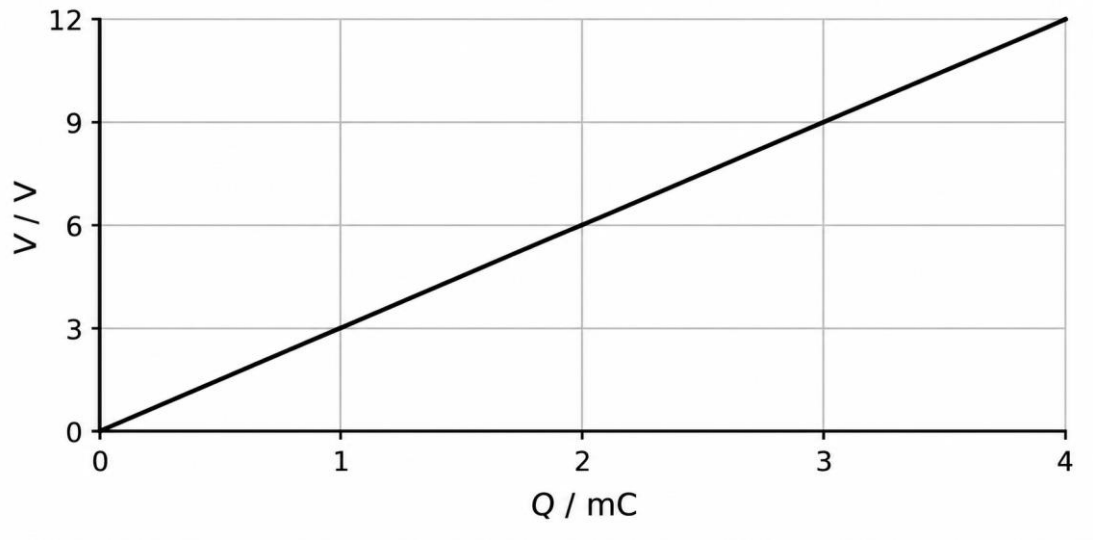


Figure 4 Voltage against charge for the sensor capacitor.

(a) Find the energy stored when the initially uncharged capacitor reaches Q = 4.0 mC. Give your answer in joules. [2 marks]

(b) Compare the energy gained over 0–2.0 mC with that gained over 2.0–4.0 mC. Calculate both values and state their ratio, later interval to earlier interval. [2 marks]

(c) A second capacitor has a V-against-Q line through the origin with twice this gradient. Find its capacitance as a factor of the first capacitor's, and its stored energy factor at the same final charge. [2 marks]

(d) Swap the axes to Q against V. State the gradient's meaning. For this linear capacitor, show whether the zero-to-final triangular area in SI units still equals stored energy. [2 marks]

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk

6 Explain and check a graph result [4 marks]

(a) A technician means that positive velocity increases at a constant rate, but writes “the velocity is accelerating”. Rewrite this precisely, including what happens to acceleration. [1 mark]

(b) For the smooth braking curve, explain why using more, narrower trapezia would usually improve the impulse estimate. Explain why the result would still be an estimate. [2 marks]

(c) A pupil calls the area under a force-against-velocity graph “energy” because it has the same triangular shape as Figure 4. Use the axis units to test this claim. [1 mark]

Final check Read the axes and units, choose tangent or area, keep signs, then check the physical meaning.

Continue your physics journey
Videos, practice and support for your next step
www.physicsuk.co.uk